Chapter 1 Function Transformations

Section 1.1 Horizontal and Vertical Translations

Section 1.1 Page 12 Question 1

a) For $y - 5 = f(x)$, $h = 0$ and $k = 5$.

b) For $y = f(x) - 4$, $h = 0$ and $k = -4$.

c) For $y = f(x + 1)$, $h = -1$ and $k = 0$.

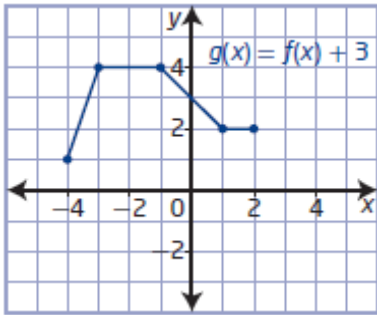
d) For $y + 3 = f(x - 7)$, $h = 7$ and $k = -3$.

e) For $y = f(x + 2) + 4$, $h = -2$ and $k = 4$.

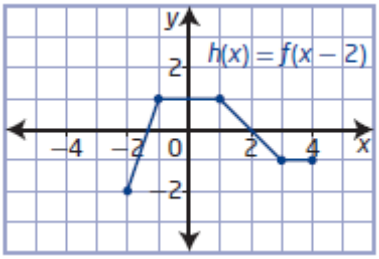
Section 1.1 Page 12 Question 2

A(–4, –2), B(–3, 1), C(–1, 1), D(1, –1), E(2, –1)

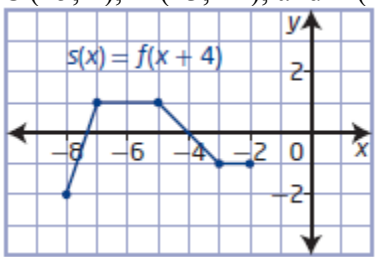
a) For $g(x) = f(x) + 3$, A′(–4, 1), B′(–3, 4), C′(–1, 4), D′(1, 2), and E′(2, 2).



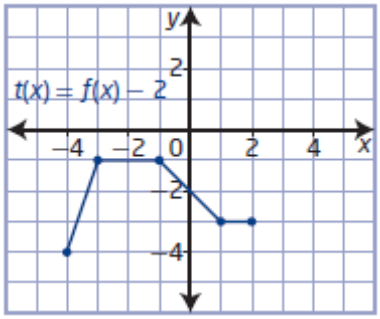
b) For $h(x) = f(x - 2)$, A′(–2, –2), B′(–1, 1), C′(1, 1), D′(3, –1), and E′(4, –1).



c) For $s(x) = f(x + 4)$, A′(–8, –2), B′(–7, 1), C′(–5, 1), D′(–3, –1), and E′(–2, –1).



d) For $t(x) = f(x) - 2$, A′(–4, –4), B′(–3, –1), C′(–1, –1), D′(1, –3), and E′(2, –3).



Section 1.1 Page 13 Question 3

For horizontal translation and vertical translation, every point (x, y) on the graph of $y = f(x)$ is transformed to $(x + h, y + k)$.

a) For $y = f(x + 10)$, $h = -10$ and $k = 0$: $(x, y) \rightarrow (x - 10, y)$.

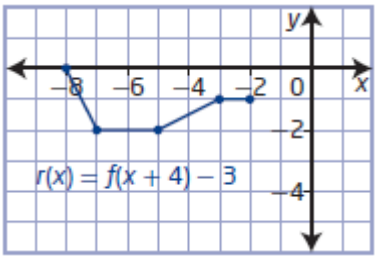
b) For $y + 6 = f(x)$, $h = 0$ and $k = -6$: $(x, y) \rightarrow (x, y - 6)$.

c) For $y = f(x - 7) + 4$, $h = 7$ and $k = 4$: $(x, y) \rightarrow (x + 7, y + 4)$.

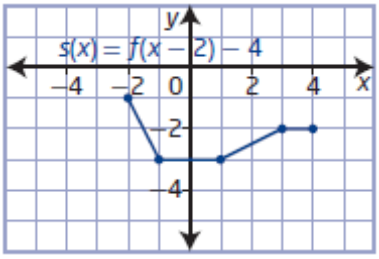
d) For $y - 3 = f(x - 1)$, $h = 1$ and $k = 3$: $(x, y) \rightarrow (x + 1, y + 3)$.

Section 1.1 Page 13 Question 4

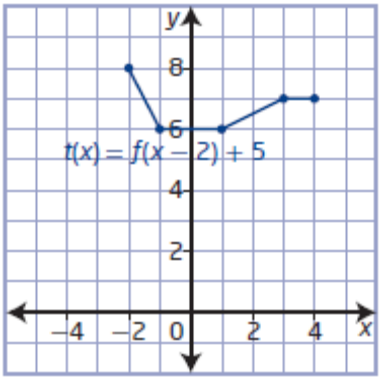
a) For $r(x) = f(x + 4) - 3$, $h = -4$ and $k = -3$. The graph of $r(x) = f(x + 4) - 3$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 3 units down and a horizontal translation of 4 units to the left.
$(x, y) \rightarrow (x - 4, y - 3)$



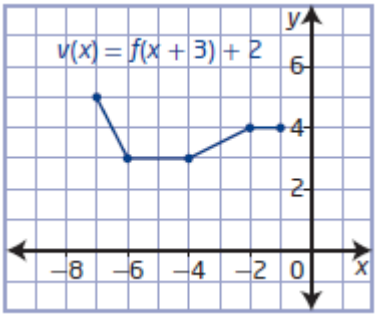
b) For $s(x) = f(x - 2) - 4$, $h = 2$ and $k = -4$. The graph of $s(x) = f(x - 2) - 4$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 4 units down and a horizontal translation of 2 units to the right.
$(x, y) \rightarrow (x + 2, y - 4)$



c) For $t(x) = f(x - 2) + 5$, $h = 2$ and $k = 5$. The graph of $t(x) = f(x - 2) + 5$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 5 units up and a horizontal translation of 2 units to the right.
$(x, y) \rightarrow (x + 2, y + 5)$



d) For $v(x) = f(x + 3) + 2$, $h = -3$ and $k = 2$. The graph of $v(x) = f(x + 3) + 2$ can be obtained from the graph of $y = f(x)$ by a vertical translation of 2 units up and a horizontal translation of 3 units to the left.
$(x, y) \rightarrow (x - 3, y + 2)$



Section 1.1 Page 13 Question 5

a) Translated 5 units to the left and 4 units up represents $h = -5$ and $k = 4$. The equation of the transformed function is $y - 4 = f(x + 5)$.

b) Translated 8 units to the right and 6 units up represents $h = 8$ and $k = 6$. The equation of the transformed function is $y - 6 = f(x - 8)$.

c) Translated 10 units to the right and 8 units down represents $h = 10$ and $k = -8$. The equation of the transformed function is $y + 8 = f(x - 10)$.

d) Translated 7 units to the left and 12 units down represents $h = -7$ and $k = -12$. The equation of the transformed function is $y + 12 = f(x + 7)$.

Section 1.1 Page 13 Question 6

The graph of $y = x^2$ passes through (4, 16). If the transformed graph passes through (4, 19), then a vertical translation of 3 units up has been applied.

Section 1.1 Page 13 Question 7

The graph of $y = x^2$ passes through (4, 16). If the transformed graph passes through (5, 16), then a horizontal translation of 1 unit to the right has been applied.

Section 1.1 Page 13 Question 8

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
horizontal	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
horizontal	$y = f(x - 3)$	$\mathbf{(x, y) \rightarrow (x + 3, y)}$
vertical	$y = f(x) - 6$	$\mathbf{(x, y) \rightarrow (x, y - 6)}$
horizontal and vertical	$y + 9 = f(x + 4)$	$\mathbf{(x, y) \rightarrow (x - 4, y - 9)}$
horizontal and vertical	$\mathbf{y + 6 = f(x - 4)}$	$(x, y) \rightarrow (x + 4, y - 6)$
horizontal and vertical	$\mathbf{y - 3 = f(x + 2)}$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$\mathbf{(x, y) \rightarrow (x + h, y + k)}$

Section 1.1 Page 13 Question 9

a) Translated 4 units to the left and 5 units up represents $h = -4$ and $k = 5$. The equation of the transformed function is $y = (x + 4)^2 + 5$.

b) For $y = (x + 4)^2 + 5$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 5, y \in \mathrm{R}\}$.

c) To determine the image function's domain and range, add the horizontal and vertical translations to the domain and range, respectively, of the base function. Since the domain is the set of real numbers, nothing changes, but the range does change.

Section 1.1 Page 13 Question 10

Given $f(x) = |x|$ and $g(x) = f(x - 9) + 5$.
a) The equation of the transformed function is $g(x) = |x - 9| + 5$.

b) The graph of $g(x)$ is a vertical and horizontal translation of the graph of $f(x)$ by 5 units up and 9 units to the right.

c) Example:

Graph of $f(x)$	**Graph of $g(x)$**	
	Horizontal translation	Vertical translation
(–2, 2)	(7, 2)	(7, 7)
(0, 0)	(9, 0)	(9, 5)
(2, 2)	(11, 2)	(11, 7)

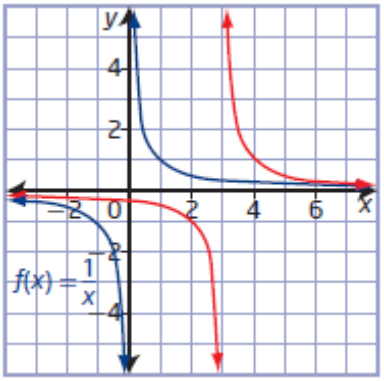
d) Example:

Graph of $f(x)$	**Graph of $g(x)$**	
	Vertical translation	Horizontal translation
(–2, 2)	(–2, 7)	(7, 7)
(0, 0)	(0, 5)	(9, 5)
(2, 2)	(2, 7)	(11, 7)

e) The coordinates of the image points from parts c) and d) are the same. The order in which the translations occur does not matter.

Section 1.1 Page 14 Question 11

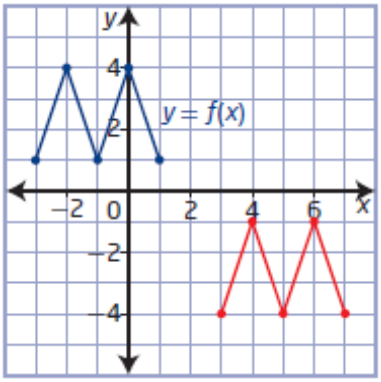
a) Choose key points on the graph of $f(x) = \frac{1}{x}$ and locate the corresponding image points on the graph of the translated function in red ($g(x)$).



$f(x)$	$g(x)$
(–2, –0.5)	→ (1, −0.5)
(−1, –1)	→ (2, −1)
(1, 1)	→ (4, 1)
(2, 0.5)	→ (5, 0.5)
(x, y)	→ $(x + 3, y)$

The equation of the translated function is $y = f(x – 3)$.

b) Choose key points on the graph of $y = f(x)$ and locate the corresponding image points on the graph of the translated function in red ($g(x)$).

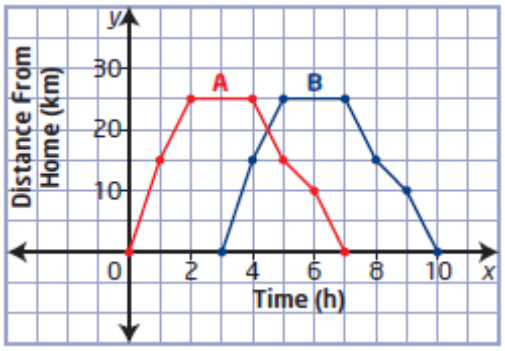


$f(x)$	$g(x)$
(–3, 1)	→ (3, −4)
(–2, 4)	→ (4, −1)
(−1, 1)	→ (5, −4)
(0, 4)	→ (6, –1)
(1, 1)	→ (7, –4)
(x, y)	→ $(x + 6, y – 5)$

The equation of the translated function is $y + 5 = f(x – 6)$.

Section 1.1 Page 14 Question 12

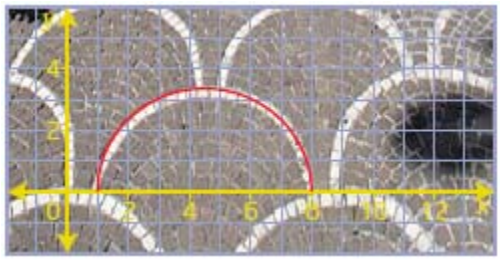
a) Example: Graph A shows that it took Janine 2 h to cycle to the lake, 25 km away. She rests at the lake for 2 h and then returns home in 3 h. If she left her home at 12 non, she returned at 7 p.m.



b) Example: Graph B shows the same trip as Graph A, but Janine does not leave her home until 3 p.m. and returns at 10 p.m.

c) The equation of graph B is $y = f(x – 3)$.

Section 1.1 Page 14 Question 13

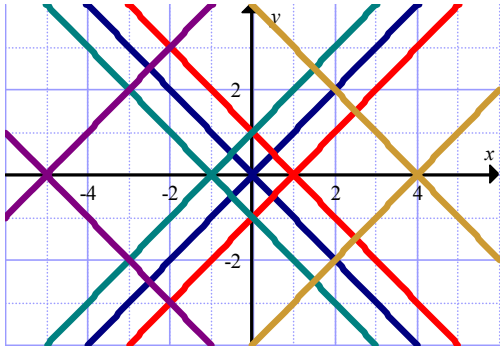


a) Example: The semicircle directly to the right is a translation of 8 units to the right of the base semicircle.

b) Example: The equation of the semicircle directly to the right is $y = f(x - 8)$. The equation of the semicircle to the right and up is $y = f(x - 4) + 3.5$. The equation of the semicircle to the left and up is $y = f(x + 4) + 3.5$.

Section 1.1 Page 14 Question 14

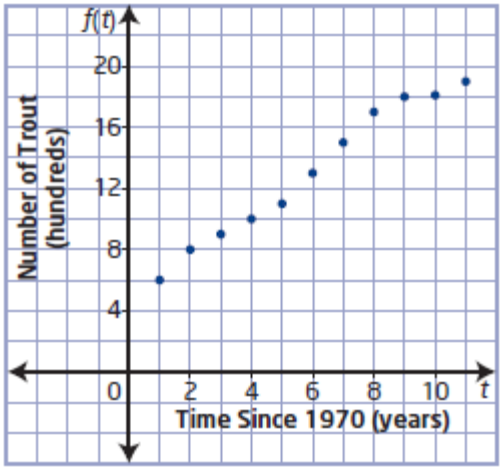
a) Example: A repeating X by using two linear functions $y = x$ and $y = -x$.



b) The original design is shown in blue.
Red: $y = f(x - 1)$
Green: $y = f(x + 1)$
Purple: $y = f(x + 5)$
Gold: $y = f(x - 4)$

Section 1.1 Page 15 Question 15

a) A vertical translation of 2 units up of the graph would result from having 200 more trout in 1970. $y = f(t) + 2$



b) A horizontal translation of 3 units to the right of the graph would result if recording the number of trout started in 1973. $y = f(t - 3)$

Section 1.1 Page 15 Question 16

If $n = f(A)$ gives the number of gallons, n, of paint needed to cover an area, A, in square metres, then $n = f(A) + 10$ represents the number of gallons Paul needs for a given area plus 10 more gallons. In this same context, $n = f(A + 10)$ represents how many gallons he needs to cover an area A less 10 units of area.

Section 1.1 Page 15 Question 17

a) The translated parabola will have an equation of the form $y = (x - h)^2 + k$. For an image parabola with zeros 7 and 1, the equation of the axis of symmetry is $x = 4$.
So, $h = 4$ and $y = (x - 4)^2 + k$. Use the coordinates of a given point to solve for k.
$y = (x - 4)^2 + k$
$0 = (1 - 4)^2 + k$
$0 = 9 + k$
$k = -9$
The equation of the image function is $y = (x - 4)^2 - 9$.

b) The graph of $y = x^2$ has been translated 4 units to the right and 9 units down.

c) Substitute $x = 0$.
$y = (x - 4)^2 - 9$
$y = (0 - 4)^2 - 9$
$y = 16 - 9$
$y = 7$
The y-intercept of the translated function is 7.

Section 1.1 Page 15 Question 18

a) The graph of $y - 4 = \frac{1}{x}$ is a vertical translation of 4 units up of the graph of $y = \frac{1}{x}$.

b) The graph of $y = \frac{1}{x+2}$ is a horizontal translation of 2 units to the left of the graph of $y = \frac{1}{x}$.

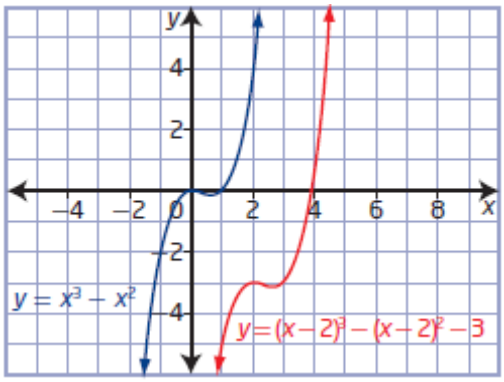
c) The graph of $y - 3 = \frac{1}{x-5}$ is a vertical translation of 3 units up and horizontal translation of 5 units to the right of the graph of $y = \frac{1}{x}$.

d) The graph of $y = \frac{1}{x+3} - 4$ is a vertical translation of 4 units down and horizontal translation of 3 units to the left of the graph of $y = \frac{1}{x}$.

Section 1.1 Page 15 Question 19

a) Prediction: The graph of $y + 3 = (x - 2)^3 - (x - 2)^2$ will be a vertical translation of 3 units down and horizontal translation of 2 units to the right of the graph of $y = x^3 - x^2$.

b) The prediction was correct.



Section 1.1 Page 15 Question C1

a) If the horizontal translation is performed before the vertical translation, the function $y = f(x)$ becomes $y = f(x - h)$ and then $y = f(x - h) + k$.
If the vertical translation is performed before the horizontal translation, the function $y = f(x)$ becomes $y = f(x) + k$ and then $y = f(x - h) + k$.
The final function equations are the same. So, translations can be applied in either order.

b) Potentially, the domain is shifted by h and the range is shifted by k.

Section 1.1 Page 15 Question C2

a) Completing the square reveals the location of the vertex and the translations of the graph of $y = x^2$.
$f(x) = x^2 + 2x + 1$
$f(x) = (x^2 + 2x + 1 - 1) + 1$
$f(x) = (x + 1)^2$
The graph of $y = x^2$ is translated 1 unit to the left to obtain the graph of $f(x) = x^2 + 2x + 1$.

b) Complete the square.
$g(x) = x^2 - 4x + 3$
$g(x) = (x^2 - 4x + 4 - 4) + 3$
$g(x) = (x - 2)^2 - 1$
The graph of $y = x^2$ is translated 2 units to the right and 1 unit down to obtain the graph of $g(x) = x^2 - 4x + 3$.

Section 1.1 Page 15 Question C3

Comparing the graphs of the corresponding functions for the equations $x^2 - x - 12 = 0$ and $(x - 5)^2 - (x - 5) - 12 = 0$ shows that the second is a horizontal translation of the first by 5 units to the right. Since the roots of the first equation are –3 and 4, the roots of the second equation will be –3 + 5 and 4 + 5, or 2 and 9.

Section 1.1 Page 15 Question C4

If the base function $f(x) = x$ is translated vertically, the function becomes $y = x + k$. So for 4 units up, the equation is $y = x + 4$.
If the base function $f(x) = x$ is translated horizontally, the function becomes $y = (x - h)$.
So for 4 units to the left, the equation is $y = (x + 4)$ or $y = x + 4$.

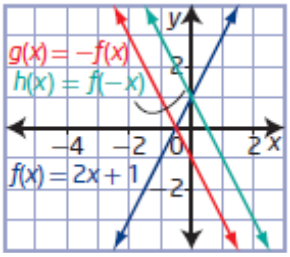
Section 1.2 Reflections and Stretches

Section 1.2 Page 28 Question 1

a)

x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
–4	–7	7	9
–2	–3	3	5
0	1	–1	1
2	5	–5	–3
4	9	–9	–7

b)



c) The y-coordinates of the points on the graph of $g(x)$ have changed sign compared to the corresponding points on the graph of $f(x)$. The invariant point is (–0.5, 0).
The x-coordinates of the points on the graph of $h(x)$ have changed sign compared to the corresponding points on the graph of $f(x)$. The invariant point is (0, 1).

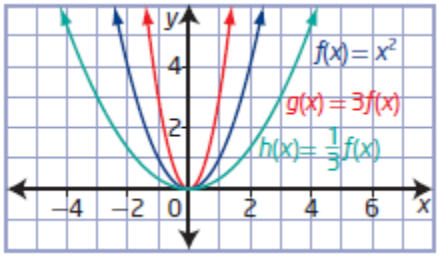
d) The graph of $g(x)$ is a reflection in the x-axis of the graph of $f(x)$. The graph of $h(x)$ is a reflection in the y-axis of the graph of $f(x)$.

Section 1.2 Page 28 Question 2

a)

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

b)



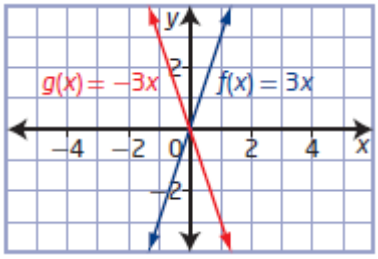
c) The y-coordinates of the points on the graph of $g(x)$ are three times far from the x-axis as the corresponding points on the graph of $f(x)$. The invariant point is (0, 0). The y-coordinates on the graph of $h(x)$ are one-third as far from the x-axis as the corresponding points on the graph of $f(x)$. The invariant point is (0, 0).

d) The graph of $g(x)$ is a vertical stretch by a factor of 3 of the graph of $f(x)$, while the graph of $h(x)$ is a vertical stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$.

Section 1.2 Page 28 Question 3

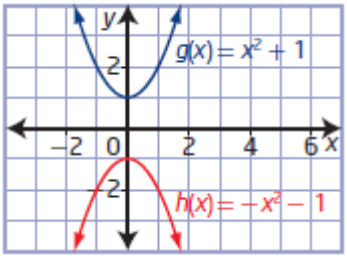
a) A reflection in the x-axis of $f(x) = 3x$ results in the function $g(x) = -3x$.

$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

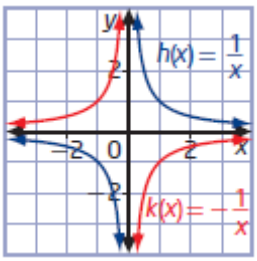


b) A reflection in the x-axis of $g(x) = x^2 + 1$ results in the function $h(x) = -x^2 - 1$.

$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq -1, y \in \mathrm{R}\}$



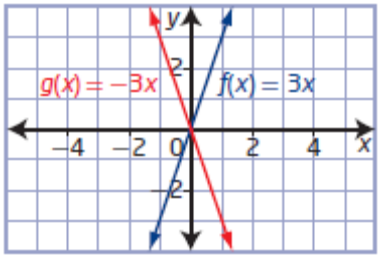
c) A reflection in the x-axis of $h(x) = \frac{1}{x}$ results in the function $k(x) = -\frac{1}{x}$.



$h(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

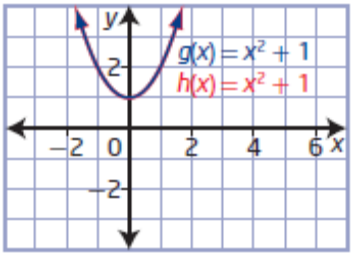
Section 1.2 Page 28 Question 4

a) A reflection in the y-axis of $f(x) = 3x$ results in the function $g(x) = -3x$.



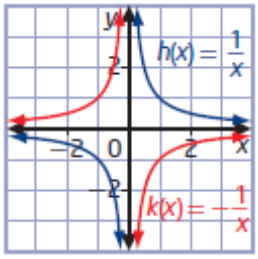
$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

b) A reflection in the y-axis of $g(x) = x^2 + 1$ results in the function $h(x) = x^2 + 1$.



$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

c) A reflection in the y-axis of $h(x) = \frac{1}{x}$ results in the function $k(x) = -\frac{1}{x}$.



$h(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

Section 1.2 Page 29 Question 5

a) Since $a = 4$, the graph of $y = 4f(x)$ is a vertical stretch of the graph of $y = f(x)$ about the x-axis by a factor of 4. The points on the graph of $y = 4f(x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (x, 4y)$.

b) Since $b = 3$, the graph of $y = f(3x)$ is a horizontal stretch of the graph of $y = f(x)$ about the y-axis by a factor of $\frac{1}{3}$. The points on the graph of $y = f(3x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow \left(\frac{1}{3}x, y\right)$.

c) Since $a = -1$, the graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis. The points on the graph of $y = -f(x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (x, -y)$.

d) Since $b = -1$, the graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis. The points on the graph of $y = f(-x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (-x, y)$.

Section 1.2 Page 29 Question 6

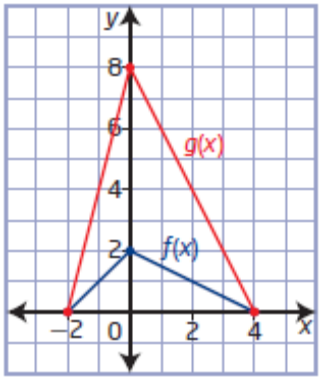
a) For the given graph of $y = f(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$ and the range is $\{y \mid -4 \leq y \leq 4, y \in \mathrm{R}\}$. If the graph of $y = f(x)$ is vertically stretched about the x-axis by a factor of 2, the domain remains the same but the range becomes $\{y \mid -8 \leq y \leq 8, y \in \mathrm{R}\}$.

b) Since all points on a graph that is vertically stretched relate to the points on the original graph by the mapping $(x, y) \rightarrow (x, ay)$, the domain remains the same but the range changes by a factor of a.

Section 1.2 Page 29 Question 7

a) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

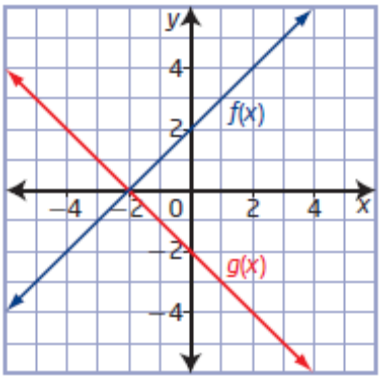
x	$y = f(x)$	$y = g(x)$
–2	0	0
–1	1	4
0	2	8
2	1	4
4	0	0



The transformation can be described by the mapping $(x, y) \rightarrow (x, 4y)$. This indicates a vertical stretch about the x-axis by a factor of 4. The equation of the transformed function is $g(x) = 4f(x)$.

b) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

x	$y = f(x)$	$y = g(x)$
–4	–2	2
–2	0	0
0	2	–2
2	4	–4

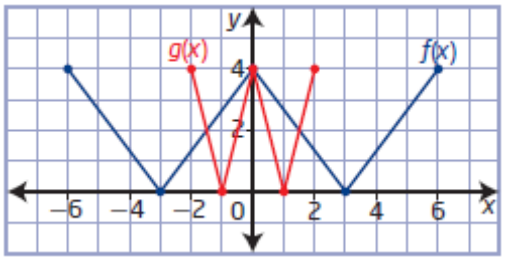


The transformation can be described by the mapping $(x, y) \rightarrow (x, -y)$. This indicates a reflection in the x-axis. The equation of the transformed function is $g(x) = -f(x)$.

c) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

x	$y = f(x)$
–6	4
–3	0
0	4
3	0
6	4

x	$y = g(x)$
–2	0
1	4
0	8
1	4
2	0

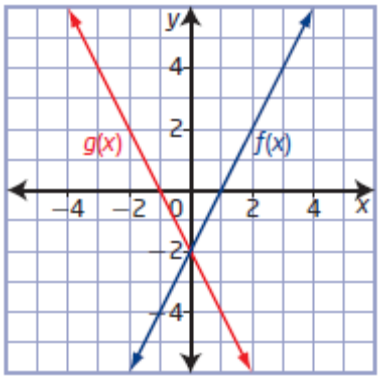


The transformation can be described by the mapping $(x, y) \rightarrow \left(\frac{1}{3}x, y\right)$. This indicates a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$. The equation of the transformed function is $g(x) = f(3x)$.

d) Compare key points on the graphs of $y = f(x)$ and $y = g(x)$.

x	$y = f(x)$
–2	–6
–1	–4
0	–2
2	2

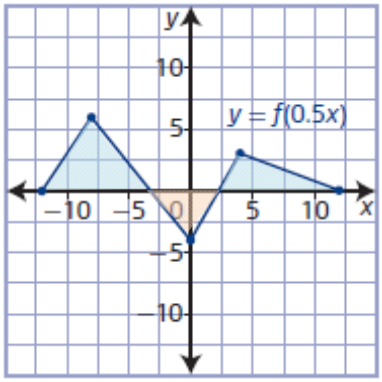
x	$y = g(x)$
2	–6
1	–4
0	–2
–2	–6



The transformation can be described by the mapping $(x, y) \rightarrow (-x, y)$. This indicates a reflection in the y-axis. The equation of the transformed function is $g(x) = f(-x)$.

Section 1.2 Page 29 Question 8

The transformation $y = f(0.5x)$ can be described by the mapping $(x, y) \rightarrow (2x, y)$.



Section 1.2 Page 30 Question 9

a) When x is replaced by $4x$, the equation becomes $y = f(4x)$ and the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{4}$.

b) When x is replaced by $\frac{1}{4}x$, the equation becomes $y = f\left(\frac{1}{4}x\right)$ and the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 4.

c) When y is replaced by $2y$, the equation becomes $y = \frac{1}{2}f(x)$ and the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of $\frac{1}{2}$.

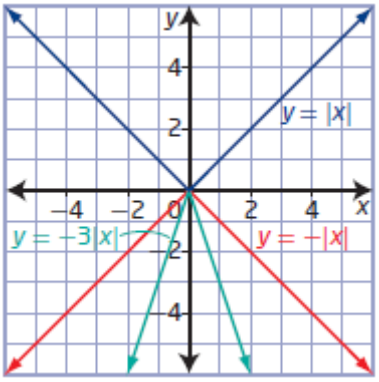
d) When y is replaced by $\frac{1}{4}y$, the equation becomes $y = 4f(x)$ and the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of 4.

e) When x is replaced by $-3x$, the equation becomes $y = f(-3x)$ and the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$ and reflected in the y-axis.

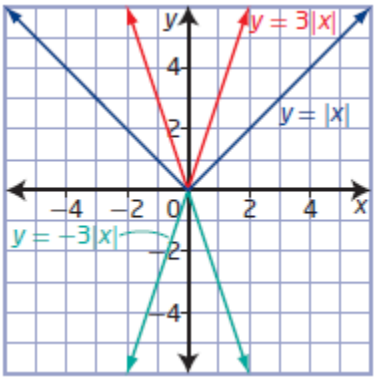
f) When y is replaced by $-\frac{1}{3}y$, the equation becomes $y = -3f(x)$ and the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of 3 and reflected in the x-axis.

Section 1.2 Page 30 Question 10

a) Apply reflection first.



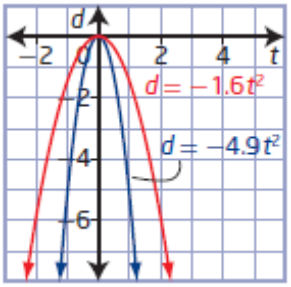
b) Apply stretch first.



c) Both Thomas and Sharyn are correct. Reflections and stretches can be applied in any order.

Section 1.2 Page 30 Question 11

a)



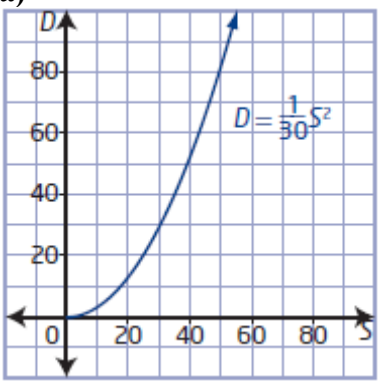
b) Both functions are reflections in the t-axis and vertical stretches of the base function $d = t^2$. The object falling on Earth ($d = –4.9t^2$) is stretched vertically more than the object falling on the moon.

Section 1.2 Page 30 Question 12

Example: When the graph of $y = f(x)$ is transformed to the graph of $y = f(bx)$, it undergoes a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$ and only the x-coordinates are affected. When the graph of $y = f(x)$ is transformed to the graph of $y = af(x)$, it undergoes a vertical stretch about the x-axis by a factor of $|a|$ and only the y-coordinates are affected.

Section 1.2 Page 30 Question 13

a)

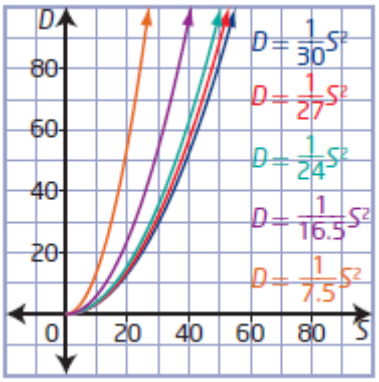


b) For asphalt, substitute $f = 0.9$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{27}S^2$.

For gravel, substitute $f = 0.8$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{24}S^2$.

For snow, substitute $f = 0.55$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{16.5}S^2$.

For ice, substitute $f = 0.25$ and $n = 1$ into $D = \frac{1}{30fn}S^2$: $D = \frac{1}{7.5}S^2$.



As the drag factor decreases, the length of the skid mark increases for the same speed.

Section 1.2 Page 31 Question 14

The zeros of the function $f(x) = (x + 4)(x - 3)$ are -4 and 3.
a) Since the points on the graph $y = 4f(x)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow (x, 4y)$, the zeros do not change.

b) Since the points on the graph $y = f(-x)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow (-x, y)$, the zeros will occur at $x = 4$ and $x = -3$.

c) Since the points on the graph $y = f\left(\frac{1}{2}x\right)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow (2x, y)$, the zeros will occur at $x = -8$ and $x = 6$.

d) Since the points on the graph $y = f(2x)$ relate to the graph of $f(x)$ by $(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$, the zeros will occur at $x = -2$ and $x = 1.5$.

Section 1.2 Page 31 Question 15

The graph of a function $y = f(x)$ is contained completely in quadrant IV.
a) A transformation to $y = -f(x)$ represents a reflection in the x-axis: $(x, y) \rightarrow (x, -y)$.
If $y = f(x)$ is transformed to $y = -f(x)$, it will be in quadrant **I**.

b) A transformation to $y = f(-x)$ represents a reflection in the y-axis: $(x, y) \rightarrow (-x, y)$.
If $y = f(x)$ is transformed to $y = f(-x)$, it will be in quadrant **III**.

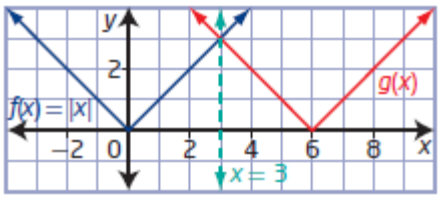
c) A transformation to $y = 4f(x)$ represents a vertical stretch about the x-axis: $(x, y) \rightarrow (x, 4y)$.
If $y = f(x)$ is transformed to $y = 4f(x)$, it will be in quadrant **IV**.

d) A transformation to $y = f\left(\frac{1}{4}x\right)$ represents a horizontal stretch about the y-axis:

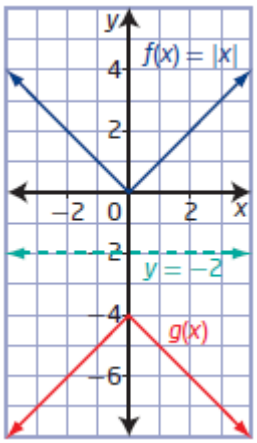
$(x, y) \rightarrow (4x, y)$.
If $y = f(x)$ is transformed to $y = 4f(x)$, it will be in quadrant **IV**.

Section 1.2 Page 31 Question 16

a) To reflect the graph of $f(x) = |x|$ in the line $x = 3$, each image point is the same distance from the line $x = 3$ as its corresponding key point.



b) To reflect the graph of $f(x) = |x|$ in the line $y = -2$, each image point is the same distance from the line $y = -2$ as its corresponding key point.



Section 1.2 Page 31 Question C1

Example: When the input values for $g(x)$ are b times the input values for $f(x)$, the scale factor must be $\frac{1}{b}$ (for $b > 0$) to maintain the same output values.

$g(x) = f\left(\frac{1}{b}(bx)\right) = f(x)$

Section 1.2 Page 31 Question C2

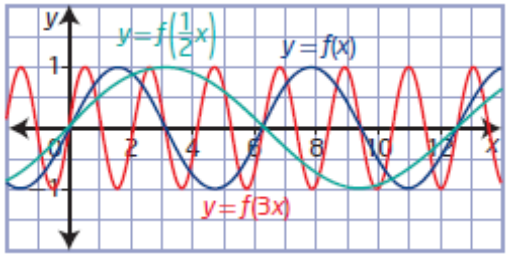
a) Transformations that result in the x-intercepts being invariant points are vertical stretches or a reflection in the x-axis.

b) Transformations that result in the y-intercepts being invariant points are horiontal stretches or a reflection in the y-axis.

Section 1.2 Page 31 Question C3

$f(x)$	$g(x)$	Transformation
(5, 6)	(5, –6)	reflection in the x-axis
(4, 8)	(–4, 8)	reflection in the y-axis
(2, 3)	(2, 12)	vertical stretch by a factor of 4
(4, –12)	(2, –6)	horizontal stretch by a factor of $\frac{1}{2}$ and vertical stretch by a factor of $\frac{1}{2}$

Section 1.2 Page 31 Question C4



Section 1.2 Page 31 Question C5

a) For the sequence –10, –6, –2, 2, 6, … $a = -10$ and $d = 4$. So, the general term is
$t_n = a + (n - 1)d$
$t_n = -10 + (n - 1)(4)$
$t_n = -14 + 4n$

b) For the sequence 10, 6, 2, –2, –6, … $a = 10$ and $d = -4$. So, the general term is
$t_n = a + (n - 1)d$
$t_n = 10 + (n - 1)(-4)$
$t_n = 14 - 4n$

c) Since the general term for part a) equals –1 times the general term in part b), the graphs will reflections of each other in the x-axis.

Section 1.3 Combining Transformations

Section 1.3 Page 38 Question 1

a) The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of 2 and then reflected in the x-axis: $b = 0.5$ and $a = -1$. Then, the transformed equation is $y = -f(0.5x)$ or
$y = -(0.5x)^2$
$= -0.25x^2$

b) The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, and then stretched vertically about the x-axis by a factor of $\frac{1}{4}$: $b = -4$ and $a = \frac{1}{4}$. Then, the transformed equation is $y = \frac{1}{4}f(-4x)$. Then, $y = \frac{1}{4}(-4x)^2$, or $y = 4x^2$.

Section 1.3 Page 38 Question 2

a) Write $g(x) = -3f(4x - 16) - 10$ in the form $g(x) = af(b(x - h)) + k$:
$g(x) = -3f(4(x - 4)) - 10$
Then, $a = -3$, $b = 4$, $h = 4$, and $k = -10$.
The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **y-axis** by a factor of $\frac{1}{4}$. It is vertically stretched about the **x-axis** by a factor of **3**. It is reflected in the **x-axis**, and then translated **4** units to the right and **10** units down.

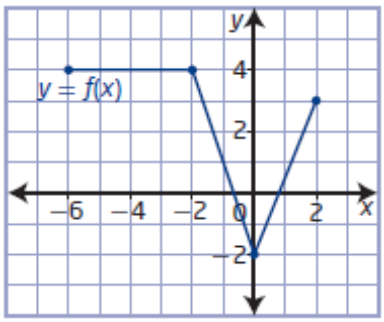
Section 1.3 Page 39 Question 3

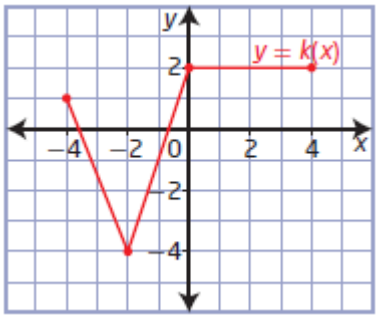
For $y - 4 = f(x - 5)$, $a = 1$, $b = 1$, $h = 5$, and $k = 4$.
For $y + 5 = 2f(3x)$, $a = 2$, $b = 3$, $h = 0$, and $k = -5$.
For $y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$, $a = \frac{1}{2}$, $b = \frac{1}{2}$, $h = 4$, and $k = 0$.
For $y + 2 = -3f(2(x + 2))$, $a = -3$, $b = 2$, $h = -2$, $k = -2$.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	-5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	-2	-2

Section 1.3 Page 39 Question 4

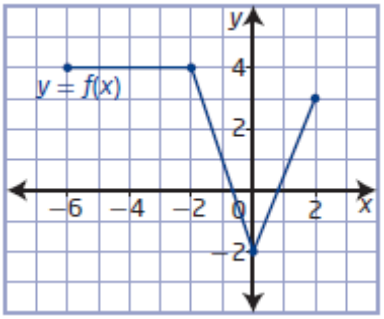
a) Locate key points on the graph of $f(x)$ and their image points on the graph of $k(x)$.
$(-6, 4) \rightarrow (4, 2)$
$(-2, 4) \rightarrow (0, 2)$
$(0, -2) \rightarrow (-2, -4)$
$(2, 3) \rightarrow (-4, 1)$

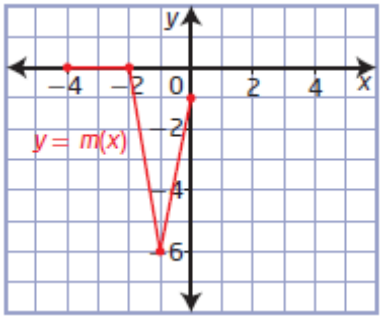




Since the x-coordinates have all changed sign, the graph has been reflected in the y-axis. The overall width and height have not changed, so the graph has not been vertically or horizontally stretched. Since the y-intercept has changed, the graph has been translated 2 units to the left and 2 units down. So, $a = 1$, $b = -1$, $h = -2$, $k = -2$, and the equation of the transformed graph is $y = f(-(x + 2)) - 2$.

b) Locate key points on the graph of $f(x)$ and their image points on the graph of $m(x)$.
$(-6, 4) \rightarrow (-4, 0)$
$(-2, 4) \rightarrow (-2, 0)$
$(0, -2) \rightarrow (-1, -6)$
$(2, 3) \rightarrow (0, -1)$

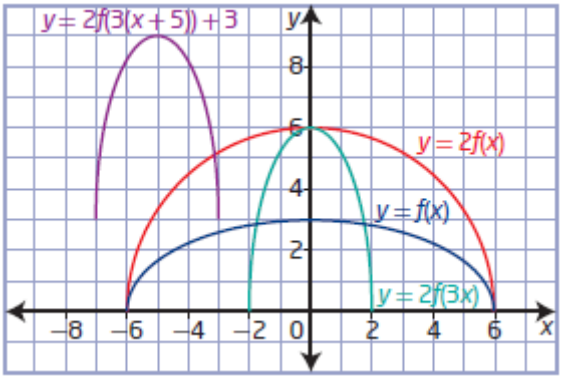




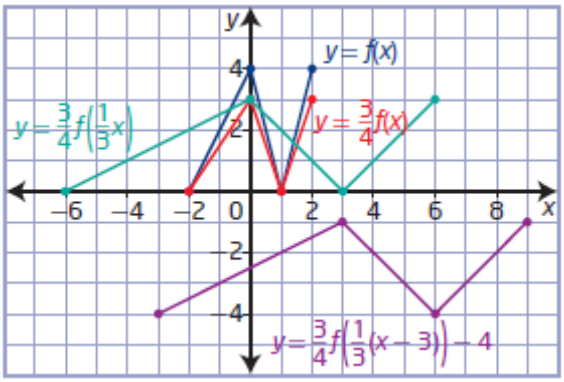
Since the orientation is unchanged, the graph has not been reflected in either axis. The overall width has changed, so the graph has been horizontally stretched by a factor of 0.5. Since the y-intercept has changed, the graph has been translated 1 unit to the left and 4 units down. So, $a = 1$, $b = 2$, $h = -1$, $k = -4$, and the equation of the transformed graph is $y = f(2(x + 1)) - 4$.

Section 1.3 Page 39 Question 5

a) vertical stretch about the x-axis by a factor of 2: $a = 2$, $y = 2f(x)$
horizontal stretch about the y-axis by a factor of $\frac{1}{3}$: $b = 3$, $y = 2f(3x)$
translation of 5 units to the left and 3 units up: $h = -5$ and $k = 3$, $y = 2f(3(x + 5)) + 3$



b) vertical stretch about the x-axis by a factor of $\frac{3}{4}$: $a = \frac{3}{4}$, $y = \frac{3}{4}f(x)$
horizontal stretch about the y-axis by a factor of 3: $b = \frac{1}{3}$, $y = \frac{3}{4}f\left(\frac{1}{3}x\right)$
translation of 3 units to the right and 4 units down: $h = 3$ and $k = -4$,
$y = \frac{3}{4}f\left(\frac{1}{3}(x-3)\right) - 4$



Section 1.3 Page 39 Question 6

a) The graph of $y + 6 = f(x - 4)$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (x + 4, y - 6)$. So, the key point (–12, 18) becomes the image point (–8, 12).

b) The graph of $y = 4f(3x)$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow \left(\frac{1}{3}x, 4y\right)$. So, the key point (–12, 18) becomes the image point (–4, 72).

c) The graph of $y = -2f(x - 6) + 4$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow (x + 6, -2y + 4)$. So, the key point (–12, 18) becomes the image point (–6, –32).

d) The graph of $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$ or $y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow \left(-\frac{3}{2}x - 9, -2y + 4\right)$. So, the key point (–12, 18) becomes the image point (9, –32).

e) The graph of $y + 3 = -\frac{1}{3}f\left(2(x + 6)\right)$ is related to the graph of $y = f(x)$ by the mapping $(x, y) \rightarrow \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right)$. So, the key point (–12, 18) becomes the image point (–12, –9).

Section 1.3 Page 40 Question 7

a) For $y = 2f(x - 3) + 4$, $a = 2$, $b = 1$, $h = 3$, and $k = 4$. The graph of $y = f(x)$ is vertically stretched about the x-axis by a factor of 2 and then translated 3 units to the right and 4 units up.
$(x, y) \rightarrow (x + 3, 2y + 4)$

b) For $y = -f(3x) - 2$, $a = -1$, $b = 3$, $h = 0$, and $k = -2$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$, reflected in the x-axis, and then translated 2 units down.

$(x, y) \rightarrow \left(\frac{1}{3}x, -y - 2\right)$

c) For $y = -\frac{1}{4}f(-(x + 2))$, $a = -\frac{1}{4}$, $b = -1$, $h = -2$, and $k = 0$. The graph of $y = f(x)$ is reflected in the y-axis, vertically stretched about the x-axis by a factor of $\frac{1}{4}$, reflected in the x-axis, and then translated 2 units to the left.

$(x, y) \rightarrow \left(-x - 2, -\frac{1}{4}y\right)$

d) For $y - 3 = -f(4(x - 2))$, $a = -1$, $b = 4$, $h = 2$, and $k = 3$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{4}$, reflected in the x-axis, and then translated 2 units to the right and 3 units up.

$(x, y) \rightarrow \left(\frac{1}{4}x + 2, -y + 3\right)$

e) For $y = -\frac{2}{3}f\left(-\frac{3}{4}x\right)$, $a = -\frac{2}{3}$, $b = -\frac{3}{4}$, $h = 0$, and $k = 0$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{4}{3}$, reflected in the y-axis, vertically stretched about the x-axis by a factor of $\frac{2}{3}$, and then reflected in the x-axis.

$(x, y) \rightarrow \left(-\frac{4}{3}x, -\frac{2}{3}y\right)$

f) Write $3y - 6 = f(-2x + 12)$ in the form $y - k = af(b(x - h))$:

$3y - 6 = f(-2x + 12)$

$3(y - 2) = f(-2(x - 6))$

$y - 2 = \frac{1}{3}f(-2(x - 6))$

Then, $a = \frac{1}{3}$, $b = -2$, $h = 6$, and $k = 2$. The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$, reflected in the y-axis, vertically stretched about the x-axis by a factor of $\frac{1}{3}$, and then translated 6 units to the right and 2 units up.

$$(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$$

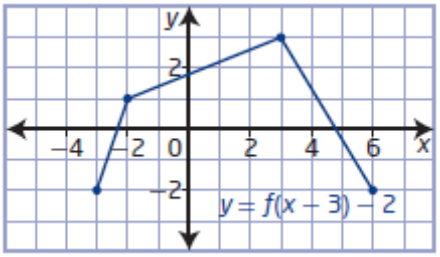
Section 1.3 Page 40 Question 8

a) For a vertical stretch about the x-axis by a factor of 3, a reflection in the x-axis, a horizontal translation of 4 units to the left, and a vertical translation of 5 units down, $a = -3$, $b = 1$, $h = -4$, and $k = -5$. So, the equation of the transformed function is $y + 5 = -3f(x + 4)$.

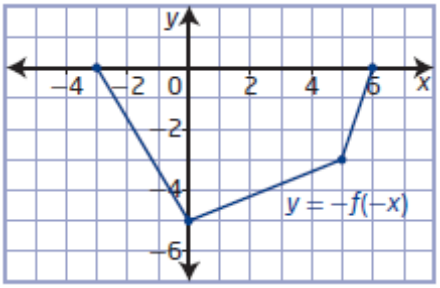
b) For a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$, a vertical stretch about the x-axis by a factor of $\frac{3}{4}$, a reflection in both the x-axis and the y-axis, and a translation of 6 units to the right and 2 units up, $a = -\frac{3}{4}$, $b = -3$, $h = 6$, and $k = 2$. So, the equation of the transformed function is $y - 2 = -\frac{3}{4}f(-3(x - 6))$.

Section 1.3 Page 40 Question 9

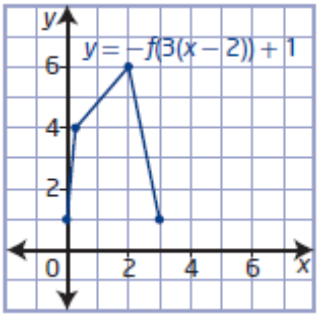
a) To obtain the graph of $y + 2 = f(x - 3)$, the graph of $y = f(x)$ is translated 3 units to the left and 2 units down.



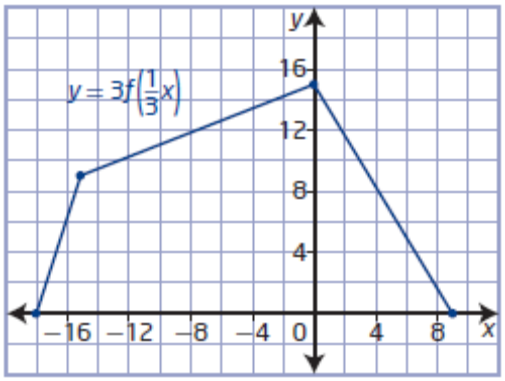
b) To obtain the graph of $y = -f(-x)$, the graph of $y = f(x)$ is reflected in the x-axis and the y-axis.



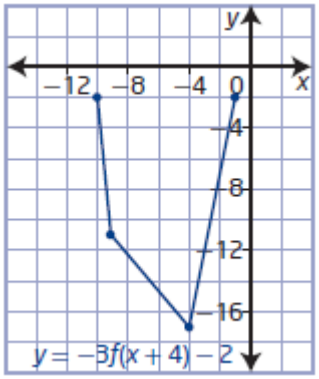
c) To obtain the graph of $y = f(3(x - 2)) + 1$, the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$ and translated 2 units to the right and 1 unit up.



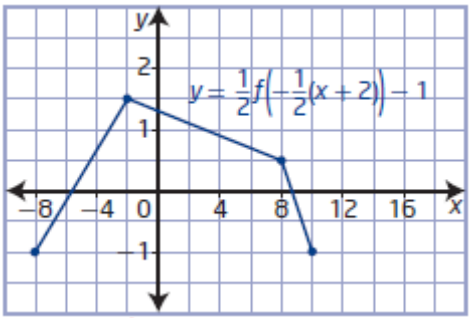
d) To obtain the graph of $y = 3f\left(\frac{1}{3}x\right)$, the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 3 and stretched vertically about the x-axis by a factor of 3.



e) To obtain the graph of $y + 2 = -3f(x + 4)$, the graph of $y = f(x)$ is stretched vertically about the x-axis by a factor of 3, reflected in the x-axis, and translated 4 units to the left and 2 units down.

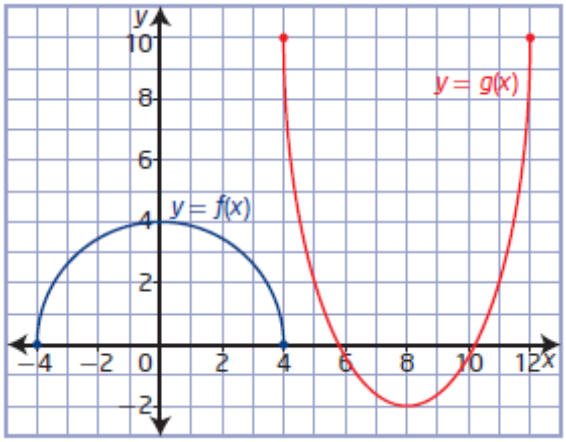


f) To obtain the graph of $y = \frac{1}{2}f\left(-\frac{1}{2}(x+2)\right) - 1$, the graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 2, reflected in the y-axis, stretched vertically about the x-axis by a factor of $\frac{1}{2}$, and translated 2 units to the left and 1 unit down.

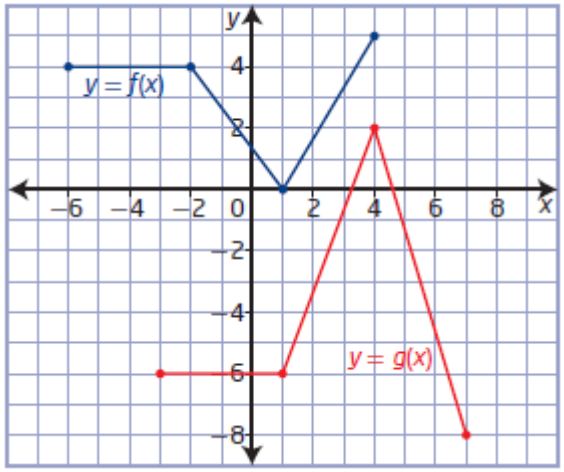


Section 1.3 Page 40 Question 10

a) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-4, 0) \rightarrow (4, 10)$
$(0, 4) \rightarrow (8, -2)$
$(4, 0) \rightarrow (12, 10)$
The orientation is changed, the graph has been reflected in the x-axis. The overall height has changed, so the graph has been vertically stretched by a factor of 3. Since the y-intercept has changed, the graph has been translated 8 units to the right and 10 units up. So, $a = -3$, $b = 1$, $h = 8$, $k = 10$, and the equation of the transformed graph is $y = -3f(x - 8) + 10$.

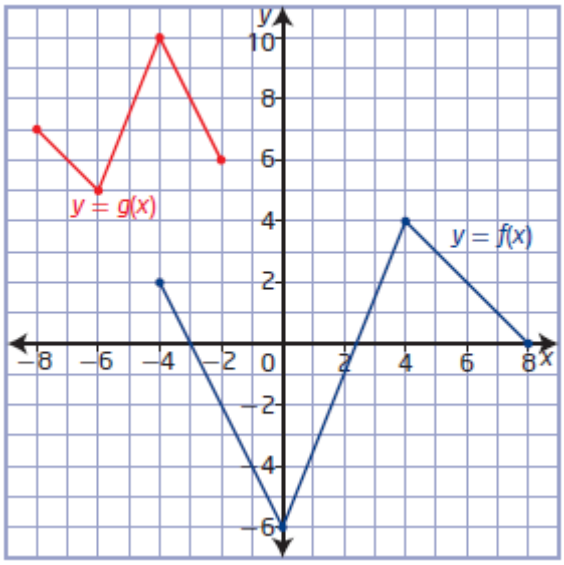


b) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-6, 4) \rightarrow (-3, -6)$
$(-2, 4) \rightarrow (1, -6)$
$(1, 0) \rightarrow (4, 2)$
$(4, 5) \rightarrow (7, -8)$



The orientation is changed, the graph has been reflected in the x-axis. The overall height has changed, so the graph has been vertically stretched by a factor of 2. Since the x-intercept has changed, the graph has been translated 3 units to the right and 2 units up. So, $a = -2$, $b = 1$, $h = 3$, $k = 2$, and the equation of the transformed graph is $y = -2f(x - 3) + 2$.

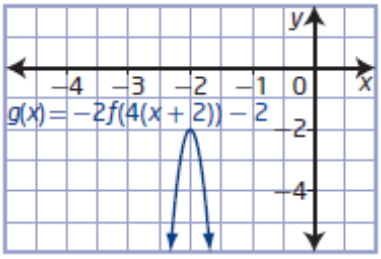
c) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(8, 0) \rightarrow (-8, 7)$
$(4, 4) \rightarrow (-6, 5)$
$(0, -6) \rightarrow (-4, 10)$
$(-4, 2) \rightarrow (-2, 6)$



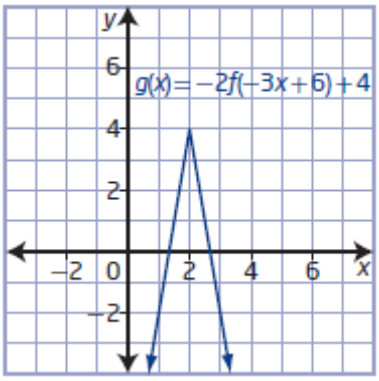
The orientation is changed, the graph has been reflected in the x-axis and in the y-axis. The overall width and height have changed, so the graph has been vertically stretched by a factor of $\frac{1}{2}$ and horizontally stretched by a factor of $\frac{1}{2}$.
The graph has been translated 4 units to the left and 7 units up. So, $a = -\frac{1}{2}$, $b = -2$, $h = -4$, and $k = 7$, and the equation of the transformed graph is $y = -\frac{1}{2} f\left(-2(x+4)\right) + 7$.

Section 1.3 Page 40 Question 11

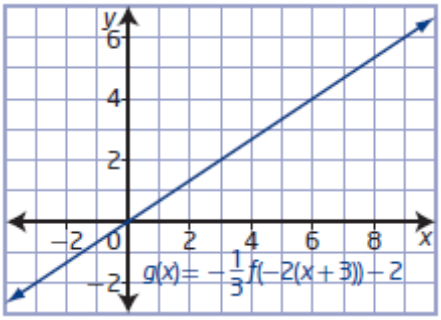
a) For $g(x) = -2f(4(x + 2)) - 2$, $a = -2$, $b = 4$, $h = -2$, and $k = -2$. The graph of $f(x) = x^2$ is stretched horizontally about the y-axis by a factor of 0.25, stretched vertically by a factor of 2, reflected in the x-axis, and translated 2 units to the left and 2 units down.



b) Write $g(x) = -2f(-3x + 6) + 4$ in the form $y = af(b(x - h) + k$: $g(x) = -2f(-3(x - 2)) + 4$
For $g(x) = -2f(-3(x - 2)) + 4$, $a = -2$, $b = -3$, $h = 2$, and $k = 4$. The graph of $f(x) = |x|$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, reflected in the y-axis, stretched vertically by a factor of 2, reflected in the x-axis, and translated 2 units to the right and 4 units up.



c) For $g(x) = -\frac{1}{3}f(-2(x + 3)) - 2$, $a = -\frac{1}{3}$, $b = -2$, $h = -3$, and $k = -2$. The graph of $f(x) = x$ is stretched horizontally about the y-axis by a factor of 0.5, reflected in the y-axis, stretched vertically by a factor of $\frac{1}{3}$, reflected in the x-axis, and translated 3 units to the left and 2 units down.

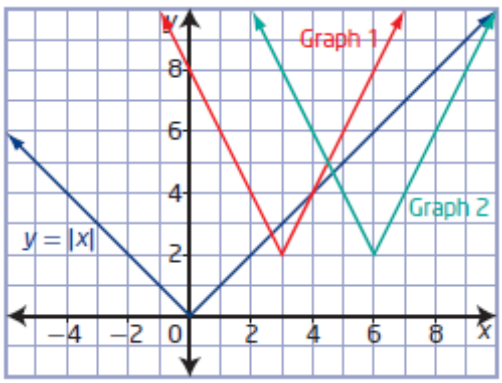


Section 1.3 Page 41 Question 12

a) To transform the original design by a horizontal stretch about the y-axis by a factor of 2, a reflection in the x-axis, and a translation of 4 units up and 3 units to the left, use the mapping $(x, y) \rightarrow (2x - 3, -y + 4)$.
A$(-4, 6) \rightarrow$ A′$(-11, -2)$
B$(-2, -2) \rightarrow$ B′$(-7, 6)$
C$(0, 0) \rightarrow$ C′$(-3, 4)$
D$(1, -1) \rightarrow$ D′$(-1, 5)$
E$(3, 6) \rightarrow$ E′$(3, -2)$
b) The equation of the design resulting from the transformations is $y = -f\left(\frac{1}{2}(x + 3)\right) + 4$.

Section 1.3 Page 41 Question 13

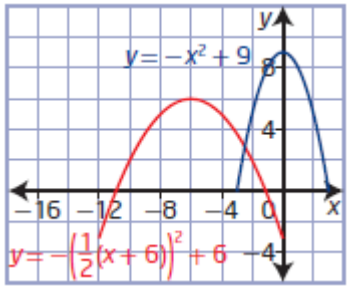
a) The graphs are in two locations because the transformations Gil performed to obtain Graph 2 do not match those in $y = |2x - 6| + 2$. Gil forgot to factor out the coefficient of the x-term, 2, from –6. The horizontal translation should have been 3 units to the right, not 6 units.



b) Gil should have rewritten the function equation $y = |2x - 6| + 2$ as $y = |2(x - 3)| + 2$.

Section 1.3 Page 41 Question 14

a) The first arch is modelled by the function $y = -x^2 + 9$, with a range of $0 \le y \le 9$. For the second arch that spans twice the distance and is translated 6 units to the left and 3 units down, use the mapping $(x, y) \rightarrow (2x - 6, y - 3)$.



b) For the second arch, $a = 1$, $b = \frac{1}{2}$, $h = -6$, and $k = -3$.

So, the equation of the second arch is $y = f\left(\frac{1}{2}(x+6)\right) - 3$

or $y = -\left(\frac{1}{2}(x+6)\right)^2 + 6$.

Section 1.3 Page 41 Question 15

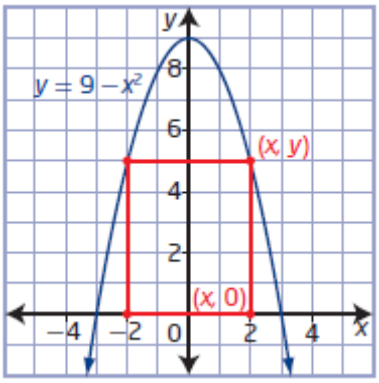
a) For $y = -f(-x)$, use the mapping $(x, y) \rightarrow (-x, -y)$.
$(a, 0) \rightarrow (-a, 0)$
$(0, b) \rightarrow (0, -b)$

b) For $y = 2f\left(\frac{1}{2}x\right)$, use the mapping $(x, y) \rightarrow (2x, 2y)$.
$(a, 0) \rightarrow (2a, 0)$
$(0, b) \rightarrow (0, 2b)$

c) and **d)** There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.

Section 1.3 Page 41 Question 16

a) The area of the rectangle can be modelled by the function $A = 2x(9 - x^2)$ or $A = -2x^3 + 18x$.



b) For a horizontal stretch by a factor of 4, the equation of the parabola becomes $y = 9 - \frac{1}{16}x^2$. So, the area of the rectangle can be modeled by the function $A = 2x\left(9 - \frac{1}{16}x^2\right)$ or $A = -\frac{1}{8}x^3 + 18x$.

c) For (2, 5), the area of the rectangle in part a) is 2(2)(5), or 20 square units.

Verify the function in part a):
$A = -2x^3 + 18x$
$A = -2(2)^3 + 18(2)$
$A = -16 + 36$
$A = 20$
For a horizontal stretch by a factor of 4, the point (2, 5) becomes the point (8, 5). Then, the area of the rectangle in part b) is 2(8)(5), or 80 square units.
Verify the function in part b):

$$A = -\frac{1}{8}x^3 + 18x$$

$$A = -\frac{1}{8}(8)^3 + 18(8)$$

$A = -64 + 144$
$A = 80$

Section 1.3 Page 42 Question 17

If the function is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down, then $a = 2$, $b = 3$, $h = 2$, $k = -4$, and $y = 2f(3(x - 2)) - 4$. Given the function $y = 2x^2 + x + 1$, the equation of the transformed function is
$y = 2[2(3(x - 2))^2 + 3(x - 2) + 1] - 4$
$y = 2[18(x - 2)^2 + 3(x - 2) + 1] - 4$
$y = 36(x - 2)^2 + 6(x - 2) + 2 - 4$
$y = 36(x - 2)^2 + 6(x - 2) - 2$
or $y = 36x^2 - 138x + 130$

Section 1.3 Page 42 Question 18

Example: Transformations in which the order does not matter include vertical stretches and horizontal stretches followed by reflections.
For example:

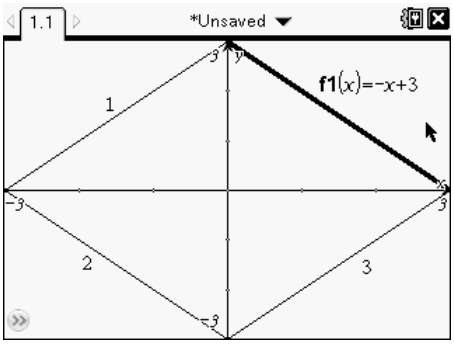
Graph of $f(x)$	**Graph of $g(x)$**	
	Horizontal stretch	Vertical stretch
(x, y)	$\left(\frac{x}{b}, y\right)$	$\left(\frac{x}{b}, ay\right)$

Graph of $f(x)$	**Graph of $g(x)$**	
	Vertical stretch	Horizontal stretch
(x, y)	(x, ay)	$\left(\frac{x}{b}, ay\right)$

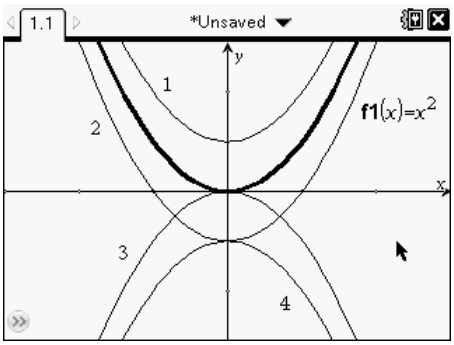
The end result is the same.

Section 1.3 Page 42 Question C1

Step 1 Graph 1: a reflection in the y-axis; $y = x + 3$
Graph 2: a reflection in the y-axis and in the x-axis: $y = -x - 3$
Graph 3: a reflection in the x-axis; $y = x - 3$



Step 2 Graph 1: a translation of 1 unit up; $y = x^2 + 1$
Graph 2: a translation of 1 unit down: $y = x^2 - 1$
Graph 3: a reflection in the x-axis; $y = -x^2$
Graph 4: a reflection in the x-axis and a translation of 1 unit down; $y = -x^2 - 1$



Section 1.3 Page 42 Question C2

They make b bracelets per week at a cost of $f(b)$.
a) Then, $f(b + 12)$ represents the cost of making $b + 12$ bracelets. This relates to transformations as a horizontal translation.

b) Then, $f(b) + 12$ represents the cost of making b bracelets plus 12 more dollars. This relates to transformations as a vertical translation.

c) Then, $3f(b)$ represents triple the cost of making b bracelets. This relates to transformations as a vertical stretch.

d) Then, $f(2b)$ represents the cost of making $0.5b$ bracelets. This relates to transformations as a horizontal stretch.

Section 1.3 Page 42 Question C3

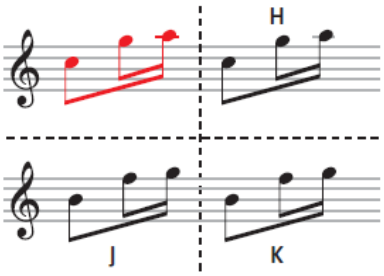
Complete the square.
$y = 2x^2 - 12x + 19$
$y = 2(x^2 - 6x) + 19$
$y = 2(x^2 - 6x + 9 - 9) + 19$
$y = 2(x^2 - 6x + 9) - 18 + 19$
$y = 2(x - 3)^2 + 1$

This form of the equation shows that the function is a vertical stretch by a factor of 2 and then a translation of 3 units to the right and 1 unit up of the graph of $y = x^2$.

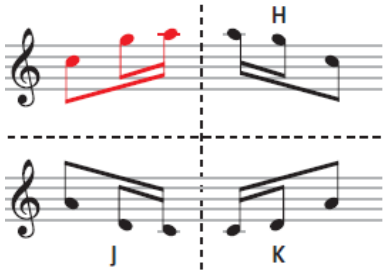
Section 1.3 Page 43 Question C4

Musical notes can be repeated (translated horizontally), transposed (translated vertically), inverted (horizontal mirror), in retrograde (vertical mirror), or in retrograde inversion (180° rotation).

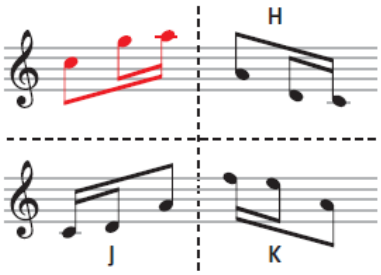
a) H is repeated; J is transposed; K is repeated and transposed



b) H is in retrograde; J is inverted; K is in retrograde and inverted



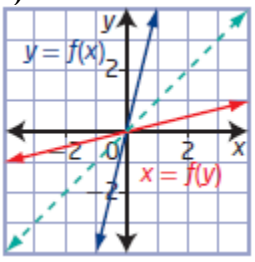
c) H is inverted, repeated, and transposed; J is in retrograde, inverted, and repeated; K is in retrograde and transposed



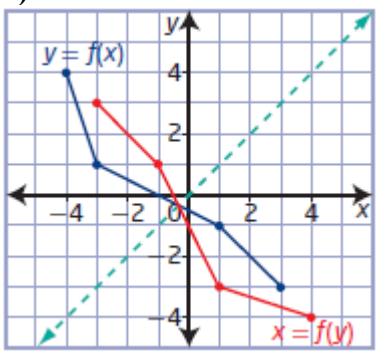
Section 1.4 Inverse of a Relation

Section 1.4 Page 51 Question 1

a)

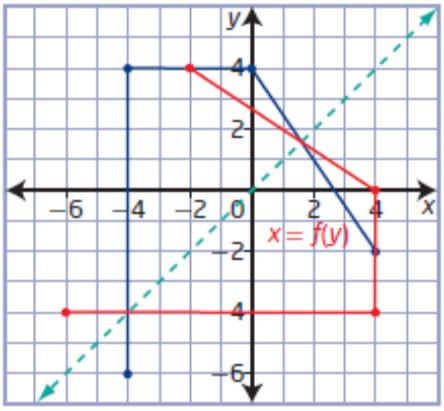


b)

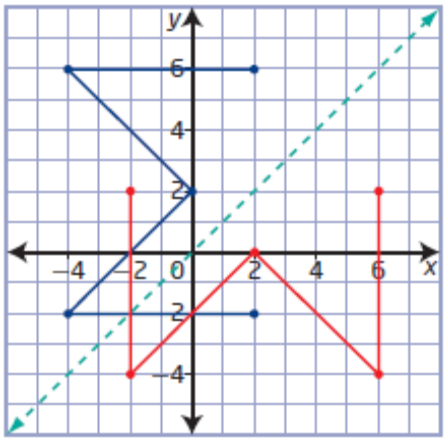


Section 1.4 Page 51 Question 2

a)

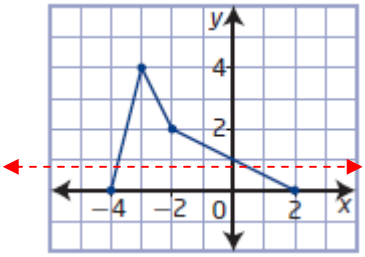


b)

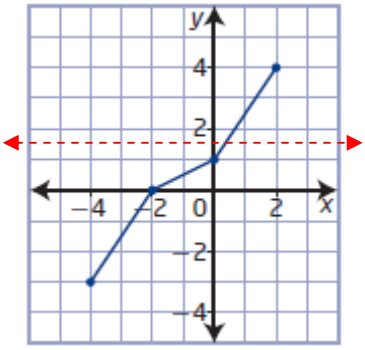


Section 1.4 Page 52 Question 3

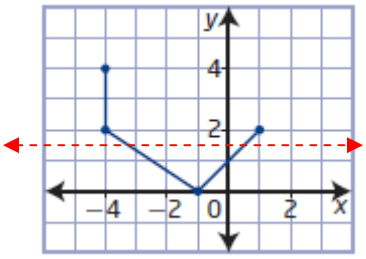
a) The graph is a function; it passes the vertical line test. The inverse will be a relation since the graph does not pass the horizontal line test.



b) The graph is a function; it passes the vertical line test. The inverse will be a function since the graph passes the horizontal line test.



c) The graph is not a function; it does not pass the vertical line test. The inverse will be a relation since the graph does not pass the horizontal line test.

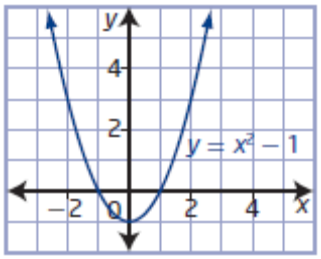


Section 1.4 Page 52 Question 4

Examples:

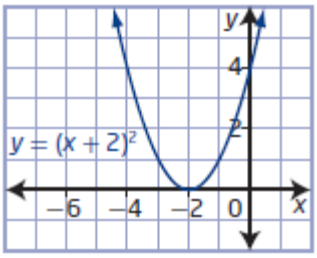
a) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:

$\{x \mid x \leq 0, x \in \mathrm{R}\}$ or $\{x \mid x \geq 0, x \in \mathrm{R}\}$.



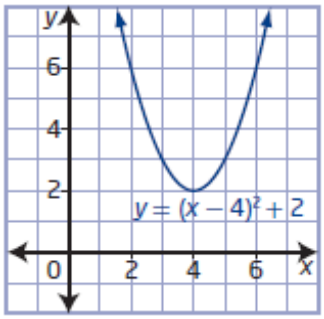
b) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:

$\{x \mid x \leq -2, x \in \mathrm{R}\}$ or $\{x \mid x \geq -2, x \in \mathrm{R}\}$.



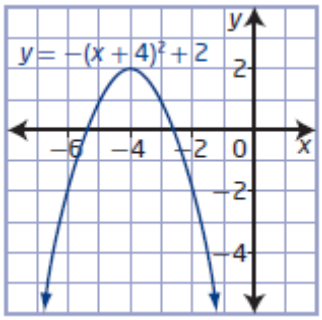
c) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:

$\{x \mid x \leq 4, x \in \mathrm{R}\}$ or $\{x \mid x \geq 4, x \in \mathrm{R}\}$.



d) A restricted domain for which the function has an inverse that is also a function is the left or right half of the parabola:

$\{x \mid x \leq -4, x \in \mathrm{R}\}$ or $\{x \mid x \geq -4, x \in \mathrm{R}\}$.



Section 1.4 Page 52 Question 5

Let $y = f(x)$. To find the equation of the inverse, interchange x and y, and then solve for y.

a) $f(x) = 7x$

$y = 7x$

$x = 7y$

$y = \frac{1}{7}x$

$f^{-1}(x) = \frac{1}{7}x$

b) $f(x) = -3x + 4$

$y = -3x + 4$

$x = -3y + 4$

$x - 4 = -3y$

$y = -\frac{1}{3}(x - 4)$

$f^{-1}(x) = -\frac{1}{3}(x - 4)$

c) $f(x) = \frac{x+4}{3}$

$y = \frac{x+4}{3}$

$x = \frac{y+4}{3}$

$3x = y + 4$

$y = 3x - 4$

$f^{-1}(x) = 3x - 4$

d) $f(x) = \frac{x}{3} - 5$

$y = \frac{x}{3} - 5$

$x = \frac{y}{3} - 5$

$x + 5 = \frac{y}{3}$

$y = 3(x + 5)$

$f^{-1}(x) = 3(x + 5)$

e) $f(x) = 5 - 2x$

$y = 5 - 2x$

$x = 5 - 2y$

$x - 5 = -2y$

$y = -\frac{1}{2}(x - 5)$

$f^{-1}(x) = -\frac{1}{2}(x - 5)$

f) $f(x) = \frac{1}{2}(x + 6)$

$y = \frac{1}{2}(x + 6)$

$x = \frac{1}{2}(y + 6)$

$2x = y + 6$

$y = 2x - 6$

$f^{-1}(x) = 2x - 6$

Section 1.4 Page 52 Question 6

a) The inverse of $y = 2x + 5$ is $y = \frac{1}{2}(x - 5)$: choice **E**.

b) The inverse of $y = \frac{1}{2}x - 4$ is $y = 2(x + 4)$: choice **C**.

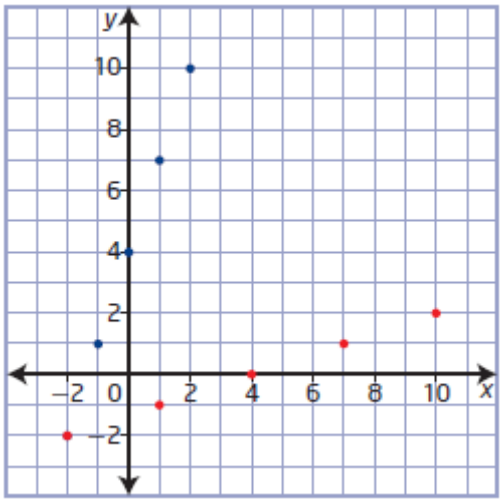
c) The inverse of $y = 6 - 3x$ is $y = -\frac{1}{3}(x - 6)$: choice **B**.

d) The inverse of $y = x^2 - 12$, $x \geq 0$ is $y = \sqrt{x + 12}$: choice **A**.

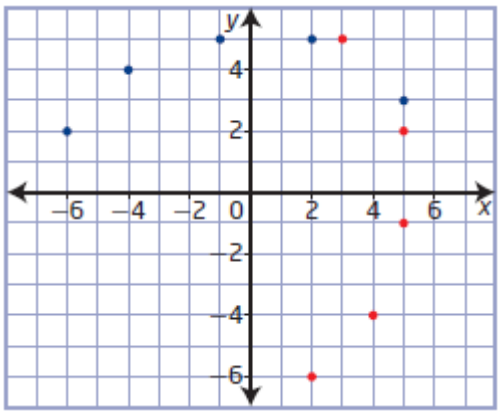
e) The inverse of $y = \frac{1}{2}(x + 1)^2$, $x \leq -1$ is $y = -\sqrt{2x} - 1$: choice **D**.

Section 1.4 Page 53 Question 7

a) Function (blue): domain {–2, –1, 0, 1, 2}, range {–2, 1, 4, 7, 10}
Inverse (red): domain {–2, 1, 4, 7, 10}, range {–2, –1, 0, 1, 2}

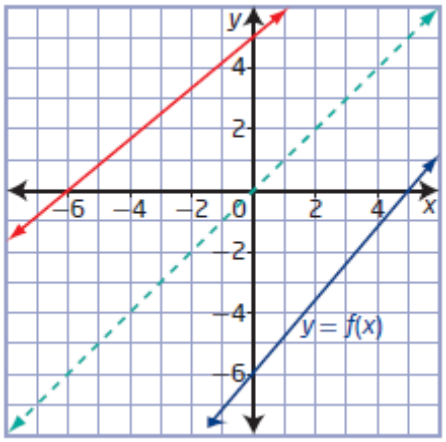


b) a) Function (blue):
domain {–6, –4, –1, 2, 5}, range {2, 4, 5, 3}
Inverse (red):
domain {2, 4, 5, 3}, range {–6, –4, –1, 2, 5}

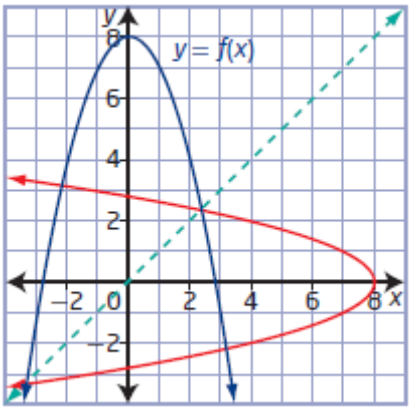


Section 1.4 Page 53 Question 8

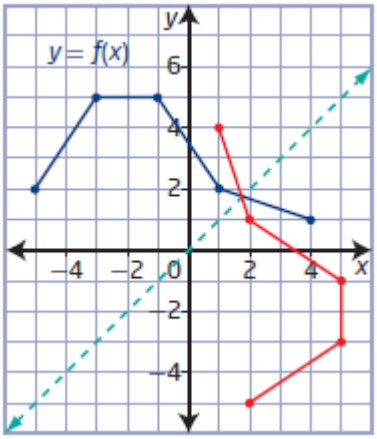
a) The inverse is a function; it passes the vertical line test.



b) The inverse is not a function; it does not pass the vertical line test.



c) The inverse is not a function; it does not pass the vertical line test.



Section 1.4 Page 53 Question 9

a) $f(x) = 3x + 2$

$y = 3x + 2$

$x = 3y + 2$

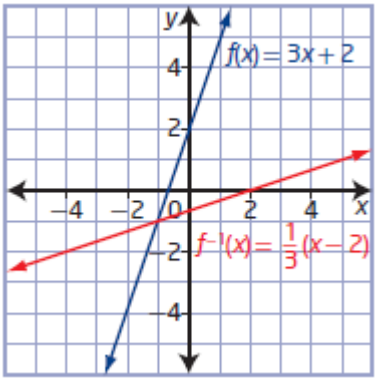
$x - 2 = 3y$

$y = \frac{1}{3}(x - 2)$

$f^{-1}(x) = \frac{1}{3}(x - 2)$

$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$



b) $f(x) = 4 - 2x$

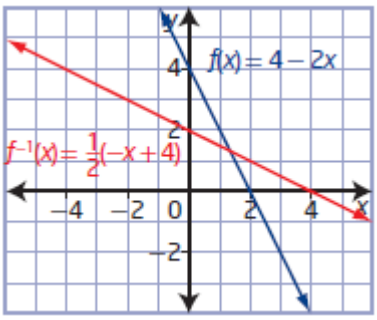
$y = 4 - 2x$

$x = 4 - 2y$

$x - 4 = -2y$

$y = -\frac{1}{2}(x - 4)$

$f^{-1}(x) = -\frac{1}{2}(x - 4)$



$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

c) $f(x) = \frac{1}{2}x - 6$

$y = \frac{1}{2}x - 6$

$x = \frac{1}{2}y - 6$

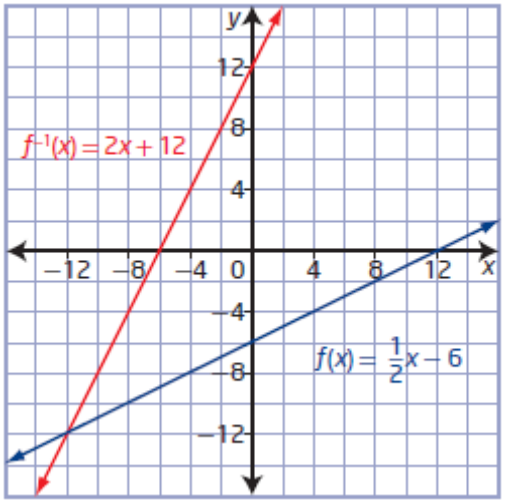
$x + 6 = \frac{1}{2}y$

$y = 2(x + 6)$

$f^{-1}(x) = 2(x + 6)$

$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$



d) $f(x) = x^2 + 2, x \leq 0$

$y = x^2 + 2$

$x = y^2 + 2$

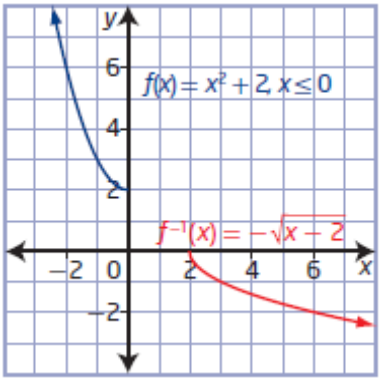
$x - 2 = y^2$

$y = -\sqrt{x - 2}$

$f^{-1}(x) = -\sqrt{x - 2}$

$f(x)$: domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 2, y \in \mathrm{R}\}$

$f^{-1}(x)$: domain $\{x \mid x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$



e) $f(x) = 2 - x^2, x \geq 0$

$y = 2 - x^2$

$x = 2 - y^2$

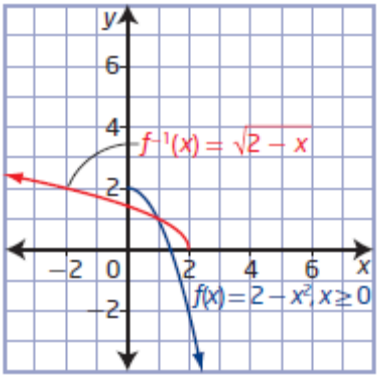
$-x - 2 = -y^2$

$y = \sqrt{-x - 2}$

$f^{-1}(x) = \sqrt{-x - 2}$

$f(x)$: domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 2, y \in \mathrm{R}\}$

$f^{-1}(x)$: domain $\{x \mid x \leq 2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$



Section 1.4 Page 53 Question 10

a) i) Complete the square.

$f(x) = x^2 + 8x + 12$

$f(x) = (x^2 + 8x + 16 - 16) + 12$

$f(x) = (x + 4)^2 - 4$

Determine the equation of the inverse of $f(x)$.

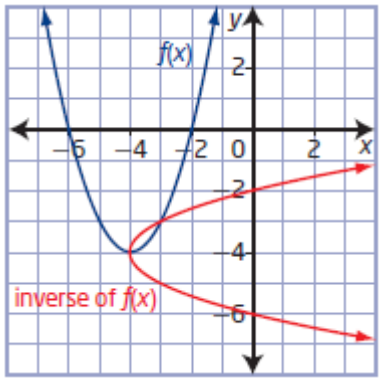
$y = (x + 4)^2 - 4$

$x = (y + 4)^2 - 4$

$x + 4 = (y + 4)^2$

$y = \pm\sqrt{x+4} - 4$

ii)



b) i) Complete the square.

$f(x) = x^2 - 4x + 2$

$f(x) = (x^2 - 4x + 4 - 4) + 2$

$f(x) = (x - 2)^2 - 2$

Determine the equation of the inverse of $f(x)$.

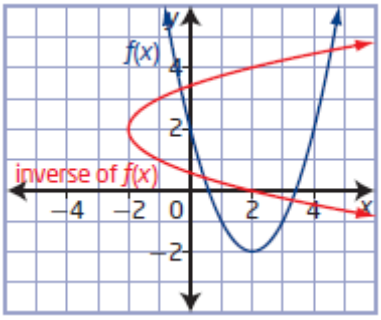
$y = (x - 2)^2 - 2$

$x = (y - 2)^2 - 2$

$x + 2 = (y - 2)^2$

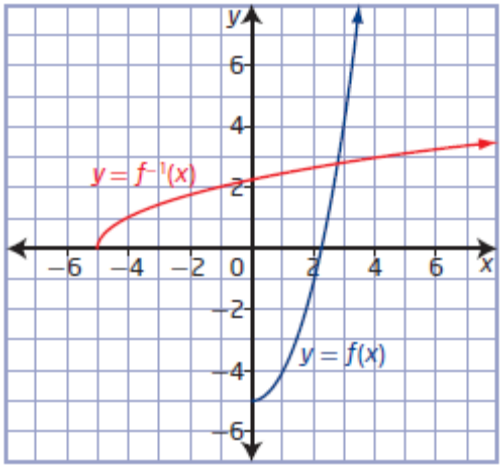
$y = \pm\sqrt{x+2} + 2$

ii)



Section 1.4 Page 53 Question 11

Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



Section 1.4 Page 54 Question 12

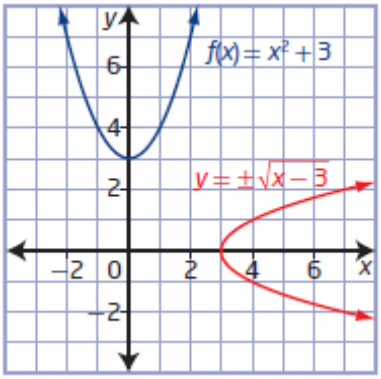
a) $f(x) = x^2 + 3$

$y = x^2 + 3$

$x = y^2 + 3$

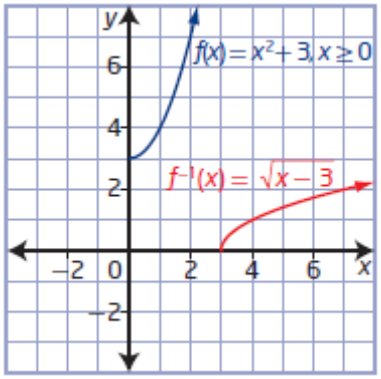
$x - 3 = y^2$

$y = \pm\sqrt{x-3}$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

$f(x) = x^2 + 3, x \geq 0$ and $f^{-1}(x) = \sqrt{x-3}$



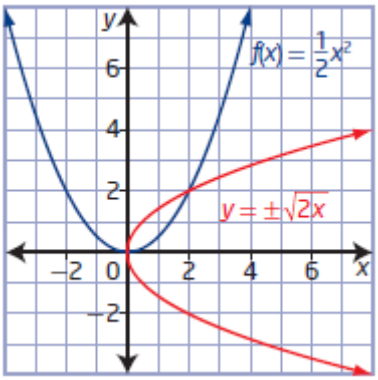
b) $f(x) = \frac{1}{2}x^2$

$y = \frac{1}{2}x^2$

$x = \frac{1}{2}y^2$

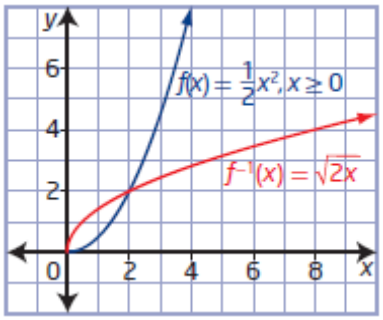
$2x = y^2$

$y = \pm\sqrt{2x}$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

$f(x) = \frac{1}{2}x^2, x \geq 0$ and $f^{-1}(x) = \sqrt{2x}$



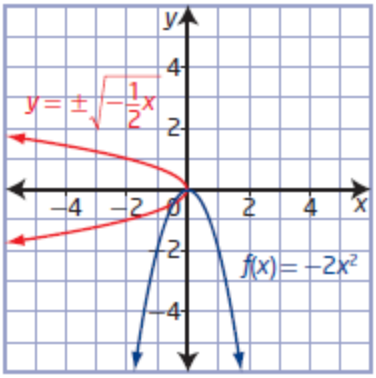
c) $f(x) = -2x^2$

$y = -2x^2$

$x = -2y^2$

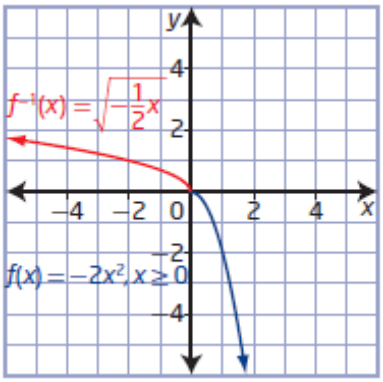
$-\frac{1}{2}x = y^2$

$y = \pm\sqrt{-\frac{1}{2}x}$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

$f(x) = -2x^2, x \geq 0$ and $f^{-1}(x) = \sqrt{-\frac{1}{2}x}$



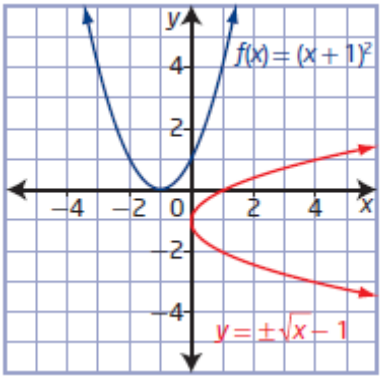
d) $f(x) = (x + 1)^2$

$y = (x + 1)^2$

$x = (y + 1)^2$

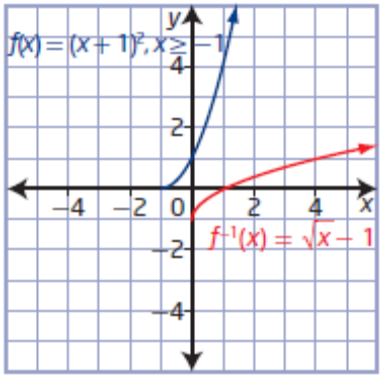
$\pm\sqrt{x} = y + 1$

$y = \pm\sqrt{x} - 1$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq -1, x \in \mathrm{R}\}$.

$f(x) = (x + 1)^2, x \geq -1$ and $f^{-1}(x) = \sqrt{x} - 1$



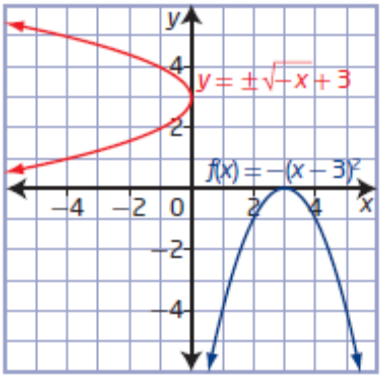
e) $f(x) = -(x-3)^2$

$y = -(x-3)^2$

$x = -(y-3)^2$

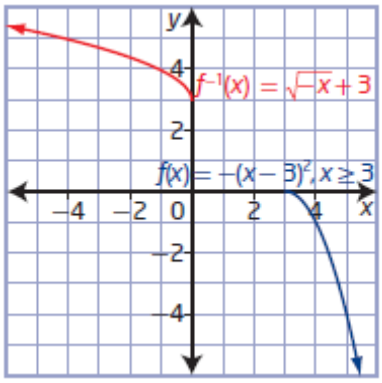
$\pm\sqrt{-x} = y - 3$

$y = \pm\sqrt{-x} + 3$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 3, x \in \mathrm{R}\}$.

$f(x) = -(x-3)^2, x \geq 3$ and $f^{-1}(x) = \sqrt{-x} + 3$



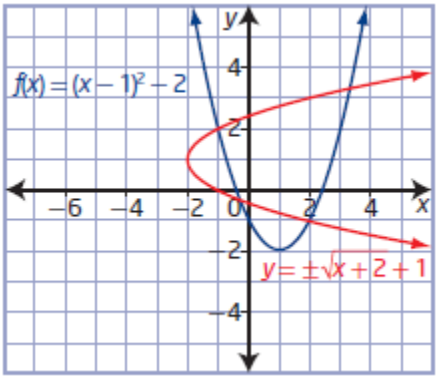
f) $f(x) = (x-1)^2 - 2$

$y = (x-1)^2 - 2$

$x = (y-1)^2 - 2$

$\pm\sqrt{x+2} = y - 1$

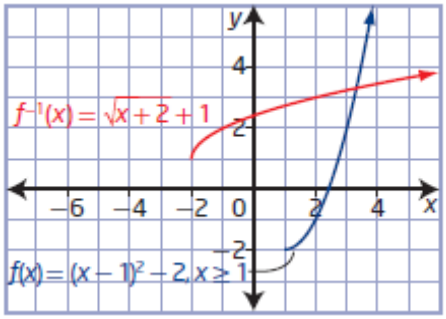
$y = \pm\sqrt{x+2} + 1$



Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola: $\{x \mid x \geq 1, x \in \mathrm{R}\}$.

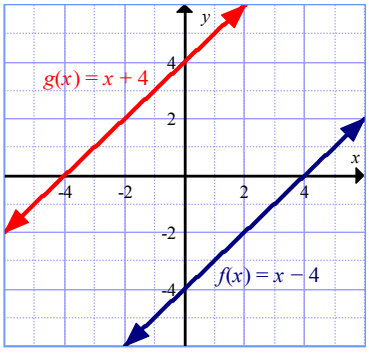
$f(x) = (x-1)^2 - 2, x \geq 1$ and

$f^{-1}(x) = \sqrt{x+2} + 1$

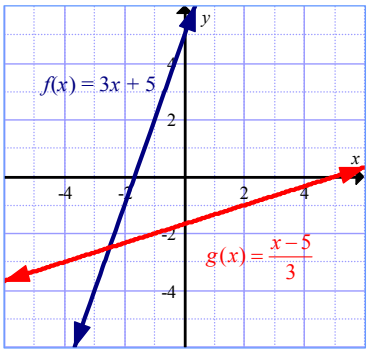


Section 1.4 Page 54 Question 13

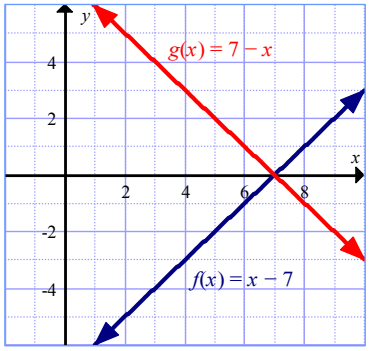
a) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



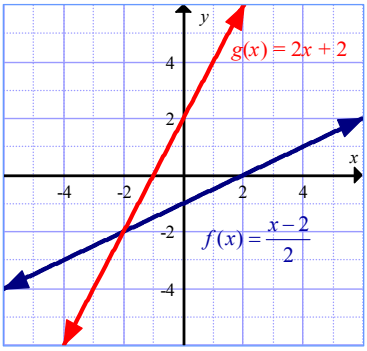
b) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



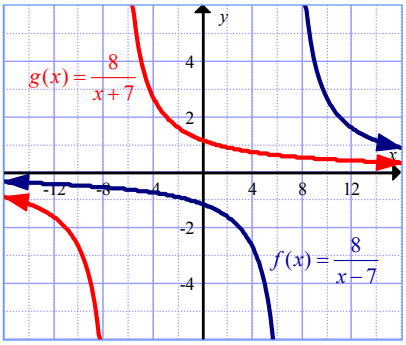
c) The graphs are not reflections of each other in the line $y = x$, so the functions are not inverses of each other.



d) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



e) The graphs are not reflections of each other in the line $y = x$, so the functions are not inverses of each other.



Section 1.4 Page 54 Question 14

Examples: A restricted domain for which these functions have an inverse that is also a function is the left or right half of the parabola.

a) For $f(x) = x^2 + 4$, the vertex is located at (0, 4). So, two ways to restrict the domain are $\{x \mid x \leq 0, x \in \mathrm{R}\}$ or $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

b) For $f(x) = 2 - x^2$, the vertex is located at (0, 2). So, two ways to restrict the domain are $\{x \mid x \leq 0, x \in \mathrm{R}\}$ or $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

c) For $f(x) = (x - 3)^2$, the vertex is located at (3, 0). So, two ways to restrict the domain are $\{x \mid x \leq 3, x \in \mathrm{R}\}$ or $\{x \mid x \geq 3, x \in \mathrm{R}\}$.

d) For $f(x) = (x + 2)^2 - 4$, the vertex is located at (–2, –4). So, two ways to restrict the domain are $\{x \mid x \leq -2, x \in \mathrm{R}\}$ or $\{x \mid x \geq -2, x \in \mathrm{R}\}$.

Section 1.4 Page 54 Question 15

$f(x) = 4x - 2$

$y = 4x - 2$

$x = 4y - 2$

$x + 2 = 4y$

$y = \frac{1}{4}(x + 2)$

$f^{-1}(x) = \frac{1}{4}(x + 2)$

a) Substitute $x = 4$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(4) = \frac{1}{4}(4 + 2)$

$f^{-1}(4) = \frac{3}{2}$

b) Substitute $x = -2$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(-2) = \frac{1}{4}(-2 + 2)$

$f^{-1}(-2) = 0$

c) Substitute $x = 8$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(8) = \frac{1}{4}(8 + 2)$

$f^{-1}(8) = \frac{5}{2}$

d) Substitute $x = 0$.

$f^{-1}(x) = \frac{1}{4}(x + 2)$

$f^{-1}(0) = \frac{1}{4}(0 + 2)$

$f^{-1}(0) = \frac{1}{2}$

Section 1.4 Page 54 Question 16

a) Substitute $x = 90$ into $y = \frac{5}{9}(x - 32)$.

$y = \frac{5}{9}(90 - 32)$

$y = \frac{5}{9}(58)$

$y \approx 32.22$

The equivalent temperature in degrees Celsius for 90 °F is approximately 32.22 °C.

b) For the inverse of the function, let x represent the temperature, in degrees Celsius, and y represent the temperature, in degrees Fahrenheit.

$$y = \frac{5}{9}(x - 32)$$

$$x = \frac{5}{9}(y - 32)$$

$$\frac{9}{5}x = y - 32$$

$$y = \frac{9}{5}x + 32$$

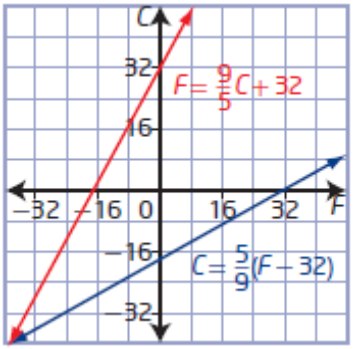
c) Substitute $x = 32$ into $y = \frac{9}{5}x + 32$.

$$y = \frac{9}{5}(32) + 32$$

$$y = 89.6$$

The equivalent temperature in degrees Fahrenheit for 32 °C is 89.6 °F.

d) The invariant point represents when the temperature is the same in both scales (–40 °C = –40 °F).



Section 1.4 Page 54 Question 17

a) Substitute $x = 45.47$ into each function.

For a male: $y = 2.32x + 65.53$

$y = 2.32(45.47) + 65.53$

$y = 171.0204$

For a female: $y = 2.47x + 54.13$

$y = 2.47(45.47) + 54.13$

$y = 166.4409$

The height of a male and of a female with a femur length of 45.47 cm is 171.02 cm and 166.44 cm, respectively.

b) For the inverse functions, let y represent the length of the femur and let x represent the height of the person, both in centimetres.

For a male: $y = 2.32x + 65.53$

$x = 2.32y + 65.53$

$x - 65.53 = 2.32y$

$$y = \frac{x - 65.53}{2.32}$$

For a female: $y = 2.47x + 54.13$

$x = 2.47y + 54.13$

$x - 54.13 = 2.47y$

$$y = \frac{x - 54.13}{2.47}$$

i) Substitute $x = 187.9$.

$$y = \frac{187.9 - 65.53}{2.32}$$

$y = 52.745\ldots$

The femur length of a male whose height is 187.9 cm is 52.75 cm.

ii) Substitute $x = 175.26$.

$$y = \frac{175.26 - 54.13}{2.47}$$

$y = 49.040\ldots$

The femur length of a female whose height is 175.26 cm is 49.04 cm.

Section 1.4 Page 54 Question 18

a) Substitute $x = 49.3$.

$$y = \frac{x - 36.5}{2.55}$$

$$y = \frac{49.3 - 36.5}{2.55}$$

$y \approx 5$

The whole-number ring size that corresponds to a finger circumference of 49.3 mm is 5.

b) For the inverse functions, let x represent the numerical ring size and let y represent finger circumference, in millimetres.

$$y = \frac{x - 36.5}{2.55}$$

$$x = \frac{y - 36.5}{2.55}$$

$2.55x = y - 36.5$

$y = 2.55x + 36.5$

c) Substitute ring sizes into $y = 2.55x + 36.5$.

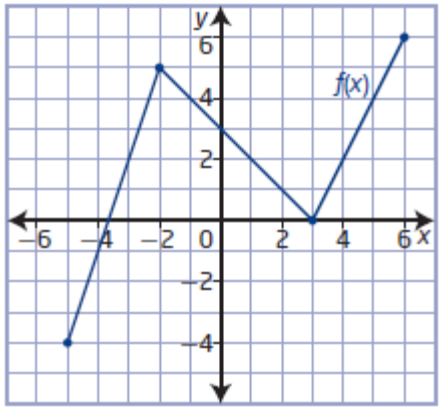
Substitute $x = 6$. $y = 2.55(6) + 36.5$ $y = 51.8$	Substitute $x = 7$. $y = 2.55(7) + 36.5$ $y = 54.35$	Substitute $x = 9$. $y = 2.55(9) + 36.5$ $y = 59.45$

The finger circumferences that correspond to ring sizes of 6, 7, and 9 are 51.8 mm, 54.35 mm, and 59.45 mm.

Section 1.4 Page 55 Question 19

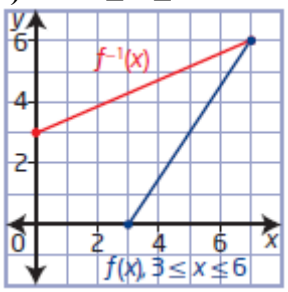
a) The function is increasing over the intervals $-5 \le x \le -2$ and $3 \le x \le 6$.

The function is decreasing over the interval $-2 \le x \le 3$.

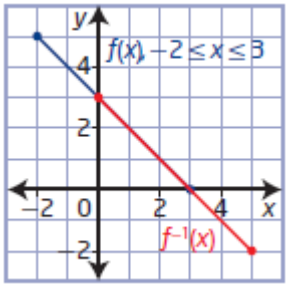


Example:

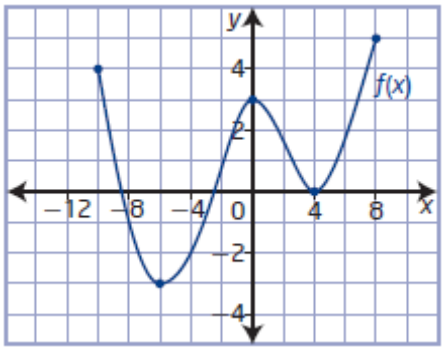
i) For $3 \le x \le 6$



ii) For $-2 \le x \le 3$

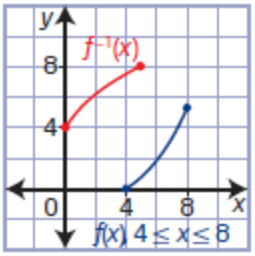


b) The function is increasing over the intervals $-6 \le x \le 0$ and $4 \le x \le 8$.
The function is decreasing over the intervals $-10 \le x \le -6$ and $0 \le x \le 4$.

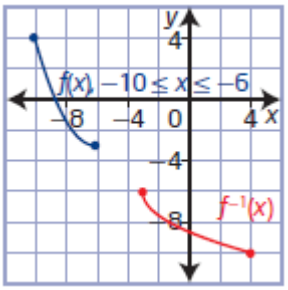


Example:

i) For $4 \le x \le 8$



ii) For $-10 \le x \le -6$



Section 1.4 Page 55 Question 20

The domain and range of $f(x)$ become the range and domain, respectively, of $f^{-1}(x)$.

a) If $f(17) = 5$, then $f^{-1}(5) = 17$.

b) If $f^{-1}\left(\sqrt{3}\right) = -2$, then $f(-2) = \sqrt{3}$.

c) Substitute $x = 1$ into $f(x) = 2x^2 + 5x + 3$, $x \ge -1.25$.
$f(1) = 2(1)^2 + 5(1) + 3$
$f(1) = 10$
If $f(1) = 10$, then $f^{-1}(10) = 1$ and $a = 10$.

Section 1.4 Page 55 Question 21

Given the point (10, 8) is on the graph of $y = f(x)$. Then, the point (8, 10) is on the graph of $y = f^{-1}(x)$.

a) The graph of $y = f^{-1}(x + 2)$ is a translation of 2 units to the left of the graph of $y = f^{-1}(x)$. The point (8, 10) on the graph of $y = f^{-1}(x)$ becomes (6, 10).

b) The graph of $y = 2f^{-1}(x) + 3$ is a vertical stretch by a factor of 2 and a translation of 3 units up of the graph of $y = f^{-1}(x)$. The point (8, 10) on the graph of $y = f^{-1}(x)$ becomes (8, 23).

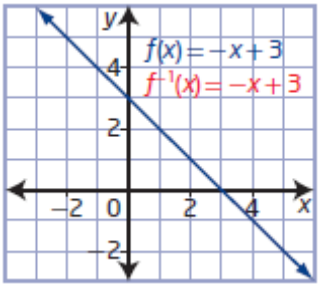
c) The graph of $y = -f^{-1}(-x) + 1$ is a reflection in the x-axis, a reflection in the y-axis, and a translation of 1 unit up of the graph of $y = f^{-1}(x)$. The point (8, 10) on the graph of $y = f^{-1}(x)$ becomes (–8, –9).

Section 1.4 Page 55 Question C1

a) The inverse sequence of operations for $f(x) = 6x + 12$ are subtract 12 and divide by 6.
b) The inverse sequence of operations for $f(x) = (x + 3)^2 - 1$ are add 1, take the positive and negative square root, and subtract 3.

Section 1.4 Page 55 Question C2

a) $f(x) = -x + 3$
$y = -x + 3$
$x = -y + 3$
$x - 3 = -y$
$y = -x + 3$
$f^{-1}(x) = -x + 3$



b) Example: The graph of the original linear function is perpendicular to $y = x$. So, after a reflection in the line $y = x$, the graph of the inverse is the same.

c) If a function and its inverse are the same, their graphs are perpendicular to the line $y = x$.

Section 1.4 Page 55 Question C3

Example: If the original function passes the vertical line test, then it is a function. If the original function passes the horizontal line test, then the inverse is a function.

Section 1.4 Page 55 Question C4

Step 1

x	$f(x) = \frac{x+5}{3}$
1	2
4	3
–8	–1
a	$\frac{a+5}{3}$

x	$g(x) = 3x - 5$
2	1
3	4
–1	–8
$\frac{a+5}{3}$	a

The output values of $g(x)$ are the same as the input values for $f(x)$. This occurs because the functions are inverses of each other.
Example: Use the output values of one function as the input values for second function. If the output values for that second function are the same as the input values of the first function, then the two functions are inverses of each other.

Step 2

x	$g(x) = 3x - 5$
1	–2
4	7
–8	–29
a	$3a - 5$

x	$f(x) = \frac{x+5}{3}$
–2	1
7	4
–29	–8
$3a - 5$	a

If two functions are inverses, then the order in which you apply them does not change the final result.

Step 3 Example:

x	$f(x) = x + 2$
1	3
4	6
–8	–6
a	$a + 2$

x	$g(x) = x - 2$
3	1
6	4
–6	–8
$a + 2$	a

x	$g(x) = x - 2$
1	–1
4	2
–8	–10
a	$a - 2$

x	$f(x) = x + 2$
–1	1
2	4
–10	–8
$a - 2$	a

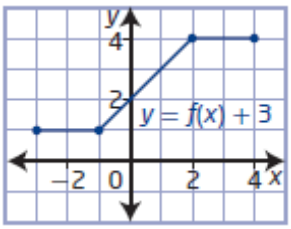
The hypothesis and conclusion from steps 1 and 2 hold.

Step 4 The statement is saying that if you have a function that when given a outputs b and another that when given b outputs a, then the functions are inverses of each other.

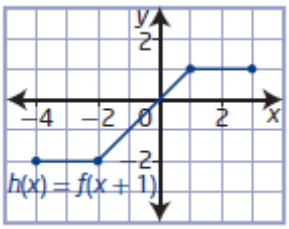
Chapter 1 Review

Chapter 1 Review Page 56 Question 1

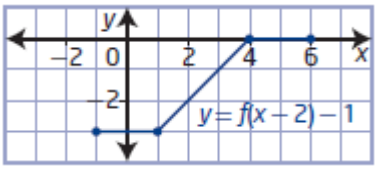
a) For the graph of $y - 3 = f(x)$, translate the graph of $y = f(x)$ up 3 units.



b) For the graph of $h(x) = f(x + 1)$, translate the graph of $y = f(x)$ to the left 1 unit.



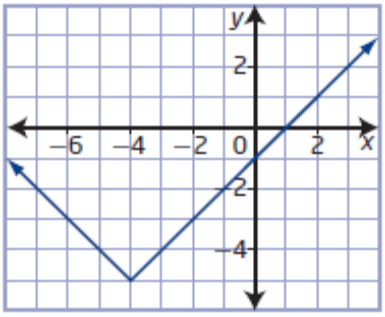
c) For the graph of $y + 1 = f(x - 2)$, translate the graph of $y = f(x)$ to the right 2 units and down 1 unit.



Chapter 1 Review Page 56 Question 2

To obtain the graph of the function shown, translate the graph of $y = |x|$ to the left 4 units and down 5 units.

The equation of the transformed function is $y + 5 = |x + 4|$.



Chapter 1 Review Page 56 Question 3

For $y = f(x - 2) + 4$, $h = 2$ and $k = 4$, representing a translation of 2 units to the right and 4 units up. If the range of the function $y = f(x)$ is $\{y \mid -2 \leq y \leq 5, y \in \mathrm{R}\}$, then the range of $y = f(x - 2) + 4$ will be $\{y \mid 2 \leq y \leq 9, y \in \mathrm{R}\}$.

Chapter 1 Review Page 56 Question 4

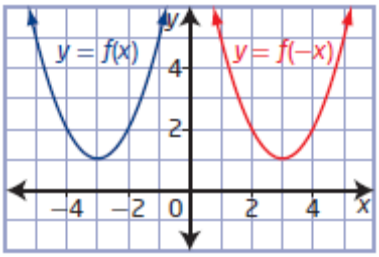
James is incorrect. If the point (a, b) is on the graph of $y = f(x)$, then the point $(a + 5, b - 4)$ is the image point on the graph of $y + 4 = f(x - 5)$, not $(a - 5, b + 4)$.
He should have compared $y + 4 = f(x - 5)$ to the form $y - k = f(x - h)$.
$y - (-4) = f(x - (+5))$

Chapter 1 Review Page 56 Question 5

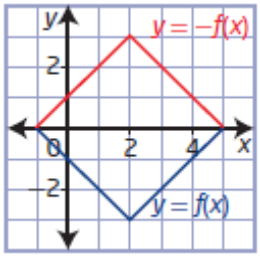
a) When the graph of $y = f(x)$ is transformed to $y = -f(x)$ it is reflected in the x-axis. The key point (3, 5) becomes the image point (3, –5).

b) When the graph of $y = f(x)$ is transformed to $y = f(-x)$ it is reflected in the y-axis. The key point (3, 5) becomes the image point (–3, 5).

Chapter 1 Review Page 56 Question 6
a) $y = f(-x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
The invariant point is (0, 10).

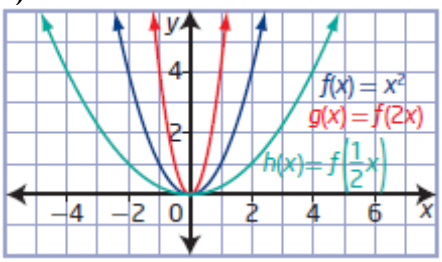


b) $y = -f(x)$: domain $\{x \mid -1 \le x \le 5, x \in \mathrm{R}\}$,
range $\{y \mid 0 \le y \le 3, y \in \mathrm{R}\}$
The invariant points are $(-1, 0)$ and $(5, 0)$.



Chapter 1 Review Page 56 Question 7

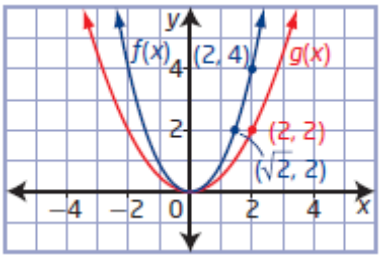
a)



b) If the coefficient of x is greater than 1, then the graph of the transformed function moves closer to the y-axis. If the coefficient of x is between 0 and 1, then the graph of the transformed function moves farther from the y-axis.

Chapter 1 Review Page 56 Question 8

a) Compare a key point on the graph of $f(x)$ to the image point on the graph of $g(x)$.
Case 1: Pattern in the y-coordinates
$f(x)$ $g(x)$
$(2, 4) \rightarrow (2, 2)$
$(x, y) \rightarrow \left(x, \frac{1}{2}y\right)$



This indicates a vertical stretch about the x-axis by a factor of $\frac{1}{2}$.

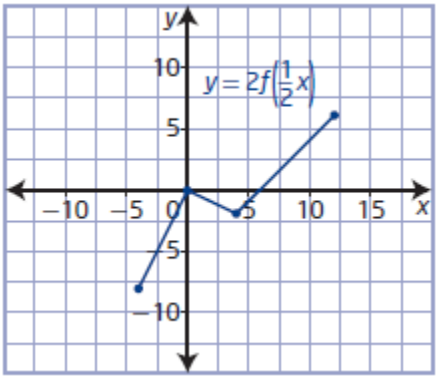
Case 2: Pattern in the x-coordinates
$f(x)$ $g(x)$
$\left(\sqrt{2}, 2\right) \rightarrow (2, 2)$
$(x, y) \rightarrow \left(\sqrt{2}x, y\right)$

This indicates a horizontal stretch about the y-axis by a factor of $\sqrt{2}$.
The graph of $g(x)$ could be a horizontal or a vertical stretch of the graph of $f(x)$.

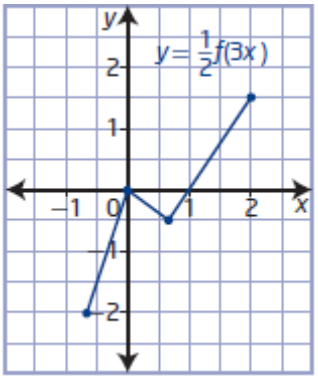
b) Example: $g(x) = \frac{1}{2}f(x)$

Chapter 1 Review Page 57 Question 9

a) The graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 2 and stretched vertically about the x-axis by a factor of 2.



b) The graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of $\frac{1}{3}$ and stretched vertically about the x-axis by a factor of $\frac{1}{2}$.



Chapter 1 Review Page 57 Question 10

Rewrite $y = f(4x + 1)$ as $y = f(4(x + 0.25))$.
Compare $y = f(4(x + 1))$ and $y = f(4(x + 0.25))$.

Both transformations represent a horizontal stretch about the y-axis by a factor of $\frac{1}{4}$.

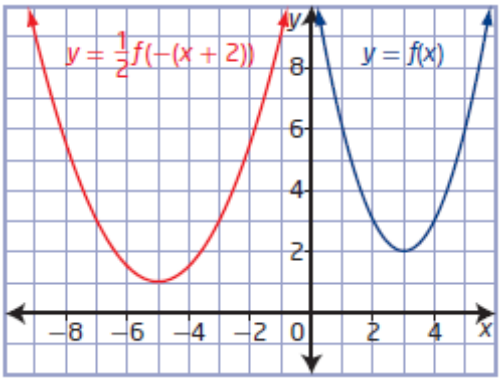
Both have also been translated to the left, but by different amounts; the first is 1 unit left and the second is 0.25 units left.

Chapter 1 Review Page 57 Question 11

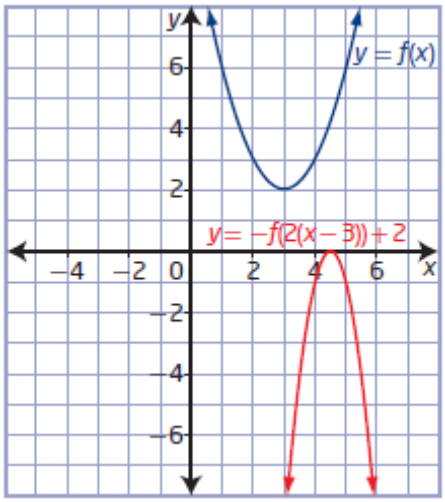
Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-2, 2) \rightarrow (4, 0)$
$(0, 0) \rightarrow (5, -2)$
$(2, 2) \rightarrow (6, 0)$
The orientation is unchanged, the graph has not been reflected.
The overall width has changed, so the graph has been horizontally stretched by a factor of 0.5. Since the point (0, 0) is not affected by stretches, the graph has been translated 5 units to the right and 2 units down.
So, $a = 1$, $b = 2$, $h = 5$, $k = -2$, and the equation of the transformed graph is $g(x) = f(2(x - 5)) - 2$.

Chapter 1 Review Page 57 Question 12

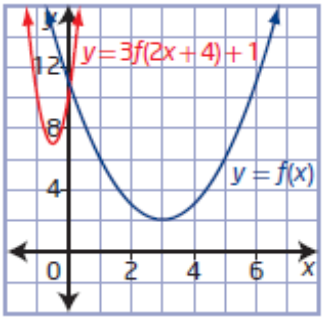
a) For $y = \frac{1}{2}f(-(x + 2))$, $a = \frac{1}{2}$, $b = -1$, $h = -2$, and $k = 0$. This represents a reflection in the y-axis, a vertical stretch about the x-axis by a factor of $\frac{1}{2}$, and a translation of 2 units to the left of the graph of $y = f(x)$.



b) For $y - 2 = -f(2(x - 3))$, $a = -1$, $b = 2$, $h = 3$, and $k = 2$. This represents a reflection in the x-axis, a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, and a translation of 3 units to the right and 2 units up of the graph of $y = f(x)$.

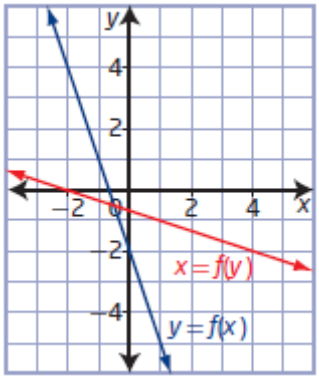


c) For $y - 1 = 3f(2x + 4)$ or $y - 1 = 3f(2(x + 2))$, $a = 3$, $b = 2$, $h = -2$, and $k = 1$. This represents a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a vertical stretch about the x-axis by a factor of 3, and a translation of 2 units to the left and 1 unit up of the graph of $y = f(x)$.



Chapter 1 Review Page 57 Question 13

a)



b) To obtain the graph of $x = f(y)$, reflect the graph of $y = f(x)$ in the line $y = x$. The invariant point is on the line of reflection, (–0.5, –0.5).

c) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f(y)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

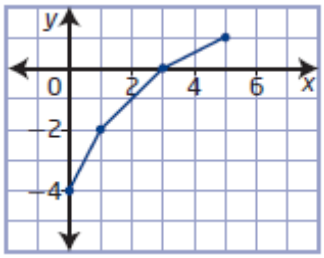
Chapter 1 Review Page 57 Question 14

Interchange the x- and y-coordinates.

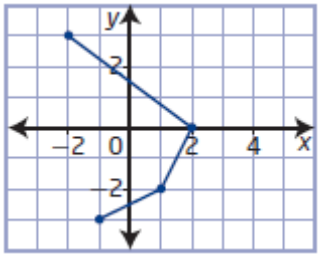
$y = f(x)$		$y = f^{-1}(x)$	
x	y	x	y
–3	7	7	–3
2	4	4	2
10	–12	–12	10

Chapter 1 Review Page 57 Question 15

a) Reflect the given graph in the line $y = x$. Both the relation and its inverse are functions, since the relation passes both the vertical line test and the horizontal line test.



b) Reflect the given graph in the line $y = x$. The relation is a function but its inverse is not, since the relation passes the vertical line test but not the horizontal line test.



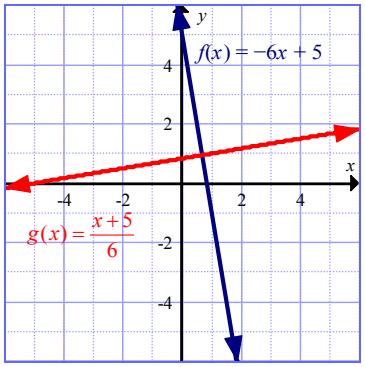
Chapter 1 Review Page 57 Question 16

$$y = (x-3)^2 + 1$$
$$x = (y-3)^2 + 1$$
$$\pm\sqrt{x-1} = y-3$$
$$y = \pm\sqrt{x-1} + 3$$

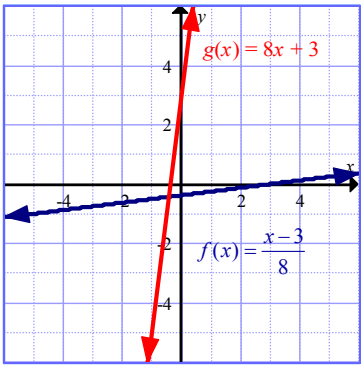
Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola.
For $y = (x-3)^2 + 1$, the vertex is located at (3, 1). So, restrict the domain to $\{x \mid x \geq 3, x \in \mathrm{R}\}$.
$y = (x-3)^2 + 1, x \geq 3$ and $y = \sqrt{x-1} + 3$

Chapter 1 Review Page 57 Question 17

a) Since the graphs are not reflections of each other in the line $y = x$, the functions are not inverses of each other.



b) Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



Chapter 1 Practice Test

Chapter 1 Practice Test Page 58 Question 1

The equation $y = (x+1)^2$ represents a translation of 1 unit to the left of the graph $y = x^2$.
The graph is the same shape but translated to the left: choice **D**.

Chapter 1 Practice Test Page 58 Question 2

Since the point (0, 0) is not affected by stretches, the graph of $y = |x|$ has been translated 4 units to the left and 6 units down.
So, $h = -4$, $k = -6$, and the equation of the transformed graph is $y + 6 = |x + 4|$: choice **D**.

Chapter 1 Practice Test Page 58 Question 3

The graph of $y = f(x + 2)$ is a translation of 2 units to the left of the graph of $y = f(x)$. The point (a, b) becomes $(a - 2, b)$: choice **B**.

Chapter 1 Practice Test Page 58 Question 4

A reflection in the y-axis is represented by $y = f(-x)$. So, the image of $y = x^2 + 2$ is $y = x^2 + 2$: choice **B**.

Chapter 1 Practice Test Page 58 Question 5

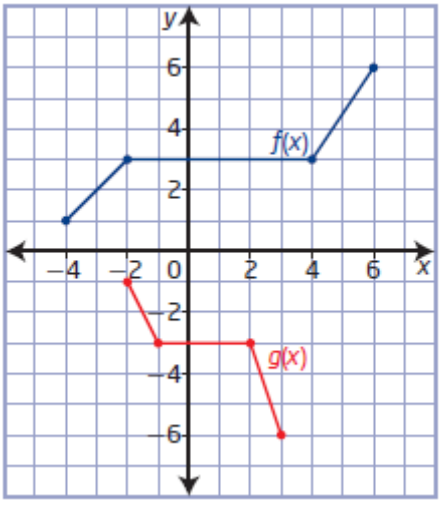
The graph of $y = \frac{1}{4}f(3x)$ is a vertical stretch by a factor of $\frac{1}{4}$ and a horizontal stretch by a factor of $\frac{1}{3}$: choice **B**.

Chapter 1 Practice Test Page 58 Question 6

The location of the y-intercept will not be affected by a horizontal stretch. The graph of $f(-9x)$ will have the same y-intercept as the graph of $f(x)$: choice **C**.

Chapter 1 Practice Test Page 58 Question 7

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-4, 1) \rightarrow (-2, -1)$
$(-2, 3) \rightarrow (-1, -3)$
$(4, 3) \rightarrow (2, -3)$
$(6, 6) \rightarrow (3, -6)$
The orientation is changed, the graph has been reflected in the x-axis. The overall width has changed, so the graph has been horizontally stretched by a factor of $\frac{1}{2}$. So, $a = -1$, $b = 2$, and the equation of the transformed graph is $g(x) = -f(2x)$: choice **C**.

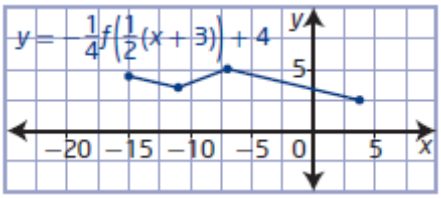


Chapter 1 Practice Test Page 59 Question 8

For $y = f(x + 2) - 1$, $h = -2$ and $k = -1$, representing a translation of 2 units to the left and 1 unit down. If the domain of the function $y = f(x)$ is $\{x \mid -3 \leq x \leq 4, x \in \mathrm{R}\}$, then the domain of $y = f(x + 2) - 1$ will be $\{x \mid -5 \leq x \leq 2, x \in \mathrm{R}\}$.

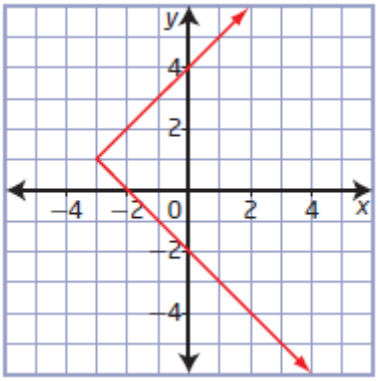
Chapter 1 Practice Test Page 59 Question 9

The graph of $y = f(x)$ is stretched horizontally about the y-axis by a factor of 2, stretched vertically by a factor of $\frac{1}{4}$, reflected in the x-axis, and translated 3 units to the left and 4 units up.



Chapter 1 Practice Test Page 59 Question 10

a)



b) The image points of an inverse are determined by interchanging the x- and y-coordinates of the key points.

c) Any invariant points lie on the line of reflection: $(-1, -1)$.

Chapter 1 Practice Test Page 59 Question 11

$y = 5x + 2$

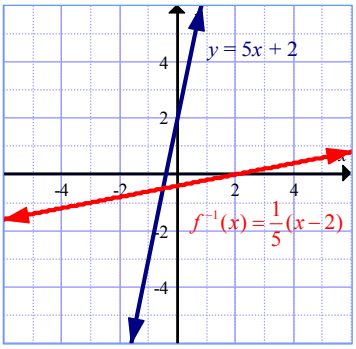
$x = 5y + 2$

$x - 2 = 5y$

$y = \frac{1}{5}(x - 2)$

$f^{-1}(x) = \frac{1}{5}(x - 2)$

Since the graphs are reflections of each other in the line $y = x$, the functions are inverses of each other.



Chapter 1 Practice Test Page 59 Question 12

For a horizontal stretch about the y-axis by a factor of 2, a horizontal reflection in the y-axis, a vertical stretch about the x-axis by a factor of 3, and a horizontal translation of 2 units to the right, $a = 3$, $b = -\frac{1}{2}$, $h = 2$, and $k = 0$. The equation of the transformed function is $y = 3f\left(-\frac{1}{2}(x-2)\right)$.

Chapter 1 Practice Test Page 59 Question 13

a) For $g(x) = f(x + 2) - 7$, $h = -2$ and $k = -7$. The graph of $f(x) = |x|$ will be translated 2 units to the left and 7 units down.

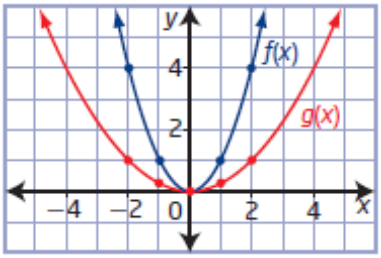
b) The equation of the transformed function is $f(x) = |x + 2| - 7$.

c) The minimum value of $g(x)$ occurs at the vertex of the V-shaped graph, $(-2, -7)$.

d) Invariant points are points that are not affected by a transformation. In this case, the points on the graph of $f(x)$ are related to the corresponding points on the graph of $g(x)$ by the mapping $(x, y) \rightarrow (x - 2, y - 7)$. So, while the domains are the same, each point is affected by the transformation, so there are no invariant points.

Chapter 1 Practice Test Page 59 Question 14

a) The equation of the base function is $f(x) = x^2$.



b) For $g(x) = af(x)$, determine the pattern in the y-coordinates of corresponding points: $a = \frac{1}{4}$. So, the equation of the transformed function is $g(x) = \frac{1}{4}f(x)$, which is a vertical stretch about the x-axis by a factor of $\frac{1}{4}$.

c) For $g(x) = f(bx)$, determine the pattern in the x-coordinates of corresponding points: $b = 2$. So, the equation of the transformed function is $g(x) = f\left(\frac{1}{2}x\right)$, which is a horizontal stretch about the y-axis by a factor of 2.

d) $g(x) = \frac{1}{4}f(x)$ $\qquad$ $g(x) = f\left(\frac{1}{2}x\right)$

$g(x) = \frac{1}{4}x^2$ $\qquad$ $g(x) = \left(\frac{1}{2}x\right)^2$

$g(x) = \frac{1}{4}x^2$

The two forms are equivalent.

Chapter 1 Practice Test Page 59 Question 15

a) If the graph of $h(x)$ passes the horizontal line test, then the inverse of $h(x)$ will be a function. A horizontal line would pass through the quadratic function $h(x)$ twice, so its inverse is not a function.

b) $h(x) = -(x + 3)^2 - 5$

$y = -(x + 3)^2 - 5$

$x = -(y + 3)^2 - 5$

$\pm\sqrt{-x - 5} = y + 3$

$y = \pm\sqrt{-x - 5} - 3$

c) Example: A restricted domain for which the function has an inverse that is also a function is right half of the parabola.
For $h(x) = -(x + 3)^2 - 5$, the vertex is located at $(-3, -5)$. So, restrict the domain to $\{x \mid x \geq -3, x \in \mathrm{R}\}$.

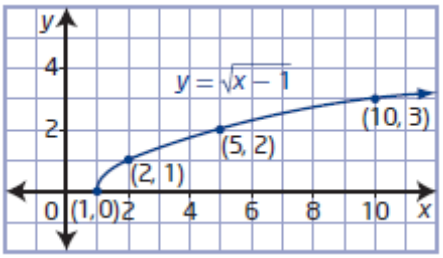
Chapter 2 Radical Functions

Section 2.1 Radical Functions and Transformations

Section 2.1 Page 72 Question 1

a)

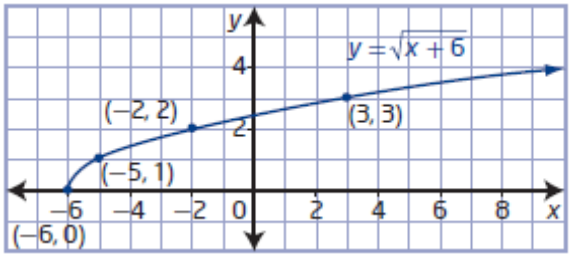
x	$y = \sqrt{x-1}$
1	0
2	1
5	2
10	3



domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)

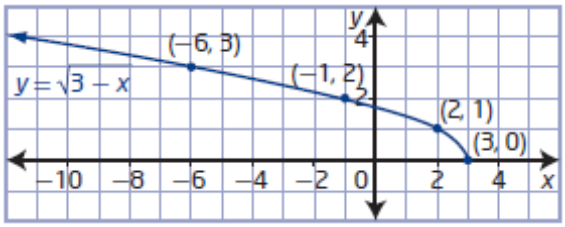
x	$y = \sqrt{x+6}$
–6	0
–5	1
–2	2
3	3



domain
$\{x \mid x \geq -6, x \in \mathrm{R}\}$
range
$\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)

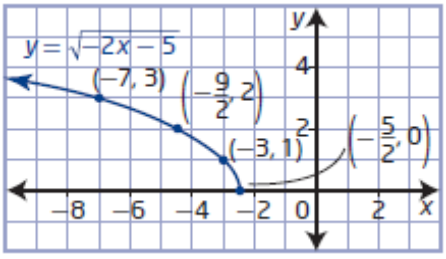
x	$y = \sqrt{3-x}$
3	0
2	1
–1	2
–6	3



domain
$\{x \mid x \leq 3, x \in \mathrm{R}\}$
range
$\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)

x	$y = \sqrt{-2x-5}$
3	0
2	1
–1	2
–6	3



domain
$\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$
range
$\{y \mid y \geq 0, y \in \mathrm{R}\}$

Section 2.1 Page 72 Question 2

a) For $y = 7\sqrt{x-9}$, $a = 7$, $b = 1$, $h = 9$, and $k = 0$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 7 and translated 9 units to the right.
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) For $y = \sqrt{-x} + 8$, $a = 1$, $b = -1$, $h = 0$, and $k = 8$. The graph of $y = \sqrt{x}$ is reflected in the y-axis and translated 8 units up.
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 8, y \in \mathrm{R}\}$

c) For $y = -\sqrt{0.2x}$, $a = -1$, $b = 0.2$, $h = 0$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis and stretched horizontally by a factor of 5.
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d) For $4 + y = \frac{1}{3}\sqrt{x+6}$, $a = \frac{1}{3}$, $b = 1$, $h = -6$, and $k = -4$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of $\frac{1}{3}$ and translated 6 units to the left and 4 units down.
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 4, y \in \mathrm{R}\}$

Section 2.1 Page 72 Question 3

a) For $y = \sqrt{x} - 2$, $a = 1$, $b = 1$, $h = 0$, and $k = -2$. The graph of $y = \sqrt{x}$ is translated 2 units down: graph **B**.

b) For $y = \sqrt{-x} + 2$, $a = 1$, $b = -1$, $h = 0$, and $k = 2$. The graph of $y = \sqrt{x}$ is reflected in the y-axis and translated 2 units up: graph **A**.

c) For $y = -\sqrt{x+2}$, $a = -1$, $b = 1$, $h = -2$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis and translated 2 units to the left: graph **D**.

d) For $y = -\sqrt{-(x-2)}$, $a = -1$, $b = -1$, $h = 2$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis and in the y-axis and translated 2 units to the right: graph **C**.

Section 2.1 Page 73 Question 4

a) For a vertical stretch by a factor of 4 and a horizontal translation of 6 units left, $a = 4$, $h = -6$, and the equation of the transformed function is $y = 4\sqrt{x+6}$.

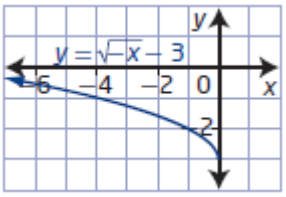
b) For a horizontal stretch by a factor of $\frac{1}{8}$ and a vertical translation of 5 units down, $b = 8$, $k = -5$, and the equation of the transformed function is $y = \sqrt{8x} - 5$.

c) For a horizontal reflection in the y-axis, a horizontal translation of 4 units right, and a vertical translation of 11 units up, $b = -1$, $h = 4$, $k = 11$, and the equation of the transformed function is $y = \sqrt{-(x-4)} + 11$.

d) For a vertical stretch by a factor of 0.25, a vertical reflection in the x-axis, and a horizontal stretch by a factor of 10, $a = -0.25$, $b = \frac{1}{10}$, and the equation of the transformed function is $y = -0.25\sqrt{\frac{1}{10}x}$.

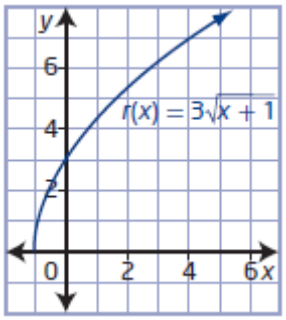
Section 2.1 Page 73 Question 5

a)



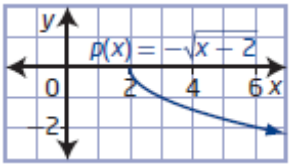
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$,
range $\{y \mid y \geq -3, y \in \mathrm{R}\}$

b)



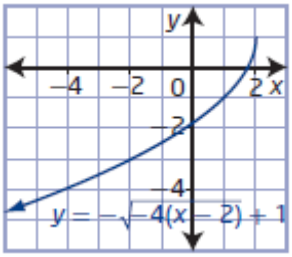
domain $\{x \mid x \geq -1, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



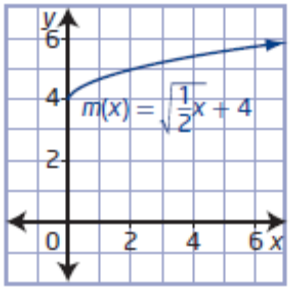
domain $\{x \mid x \geq 2, x \in \mathrm{R}\}$,
range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d)



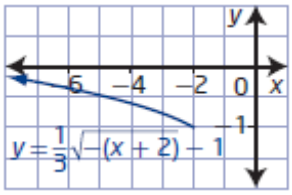
domain $\{x \mid x \leq 2, x \in \mathrm{R}\}$,
range $\{y \mid y \leq 1, y \in \mathrm{R}\}$

e)



domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 4, y \in \mathrm{R}\}$

f)



domain $\{x \mid x \leq -2, x \in \mathrm{R}\}$,
range $\{y \mid y \geq -1, y \in \mathrm{R}\}$

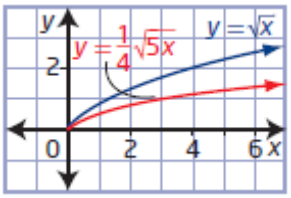
Section 2.1 Page 73 Question 6

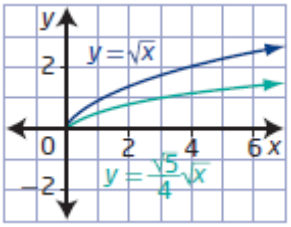
a) For $f(x) = \frac{1}{4}\sqrt{5x}$, $a = \frac{1}{4}$ and $b = 5$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of $\frac{1}{4}$ and horizontally stretched by a factor of $\frac{1}{5}$.

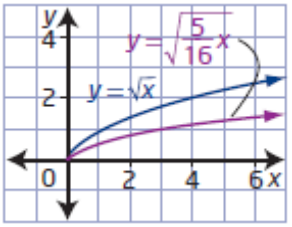
b) Two functions equivalent to $f(x)$: $y = \frac{\sqrt{5}}{4}\sqrt{x}$ and $y = \sqrt{\frac{5}{16}x}$.

c) The function $y = \frac{\sqrt{5}}{4}\sqrt{x}$ represents a vertical stretch by a factor of $\frac{\sqrt{5}}{4}$, and the function $y = \sqrt{\frac{5}{16}x}$ represents a horizontal stretch by a factor of $\frac{16}{5}$.

d) The graphs are the same.







Section 2.1 Page 73 Question 7

a) The radius, r, of a circle as a function of area, A:

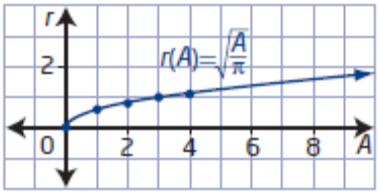
$A = \pi r^2$

$\frac{A}{\pi} = r^2$

$r = \sqrt{\frac{A}{\pi}}$

b)

A	r
0	0
1	0.6
2	0.8
3	1.0
4	1.1



Section 2.1 Page 73 Question 8

a) For $d = \sqrt{1.50h}$, $b = 1.50$. The graph of $d = \sqrt{h}$ is horizontally stretched by a factor of $\frac{1}{1.50}$ or $\frac{2}{3}$.

b) An approximate equivalent function of the form $d = a\sqrt{h}$ is $d \approx 1.22\sqrt{h}$.
Example: I prefer the original function because the values are exact.

c) Substitute $h = 20$ into $d = \sqrt{1.50h}$.
$d = \sqrt{1.50(20)}$
$d = 5.477\ldots$
The lifeguard can see approximately 5.5 miles.

Section 2.1 Page 73 Question 9

a) The function $y = -\sqrt{3x} + 4$ has domain $\{x \mid x \geq 0, x \in \text{R}\}$ and range $\{y \mid y \leq 4, y \in \text{R}\}$.
After it is translated 9 units up and reflected in the x-axis, the domain remains as $\{x \mid x \geq 0, x \in \text{R}\}$ but the range becomes $\{y \mid y \geq -13, y \in \text{R}\}$.

b) Compared to the base function $y = \sqrt{x}$, the transformed function has not been translated horizontally but has been translated vertically 13 units down.

Section 2.1 Page 74 Question 10

a) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (-3, 4)$
$(1, 1) \rightarrow (-2, 3)$
$(4, 2) \rightarrow (1, 2)$
$(9, 3) \rightarrow (6, 1)$
The overall width and height have not changed, so the graph has not been vertically or horizontally stretched. From the general shape, the graph has been reflected in the x-axis. From the endpoint, the graph has been translated 3 units to the left and 4 units up. So, $a = -1$, $b = 1$, $h = -3$, $k = 4$, and the equation of the transformed graph is $y = -\sqrt{x+3} + 4$.

b) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (-5, -3)$
$(1, 1) \rightarrow (-1, -2)$
$(4, 2) \rightarrow (11, -1)$

The overall width has changed: the graph has been horizontally stretched by a factor of 4. From the general shape, the graph has not been reflected. From the endpoint, the graph has been translated 5 units to the left and 3 units down. So, $a = 1$, $b = \frac{1}{4}$, $h = -5$, $k = -3$, and the equation of the transformed graph is $y = \sqrt{\frac{1}{4}(x+5)} - 3$ or alternatively $y = \frac{1}{2}\sqrt{x+5} - 3$.

c) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (5, -1)$
$(1, 1) \rightarrow (4, 1)$
$(4, 2) \rightarrow (1, 3)$
$(3, 9) \rightarrow (-4, 5)$
The overall height has changed: the graph has been vertically stretched by a factor of 2. From the general shape, the graph has been reflected in the y-axis. From the endpoint, the graph has been translated 5 units to the right and 1 unit down. So, $a = 2$, $b = -1$, $h = 5$, $k = -1$, and the equation of the transformed graph is $y = 2\sqrt{-(x-5)} - 1$ or alternatively $y = \sqrt{-4(x-5)} - 1$.

d) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (4, 5)$
$(1, 1) \rightarrow (3, 1)$
$(4, 2) \rightarrow (0, -3)$
$(3, 9) \rightarrow (-5, -7)$
The overall height has changed: the graph has been vertically stretched by a factor of 4. From the general shape, the graph has been reflected in both the x-axis and the y-axis. From the endpoint, the graph has been translated 4 units to the right and 5 units up. So, $a = -4$, $b = -1$, $h = 4$, $k = 5$, and the equation of the transformed graph is $y = -4\sqrt{-(x-4)} + 5$ or alternatively $y = -\sqrt{-16(x-5)} - 1$.

Section 2.1 Page 74 Question 11

Compare each given domain and range to the those of the base function $y = \sqrt{x}$:
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
Examples:
a) For domain $\{x \mid x \geq 6, x \in \mathrm{R}\}$, the function has been translated 6 units to the right: $h = 6$. For range $\{y \mid y \geq 1, y \in \mathrm{R}\}$, the function has been translated 1 unit up: $k = 1$. Then, the equation of the radical function is $y = \sqrt{x-6} + 1$.

b) For domain $\{x \mid x \geq -7, x \in \mathrm{R}\}$, the function has been translated 7 units to the left: $h = -7$. For range $\{y \mid y \leq -9, y \in \mathrm{R}\}$, the function has been translated 9 units down and reflected in the x-axis: $k = -9$ and $a = -1$. Then, the equation of the radical function is $y = -\sqrt{x+7} - 9$.

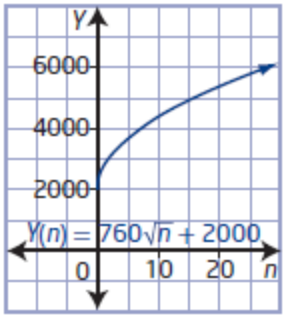
c) For domain $\{x \mid x \leq 4, x \in \mathrm{R}\}$, the function has been translated 4 units to the right and reflected in the y-axis: $h = 4$ and $b = -1$. For range $\{y \mid y \geq -3, y \in \mathrm{R}\}$, the function has been translated 3 units down: $k = -3$. Then, the equation of the radical function is $y = \sqrt{-(x-4)} - 3$.

d) For domain $\{x \mid x \leq -5, x \in \mathrm{R}\}$, the function has been translated 5 units to the left and reflected in the y-axis: $h = -5$ and $b = -1$. For range $\{y \mid y \leq 8, y \in \mathrm{R}\}$, the function has been translated 8 units up and reflected in the x-axis: $k = 8$ and $a = -1$. Then, the equation of the radical function is $y = -\sqrt{-(x+5)} + 8$.

Section 2.1 Page 74 Question 12

a) For $Y(n) = 760\sqrt{n} + 2000$, $a = 760$, $b = 1$, $h = 0$, and $k = 2000$. The graph will be vertically stretched by a factor of 760 and translated 2000 units up.

b)



c) domain $\{n \mid n \geq 0, n \in \mathrm{R}\}$, range $\{Y \mid Y \geq 2000, Y \in \mathrm{R}\}$

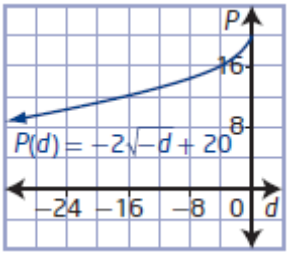
d) Example: The minimum yield is 2000 kg/hectare. The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.

Section 2.1 Page 75 Question 13

a) For $P(d) = -2\sqrt{-d} + 20$, the domain is $\{d \mid -100 \leq d \leq 0, d \in \mathrm{R}\}$ and the range is $\{P \mid 0 \leq P \leq 20, n \in \mathrm{R}\}$. The domain is negative indicating days remaining before the phone's release, and the maximum value of P, the number of pre-orders, is 20 million.

b) For $P(d) = -2\sqrt{-d} + 20$, $a = -2$, $b = -1$, $h = 0$, and $k = 20$. The graph will be vertically stretched by a factor of 2, reflected in the d-axis, reflected in the P-axis, and translated 20 units up.

c)



Since d represents the number of days remaining before release, d is negative. The function has a maximum of 20 million pre-orders.

d) Substitute $d = -30$.

$P(d) = -2\sqrt{-(-30)} + 20$

$P(d) = 9.045\ldots$

The manufacturer can expect to have approximately 9.05 million pre-orders.

Section 2.1 Page 75 Question 14

a) The graph shows that the polling error decreases as the election approaches.

b) Since the graph starts on the origin, there are no translations: $h = 0$ an $k = 0$. Since the graph is reflected in the y-axis, $b = -1$. So, the equation is of the form $y = a\sqrt{-x}$. Use the coordinates of a point, say (–150, 6) to determine the value of a.

$6 = a\sqrt{-(-150)}$

$$a = \frac{6}{\sqrt{150}}$$

$a \approx 0.49$

c) The function has been stretched vertically by a factor of approximately 0.49 and reflected in the y-axis.

Section 2.1 Page 75 Question 15

Since the graph is reflected in the y-axis, $b = -1$. So, the equation is of the form $y = a\sqrt{-x}$. Use the coordinates of a point, say (–5.4, 4.8) to determine the value of a.

$4.8 = a\sqrt{-(-5.4)}$

$$a = \frac{4.8}{\sqrt{5.4}}$$

$a \approx 2.07$

An equation to represent the shape of the greenhouse roof is $y = 2.07\sqrt{-x}$.

Section 2.1 Page 75 Question 16

Examples:

a) Since the endpoint is (2, 5), $h = 2$ and $k = 5$.

Try an equation of the form $y = a\sqrt{x-2}+5$. Use the coordinates of the point (6, 1) to determine the value of a.

$$1 = a\sqrt{6-2}+5$$

$$-4 = 2a$$

$$a = -2$$

The equation of the function is $y = -2\sqrt{x-2}+5$.

b) Since the endpoint is (3, –2), $h = 3$ and $k = -2$.

Try an equation of the form $y = a\sqrt{x-3}-2$. Use the coordinates of the point (–6, 0) to determine the value of a.

$$0 = a\sqrt{-6-3}-2$$

$$2 = a\sqrt{-9}$$

This is not possible as you cannot have a negative radicand.

Try an equation of the form $y = \sqrt{b(x-3)}-2$. Use the coordinates of the point (–6, 0) to determine the value of b.

$$0 = \sqrt{b(-6-3)}-2$$

$$2 = \sqrt{-9b}$$

$$4 = -9b$$

$$b = -\frac{4}{9}$$

The equation of the function is $y = \sqrt{-\frac{4}{9}(x-3)}-2$ or $y = \frac{2}{3}\sqrt{(3-x)}-2$.

Section 2.1 Page 76 Question 17

a) China, India, and USA might feel that the "one nation, one vote" system is unfair. The larger the country, the more unfair the this system becomes. Tuvalu, Nauru, Vatican City might feel that the "one person, one vote" system is unfair. The smaller the nation, the more unfair this system becomes.

b) For each country, calculate $\frac{\text{Population}}{\text{World Population}} \times 100\%$.

Nation	Percentage
China	18.6%
India	17.1%
US	4.5%
Canada	0.48%
Tuvalu	0.000 151%
Nauru	0.000 137%
Vatican City	0.000 014%

c) The Penrose system can be represented by the function $V(x) = \frac{1}{1000}\sqrt{x}$.

d) For each country, calculate $\frac{V(x)}{765} \times 100\%$.

Nation	Percentage
China	4.82%
India	4.62%
US	2.36%
Canada	0.77%
Tuvalu	0.014%
Nauru	0.013%
Vatican City	0.004%

e) Example: The Penrose system gives larger nations votes based on population but also provides an opportunity for smaller nations to provide influence.

Section 2.1 Page 76 Question 18

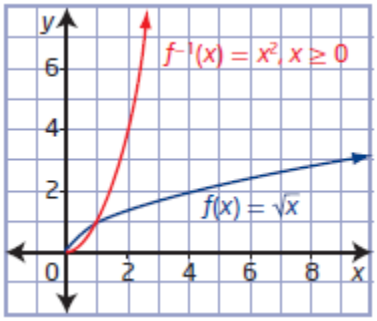
Answers may vary.
Step 2 and Step 3 In general, the periods of the pendulums should decrease as the length of the string decreases.

Step 4 In general, the graph should resemble the graph of $y = a\sqrt{x}$.

Step 5 The graph should be a vertical stretch by a factor of approximately 0.2 of the graph of $y = \sqrt{x}$. A function that approximates the graph is $T = 0.2\sqrt{L}$.

Section 2.1 Page 77 Question 19

a) The domain and range of a function become the range and domain, respectively, of the inverse of the function. The positive domain of the inverse is the same as the range of the original function.



b) i) $g(x) = -\sqrt{x-5}$, $x \geq 5$

$y = -\sqrt{x-5}$

$x = -\sqrt{y-5}$

$x^2 = y - 5$

$y = x^2 + 5$

$g^{-1}(x) = x^2 + 5, x \leq 0$

ii) $h(x) = \sqrt{-x} + 3$, $x \leq 0$

$$y = \sqrt{-x} + 3$$
$$x = \sqrt{-y} + 3$$
$$x - 3 = \sqrt{-y}$$
$$(x-3)^2 = -y$$
$$y = -(x-3)^2$$
$$h^{-1}(x) = -(x-3)^2,\ x \geq 3$$

iii) $j(x) = \sqrt{2x-7} - 6,\ x \geq \frac{7}{2}$

$$y = \sqrt{2x-7} - 6$$
$$x = \sqrt{2y-7} - 6$$
$$x + 6 = \sqrt{2y-7}$$
$$(x+6)^2 = 2y - 7$$
$$y = \frac{1}{2}(x+6)^2 + \frac{7}{2}$$
$$j^{-1}(x) = \frac{1}{2}(x+6)^2 + \frac{7}{2},\ x \geq -6$$

Section 2.1 Page 77 Question 20

For $f(x) = \frac{5}{8}\sqrt{-\frac{7}{12}x}$, the key points on the graph of $y = \sqrt{x}$ are changed by the mapping $(x, y) \rightarrow \left(-\frac{12}{7}x, \frac{5}{8}y\right)$.

For $g(x) = -\frac{2}{5}\sqrt{6(x+3)} - 4$, the key points on the graph of $y = \sqrt{x}$ are changed by the mapping $(x, y) \rightarrow \left(\frac{1}{6}x - 3, -\frac{2}{5}y - 4\right)$.

To transform the graph of $f(x)$ to create the graph of $g(x)$, determine the mapping needed to transform points $\left(-\frac{12}{7}x, \frac{5}{8}y\right)$ to $\left(\frac{1}{6}x - 3, -\frac{2}{5}y - 4\right)$.

$$-\frac{7}{72}\left(-\frac{12}{7}x\right) - 3$$
$$= \frac{1}{6}x - 3$$

Apply a horizontal stretch by a factor of $\frac{7}{72}$, a reflection in the y-axis, and a translation of 3 units to the left.

$$-\frac{16}{25}\left(\frac{5}{8}y\right) - 4$$
$$= -\frac{2}{5}y - 4$$

Apply a vertical stretch by a factor of $\frac{16}{25}$, a reflection in the x-axis, and a translation of 4 units down.

Section 2.1 Page 77 Question C1

For a radical function $y = a\sqrt{b(x-h)}+k$, the parameters b and h affect the domain. The radicand must be greater than or equal to zero.

Case 1: $b > 0$

$b(x-h) \geq 0$

$x-h \geq 0$

$x \geq h$

Case 2: $b < 0$

$b(x-h) \geq 0$

$x-h \leq 0$

$x \leq h$

For example, $y = \sqrt{x}$ has domain $x \geq 0$ but $y = \sqrt{2(x-3)}$ has domain $x \geq 3$ and $y = \sqrt{-2(x-3)}$ has domain $x \leq 3$.

For a radical function $y = a\sqrt{b(x-h)}+k$, the parameters a and k affect the range. The graph of a radical function starts with an endpoint whose value of x makes $b(x-h) = 0$. So, the range restriction value is determined by the value of k and the direction of the inequality symbol is determined by the value of a.

For example, $y = \sqrt{x}$ has range $y \geq 0$ but $y = 2\sqrt{x}-4$ has range $y \geq -4$ and $y = -2\sqrt{x}-4$ has range $y \leq -4$.

Section 2.1 Page 77 Question C2

Yes, any given radical function can be simplified so that there is no value of b, only a value of a.

$y = a\sqrt{bx}$

$y = \left(a\sqrt{b}\right)\sqrt{x}$

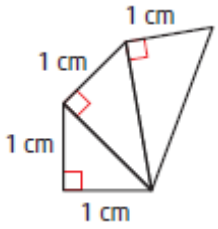
For example, $y = 2\sqrt{9x}$ can be simplified to $y = 6\sqrt{x}$.

Section 2.1 Page 77 Question C3

The processes are similar because the parameters a, b, h, and k have the same effect on radical functions and quadratic functions. The processes are different because the base functions are different: one is the shape of a parabola and the other is the shape of half of a parabola.

Section 2.1 Page 77 Question C4

Steps 1–4



Triangle Number, n	Length of Hypotenuse, L
First	$\sqrt{2} = 1.414...$
Second	$\sqrt{3} = 1.732...$
Third	$\sqrt{4} = 2$

Step 5 The function that represents the hypotenuse length as it relates to its triangle number is $L = \sqrt{n+1}$. This represents the translation of 1 unit to the left of the base square root function.

Section 2.2 Square Root of a Function

Section 2.2 Page 86 Question 1

$f(x)$	$\sqrt{f(x)}$
36	6
0.09	0.3
1	1
–9	undefined
2.56	1.6
0	0

Section 2.2 Page 86 Question 2

a) For (4, 12), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(4, \sqrt{12}\right)$ or approximately (4, 3.46).

b) For (–2, 0.4), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(-2, \sqrt{0.4}\right)$ or approximately (–2, 0.63).

c) For (10, –2), there is no corresponding point on the graph of $y = \sqrt{f(x)}$ because $\sqrt{-2}$ does not exist.

d) For (0.09, 1), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(0.09, \sqrt{1}\right)$ or (0.09, 1).

e) For (–5, 0), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(-5, \sqrt{0}\right)$ or (–5, 0).

f) For (m, n), the corresponding point on the graph of $y = \sqrt{f(x)}$ is $\left(m, \sqrt{n}\right)$ for $n \geq 0$.

Section 2.2 Page 86 Question 3

a) Since the graph of $y = f(x)$ is on or above the x-axis for $x \leq -2$ or $x \geq 2$, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **C**.

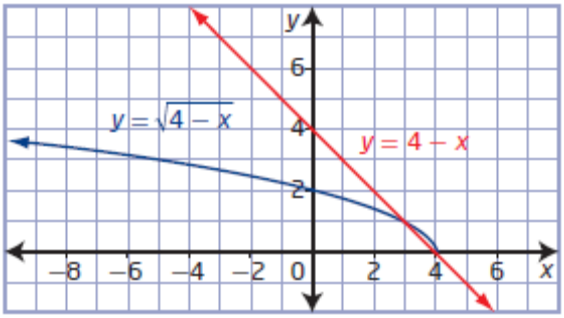
b) Since the graph of $y = f(x)$ is on or above the x-axis for $-2 \le x \le 2$, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **D**.

c) Since the graph of $y = f(x)$ is above the x-axis for all values of x, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **A**.

d) Since the graph of $y = f(x)$ is on or above the x-axis for $-4 \le x \le 4$, the graph of $y = \sqrt{f(x)}$ exists only for these values: graph **B**.

Section 2.2 Page 87 Question 4

a)

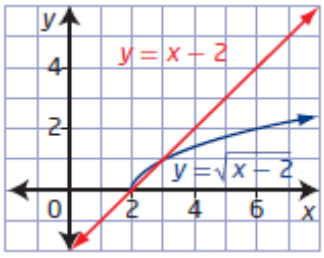


b) The graph of $y = \sqrt{4-x}$ exists for values where the graph of $y = 4 - x$ is on or above the x-axis, $x \le 4$. The graph of $y = \sqrt{4-x}$ is above the graph of $y = 4 - x$ for $3 < x < 4$, and the graph of $y = \sqrt{4-x}$ is below the graph of $y = 4 - x$ for $x > 3$.

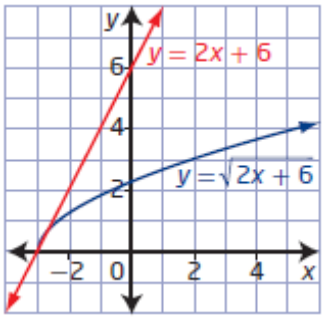
c) For $y = 4 - x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{4-x}$, the domain is $\{x \mid x \le 4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{4-x}$ is undefined for $x > 4$.
The ranges differ because $y = \sqrt{4-x}$ is undefined for $y < 0$.

Section 2.2 Page 87 Question 5

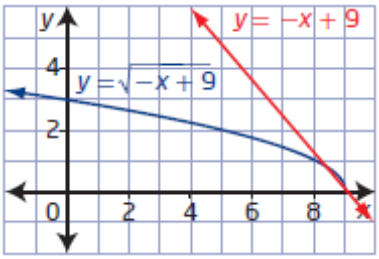
a) For $y = x - 2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{x-2}$, the domain is $\{x \mid x \ge 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x-2}$ is undefined for $x < 2$.
The ranges differ because $y = \sqrt{x-2}$ is undefined for $y < 0$.



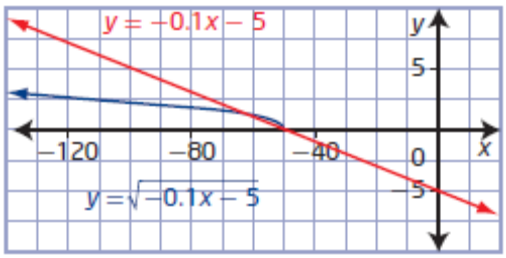
b) For $y = 2x + 6$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{2x+6}$, the domain is $\{x \mid x \geq -3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{2x+6}$ is undefined for $x < -3$. The ranges differ because $y = \sqrt{2x+6}$ is undefined for $y < 0$.



c) For $y = -x + 9$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{-x+9}$, the domain is $\{x \mid x \leq 9, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{-x+9}$ is undefined for $x > 9$. The ranges differ because $y = \sqrt{-x+9}$ is undefined for $y < 0$.



d) For $y = -0.1x - 5$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{-0.1x-5}$, the domain is $\{x \mid x \leq 50, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{-0.1x-5}$ is undefined for $x > 50$. The ranges differ because $y = \sqrt{-0.1x-5}$ is undefined for $y < 0$.



Section 2.2 Page 87 Question 6

a) For $y = x^2 - 9$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2-9}$, the domain is $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x^2-9}$ is undefined for $-3 < x < 3$. The ranges differ because $y = \sqrt{x^2-9}$ is undefined for $y < 0$.

b) For $y = 2 - x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 2, y \in \mathrm{R}\}$.
For $y = \sqrt{2-x^2}$, the domain is $\{x \mid -\sqrt{2} \leq x \leq \sqrt{2}, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq \sqrt{2}, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{2-x^2}$ is undefined for $x < -\sqrt{2}$ and $x > \sqrt{2}$.
The ranges differ because $y = \sqrt{2-x^2}$ is undefined for $y < 0$ and has a maximum value of $\sqrt{2}$.

c) For $y = x^2 + 6$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 6, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 + 6}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{6}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = x^2 + 6$ is above the x-axis.
The ranges differ because $y = \sqrt{x^2 + 6}$ has a minimum value of $\sqrt{6}$.

d) For $y = 0.5x^2 + 3$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 3, y \in \mathrm{R}\}$.
For $y = \sqrt{0.5x^2 + 3}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{3}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = 0.5x^2 + 3$ is above the x-axis.
The ranges differ because $y = \sqrt{0.5x^2 + 3}$ has a minimum value of $\sqrt{3}$.

Section 2.2 Page 87 Question 7

a) For $y = x^2 - 25$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -25, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 25}$, the domain is $\{x \mid x \leq -5 \text{ and } x \geq 5, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x^2 - 25}$ is undefined for $-5 < x < 5$. The ranges differ because $y = \sqrt{x^2 - 9}$ is undefined for $y < 0$.

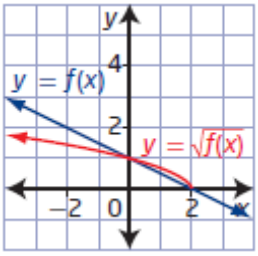
b) For $y = x^2 + 3$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 3, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 + 3}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{3}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = x^2 + 3$ is above the x-axis.
The ranges differ because $y = \sqrt{x^2 + 3}$ has a minimum value of $\sqrt{3}$.

c) For $y = 32 - 2x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 32, y \in \mathrm{R}\}$.
For $y = \sqrt{32 - 2x^2}$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq \sqrt{32}, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{2 - x^2}$ is undefined for $x < -\sqrt{2}$ and $x > \sqrt{2}$.
The ranges differ because $y = \sqrt{2 - x^2}$ is undefined for $y < 0$ and has a maximum value of $\sqrt{2}$.

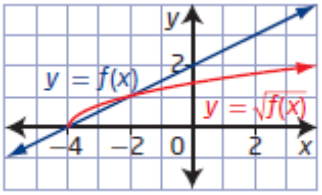
d) For $y = 5x^2 + 50$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 50, y \in \mathrm{R}\}$.
For $y = \sqrt{5x^2 + 50}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{50}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = 5x^2 + 50$ is above the x-axis.
The ranges differ because $y = \sqrt{5x^2 + 50}$ has a minimum value of $\sqrt{50}$.

Section 2.2 Page 87 Question 8

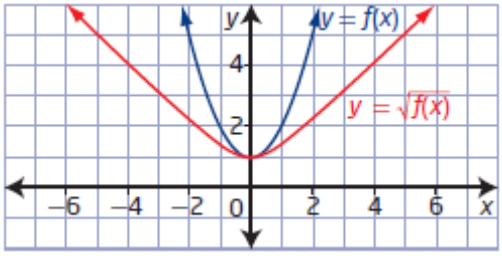
a)



b)



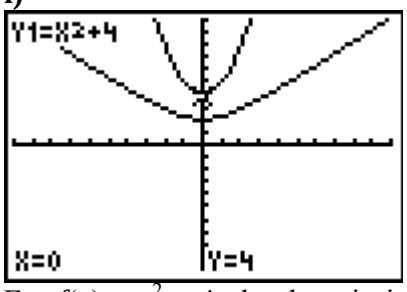
c)



Section 2.2 Page 87 Question 9

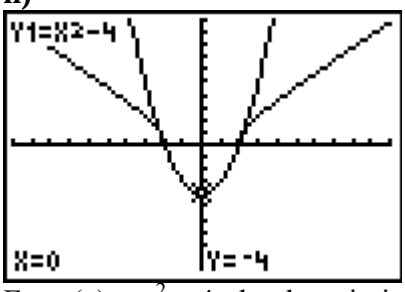
a) and **b)**

i)



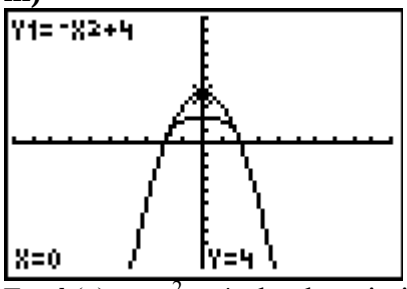
For $f(x) = x^2 + 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 4, y \in \mathrm{R}\}$.

ii)



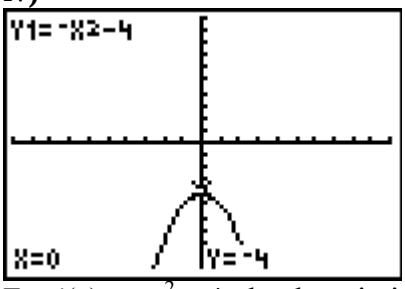
For $g(x) = x^2 - 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -4, y \in \mathrm{R}\}$.

iii)



For $h(x) = -x^2 + 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 4, y \in \mathrm{R}\}$.

iv)



For $j(x) = -x^2 - 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq -4, y \in \mathrm{R}\}$.

c) The graph of $y = \sqrt{j(x)}$ does not exist. This is because the entire graph of $y = j(x)$ lies below the x-axis. For $j(x) = -x^2 - 4$, there are no values of x for which $\sqrt{-x^2 - 4}$ is defined.

$-x^2 - 4 \geq 0$

$-x^2 \geq 4$

$x^2 \leq -4$

This is never true.

d) The domains of the square root of a function are the same as the domains of the function when the value of the function is greater than or equal to zero. The domains of the square root of a function do not exist when the value of the function is less than zero. The ranges of the square root of a function are the square root of the range of the original function, except when the value of the function is less than zero then the range is undefined.

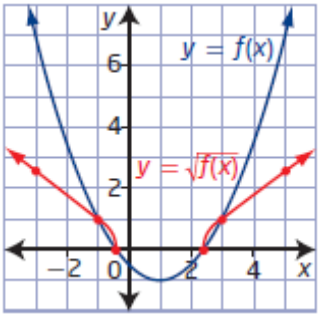
Section 2.2 Page 87 Question 10

a) For $y = x^2 - 4$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -4, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 4}$, the domain is $\{x \mid x \leq -2 \text{ and } x \geq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) The function $y = \sqrt{x^2 - 4}$ is undefined for $-2 < x < 2$ because $y = x^2 - 4$ is less than zero. So, the domain does not contain this interval.

Section 2.2 Page 87 Question 11

a) I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.



b) For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$.
Determine the equation of the parabola: $y = \frac{1}{2}(x-1)^2 - 1$ or $y = \frac{1}{2}x^2 - x - \frac{1}{2}$.
Use the quadratic formula or graphing technology to determine the zeros: $x = 1 \pm \sqrt{2}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1 - \sqrt{2} \text{ and } x \geq 1 + \sqrt{2}, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{f(x)}$ is undefined for $1 - \sqrt{2} < x < 1 + \sqrt{2}$. The ranges differ because $y = \sqrt{f(x)}$ is undefined for $y < 0$.

Section 2.2 Page 88 Question 12

a) $d^2 = (h + 6378)^2 - 6378^2$
$d^2 = h^2 + 12\,756h + 6378^2 - 6378^2$
$d^2 = h^2 + 12\,756h$

$d = \sqrt{h^2 + 12\,756h}$

The equation for the distance, in kilometres, to the horizon is $d = \sqrt{h^2 + 12\,756h}$.

b) The domain is $\{h \mid h \geq 0, h \in \mathrm{R}\}$ and the range is $\{d \mid d \geq 0, d \in \mathrm{R}\}$.

c) You can use a graph to find the distance to the horizon for a satellite that is 800 km above Earth's surface by finding the d-coordinate of the point where the h-coordinate is 800. In this case, d is approximately 3293. The distance to the horizon for this satellite is approximately 3293 km.

d) If this function is not in context then the domain is $\{h \mid h \leq -12\,756 \text{ or } h \geq 0, h \in \mathrm{R}\}$ and the range remains as $\{d \mid d \geq 0, d \in \mathrm{R}\}$.

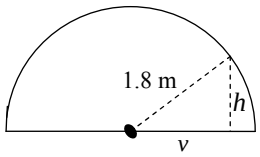
Section 2.2 Page 88 Question 13

a) No, I do not agree with Chris. The function $y = \sqrt{f(x)}$ is undefined for $f(x) < 0$, where $f(x)$ represents the range of $y = f(x)$, not the domain.

b) To determine whether a graph shows the function or the square root of the function, check for points below the x-axis. If there are such points, the graph is of the function. If there are not, then it could be a graph of the square root of the function.

Section 2.2 Page 88 Question 14

a) The radius of the iglu is 1.8 m. Any point on the iglu will be at the end of the hypotenuse of a right triangle, in which $r^2 = h^2 + v^2$. So, $v^2 = 1.8^2 - h^2$. A function that gives the vertical height in terms of the horizontal distance from the centre is $v = \sqrt{3.24 - h^2}$.



b) The domain is $\{h \mid 0 \leq h \leq 1.8, h \in \mathrm{R}\}$ and the range is $\{v \mid 0 \leq v \leq 1.8, v \in \mathrm{R}\}$. Both variables represent distances and must be non-negative.

c) 1 m in from the bottom edge of the wall: $h = 0.8$

$v = \sqrt{3.24 - 0.8^2}$

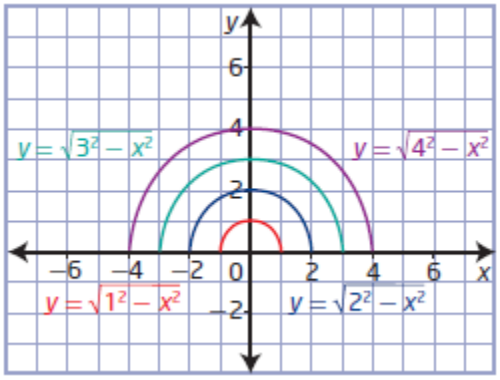
$v = \sqrt{2.6}$

$v = 1.612\ldots$

The height of the iglu at a point 1 m from the bottom edge is approximately 1.61 m.

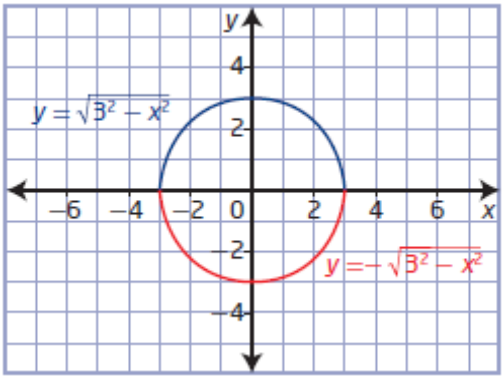
Section 2.2 Page 88 Question 15

Step 1 Example:



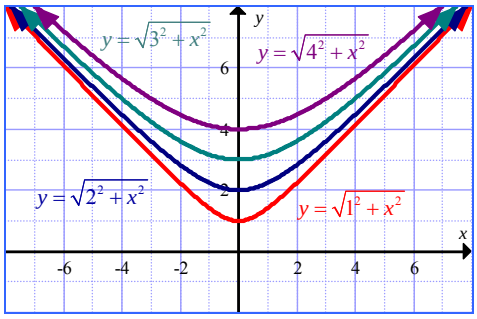
Step 2 The value of a determines the minimum value and the maximum value of the domain: $\{x \mid -a \le x \le a, x \in \mathrm{R}\}$. It also determines the maximum value of the range: $\{y \mid 0 \le y \le a, y \in \mathrm{R}\}$.

Step 3 Example: Choose $a = 3$. The equation for the function reflected in the x-axis is $y = -\sqrt{3^2 - x^2}$.

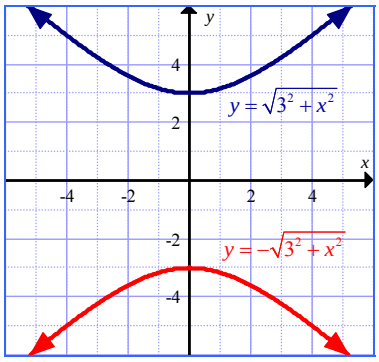


The graph forms a circle.

Step 4 Example: The value of a has no effect on the domain: $\{x \mid x \in \mathrm{R}\}$. It does determine the minimum value of the range: $\{y \mid y \ge a, y \in \mathrm{R}\}$.



Choose $a = 3$. The equation for the function reflected in the x-axis is $y = -\sqrt{3^2 + x^2}$.



Section 2.2 Page 89 Question 16

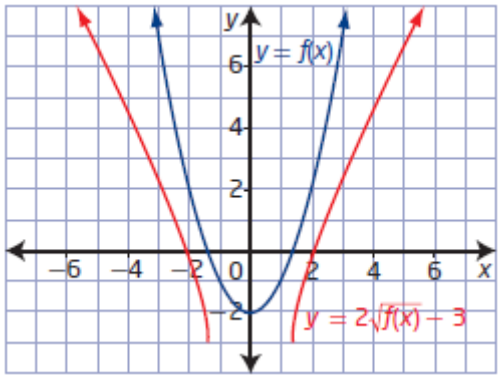
a) For $y = 4f(x + 3)$, $a = 4$, $b = 1$, $h = -3$, and $k = 0$. For $y = \sqrt{4f(x+3)}$, use the mapping $(x, y) \rightarrow \left(x-3, \sqrt{4y}\right)$. So, the point (–24, 12) becomes (–27, $\sqrt{48}$) or (–27, $4\sqrt{3}$).

b) For $y = f(4x)$, $a = 1$, $b = 4$, $h = 0$, and $k = 0$. For $y = -\sqrt{f(4x)} + 12$, use the mapping $(x, y) \rightarrow \left(\frac{1}{4}x, -\sqrt{y} + 12\right)$. So, the point (–24, 12) becomes (–6, $-\sqrt{12} + 12$) or (–27, $-2\sqrt{3} + 12$).

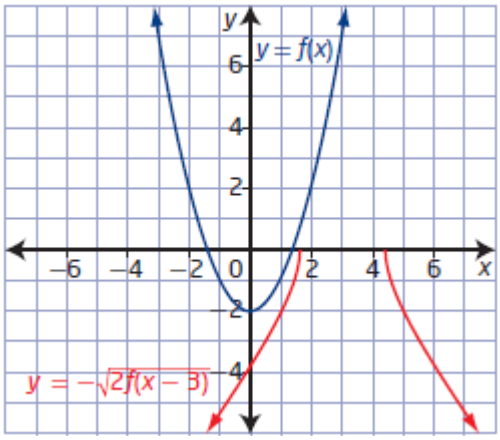
c) For $y = f(-(x - 2)) - 4$, $a = 1$, $b = -1$, $h = 2$, and $k = -4$. For $y = -2\sqrt{f(-(x-2)) - 4} + 6$, use the mapping $(x, y) \rightarrow \left(-x + 2, -2\sqrt{y} + 6\right)$. So, the point (–24, 12) becomes (26, $-2\sqrt{12} + 6$) or (26, $-4\sqrt{3} + 6$).

Section 2.2 Page 89 Question 17

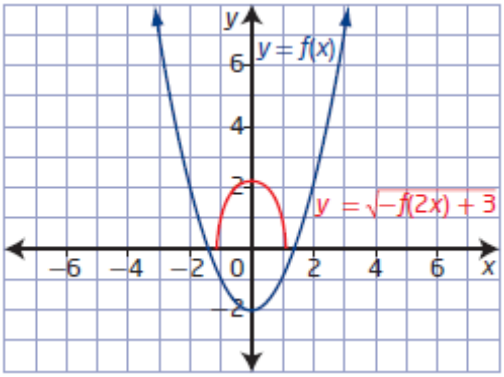
a) For $y = 2\sqrt{f(x)} - 3$, use the mapping $(x, y) \rightarrow \left(x, 2\sqrt{y} - 3\right)$.



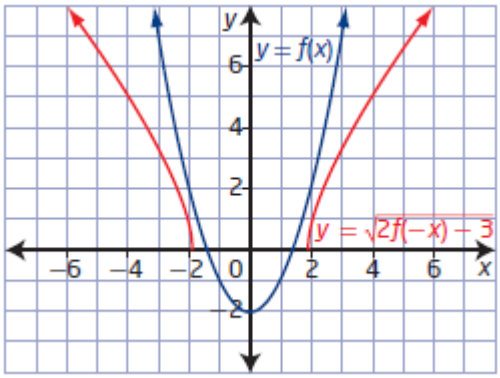
b) For $y = -\sqrt{2f(x-3)}$, use the mapping $(x, y) \rightarrow \left(x + 3, -\sqrt{2y}\right)$.



c) For $y = \sqrt{-f(2x)+3}$, use the mapping $(x, y) \rightarrow \left(\frac{1}{2}x, \sqrt{-y+3}\right)$.



d) For $y = \sqrt{2f(-x)-3}$, use the mapping $(x, y) \rightarrow \left(-x, \sqrt{2y-3}\right)$.



Section 2.2 Page 89 Question 18

Example: First sketch the graph of $y = 2f(x - 3)$. Next, sketch the graph of $y = \sqrt{2f(x-3)}$. Finally, reflect in the x-axis to get the graph of $y = -\sqrt{2f(x-3)}$.

Section 2.2 Page 89 Question 19

a) A formula for radius as a function of the surface area, SA, of a cylinder with equal diameter and height, $h = 2r$, is

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi r^2 + 2\pi r(2r)$

$SA = 6\pi r^2$

$$\frac{SA}{6\pi} = r^2$$

$$r = \sqrt{\frac{SA}{6\pi}}$$

b) A formula for radius as a function of the surface area, SA, of a cone with height three times its diameter, $h = 6r$, is

$$SA = \pi r\sqrt{h^2 + r^2} + \pi r^2$$

$$SA = \pi r\sqrt{(6r)^2 + r^2} + \pi r^2$$

$$SA = \pi r\sqrt{37r^2} + \pi r^2$$

$$SA = \pi r^2\left(\sqrt{37} + 1\right)$$

$$\frac{SA}{\pi\left(\sqrt{37}+1\right)} = r^2$$

$$r = \sqrt{\frac{SA}{\pi\left(\sqrt{37}+1\right)}}$$

Section 2.2 Page 89 Question C1

Example: Choose key points on the graph of $y = f(x)$. Transform the points using the mapping $(x, y) \rightarrow \left(x, \sqrt{y}\right)$. Plot the new points and smooth out the graph.

Section 2.2 Page 89 Question C2

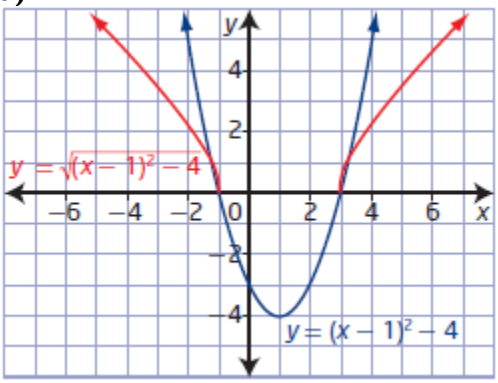
Example: The graph of $y = 16 - 4x$ is a line with slope –4, y-intercept 16, and x-intercept 4. The graph of $y = \sqrt{16-4x}$ only exists when the graph of $y = 16 - 4x$ is on or above the x-axis. The y-intercept is 4 and the x-intercept stays the same. For $x \leq 4$, the points are related by the mapping $(x, y) \rightarrow \left(x, \sqrt{y}\right)$.

Section 2.2 Page 89 Question C3

Example: No, it is not possible to completely graph the function $y = f(x)$ given only the graph of $y = \sqrt{f(x)}$. The graph of $y = f(x)$ may exist when $y < 0$ but the graph of $y = \sqrt{f(x)}$ does not.

Section 2.2 Page 89 Question C4

a)



b) Example: The function $y = (x-1)^2 - 4$ has domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \geq -4, y \in \mathrm{R}\}$.
The function $y = \sqrt{(x-1)^2 - 4}$ has domain $\{x \mid x \leq -1 \text{ and } x \geq 3, x \in \mathrm{R}\}$ and range $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The points on the graph of $y = (x-1)^2 - 4$ that are on or above the x-axis become points $\left(x, \sqrt{y}\right)$ on the graph of $y = \sqrt{(x-1)^2 - 4}$.

Section 2.3 Solving Radical Equations Graphically

Section 2.3 Page 96 Question 1

a) The equation $2+\sqrt{x+4}=4$ can be solved by graphing the single function $y=-2+\sqrt{x+4}$: choice **B**.

b) The equation $x-4=\sqrt{x+4}$ can be solved by graphing the single function $y=x-4-\sqrt{x+4}$: choice **A**.

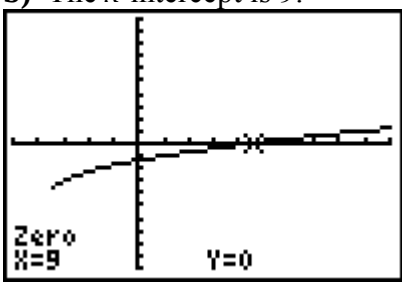
c) The equation $2=\sqrt{x+4}-4$ can be solved by graphing the single function $y=\sqrt{x+4}-6$: choice **D**.

d) The equation $\sqrt{x+4}+2=x+6$ can be solved by graphing the single function $y=\sqrt{x+4}-4-x$: choice **C**.

Section 2.3 Page 96 Question 2

a)
$$\begin{aligned}\sqrt{x+7}-4&=0\\ \sqrt{x+7}&=4\\ x+7&=16\\ x&=9\end{aligned}$$

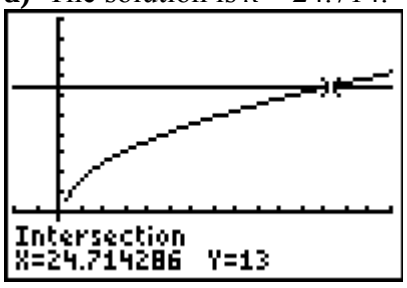
b) The *x*-intercept is 9.



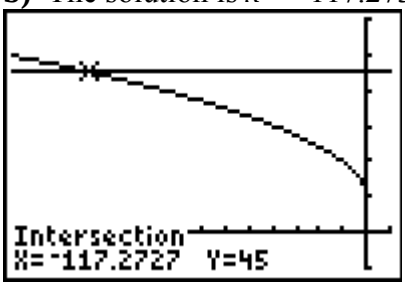
c) The solutions or roots of a radical equation are equivalent to the *x*-intercept(s) of the graph of the corresponding radical function.

Section 2.3 Page 96 Question 3

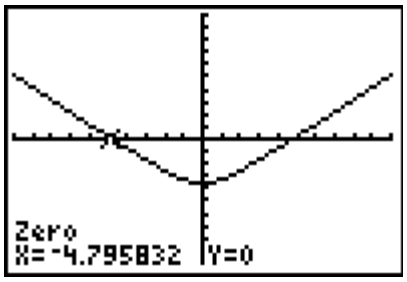
a) The solution is $x \approx 24.714$.

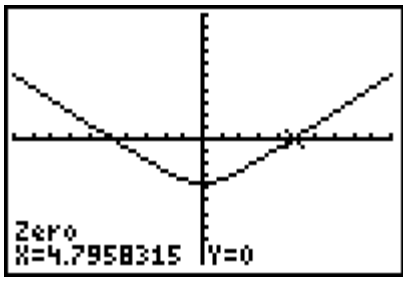


b) The solution is $x \approx -117.273$.

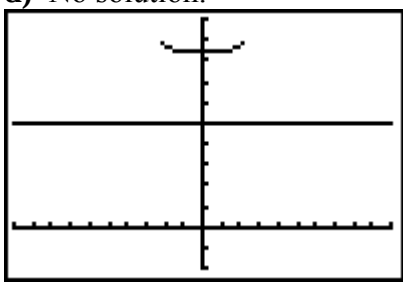


c) The solution is $x \approx \pm 4.796$.





d) No solution.



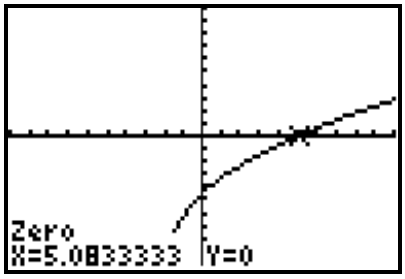
Section 2.3 Page 96 Question 4

a)

$$2\sqrt{3x+5}+7=16$$
$$2\sqrt{3x+5}=9$$
$$\sqrt{3x+5}=\frac{9}{2}$$
$$3x+5=\frac{81}{4}$$
$$3x=\frac{61}{4}$$
$$x=\frac{61}{12}$$

The solution is $x=\frac{61}{12}$ or $x \approx 5.083$.

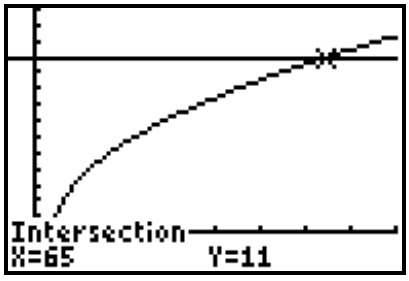
b) The solution will be the x-intercept of the graph, $x \approx 5.083$.



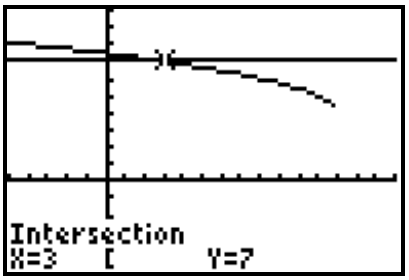
Section 2.3 Page 96 Question 5

a) For $\sqrt{2x-9}=11$, $x \geq \frac{9}{2}$.

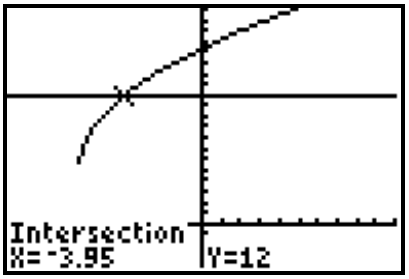
The solution is $x = 65$.



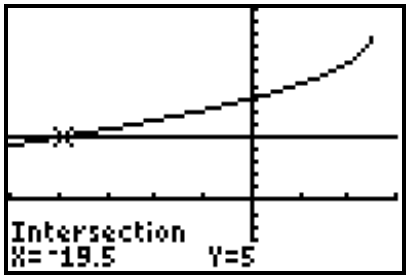
b) For $7 = \sqrt{12 - x} + 4$, $x \le 12$.
The solution is $x = 3$.



c) For $5 + 2\sqrt{5x + 32} = 12$, $x \ge -\frac{32}{5}$.
The solution is $x = -3.95$.



d) For $5 = 13 - \sqrt{25 - 2x}$, $x \le \frac{25}{2}$. The solution is $x = -19.5$.



Section 2.3 Page 97 Question 6

a) For $\sqrt{5x^2 + 11} = x + 5$, there are no restrictions on the variable.

$$\sqrt{5x^2 + 11} = x + 5$$
$$5x^2 + 11 = x^2 + 10x + 25$$
$$4x^2 - 10x - 14 = 0$$
$$(2x + 2)(2x - 7) = 0$$

$2x + 2 = 0$ or $2x - 7 = 0$

$2x = -2$ $\qquad$ $2x = 7$

$x = -1$ $\qquad$ $x = \frac{7}{2}$

b) For $x+3=\sqrt{2x^2-7}$, the restrictions on the variable are $x \le -\frac{\sqrt{14}}{2}$ or $x \ge \frac{\sqrt{14}}{2}$.

$$x+3=\sqrt{2x^2-7}$$
$$x^2+6x+9=2x^2-7$$
$$0=x^2-6x-16$$
$$0=(x+2)(x-8)$$

$x+2=0$ or $x-8=0$

$x=-2$ $\quad$ $x=8$

c) For $\sqrt{13-4x^2}=2-x$, the restrictions on the variable are $-\frac{\sqrt{13}}{2} \le x \le \frac{\sqrt{13}}{2}$.

$$\sqrt{13-4x^2}=2-x$$
$$13-4x^2=4-4x+x^2$$
$$0=5x^2-4x-9$$
$$0=(5x-9)(x+1)$$

$5x-9=0$ or $x+1=0$

$5x=9$ $\quad$ $x=-1$

$x=\frac{9}{5}$ or 1.8

d) For $x+\sqrt{-2x^2+9}=3$, the restrictions on the variable are $-\frac{3\sqrt{2}}{2} \le x \le \frac{3\sqrt{2}}{2}$.

$$x+\sqrt{-2x^2+9}=3$$
$$\sqrt{-2x^2+9}=3-x$$
$$-2x^2+9=9-6x+x^2$$
$$0=3x^2-6x$$
$$0=3x(x-2)$$

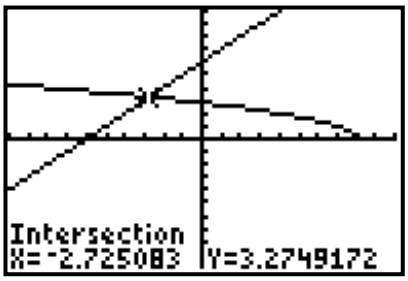
$3x=0$ or $x-2=0$

$x=0$ $\quad$ $x=2$

Section 2.3 Page 97 Question 7

a) For $\sqrt{8-x} = x+6$, the restrictions on the variable are $x \le 8$.

$$\sqrt{8-x} = x+6$$
$$8-x = x^2+12x+36$$
$$0 = x^2+13x+28$$
$$x = \frac{-13 \pm \sqrt{13^2-4(1)(28)}}{2(1)}$$
$$x = \frac{-13 \pm \sqrt{57}}{2}$$

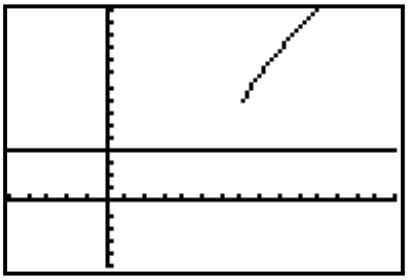


$$x = \frac{-13+\sqrt{57}}{2} \quad \text{or} \quad x = \frac{-13-\sqrt{57}}{2}$$
$$x \approx -2.7 \qquad x \approx -10.3$$

The solution is $x \approx -2.7$, as $x \approx -10.3$ is an extraneous root.

b) For $4 = x+2\sqrt{x-7}$ the restrictions on the variable are $x \ge 7$.

$$4 = x+2\sqrt{x-7}$$
$$4-x = 2\sqrt{x-7}$$
$$16-8x+x^2 = 4x^2-14$$
$$0 = 3x^2+8x-30$$
$$x = \frac{-8 \pm \sqrt{8^2-4(3)(-30)}}{2(3)}$$
$$x = \frac{-8 \pm \sqrt{424}}{6}$$
$$x = \frac{-8 \pm 2\sqrt{106}}{6}$$
$$x = \frac{-4 \pm \sqrt{106}}{3}$$

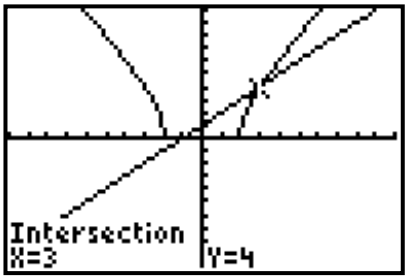


$$x = \frac{-4+\sqrt{106}}{3} \quad \text{or} \quad x = \frac{-4-\sqrt{106}}{3}$$
$$x \approx 2.1 \qquad x \approx -4.8$$

There is no solution, as neither value of x meets the restriction.

c) For $\sqrt{3x^2-11} = x+1$ the restrictions on the variable are $x \le -\frac{\sqrt{33}}{3}$ or $x \ge \frac{\sqrt{33}}{3}$.

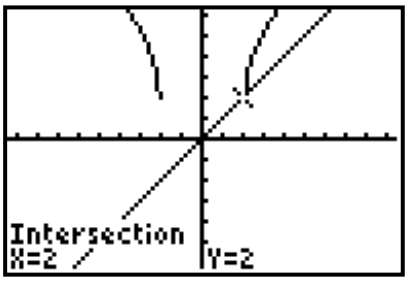
$$\sqrt{3x^2-11} = x+1$$
$$3x^2-11 = x^2+2x+1$$
$$2x^2-2x-12 = 0$$
$$x^2-x-6 = 0$$
$$(x+2)(x-3) = 0$$
$x+2=0$ or $x-3=0$
$x=-2$ $x=3$



The solution is $x = 3$, as $x = -2$ is an extraneous root.

d) For $x = \sqrt{2x^2-8}+2$ the restrictions on the variable are $x \le -2$ or $x \ge 2$.

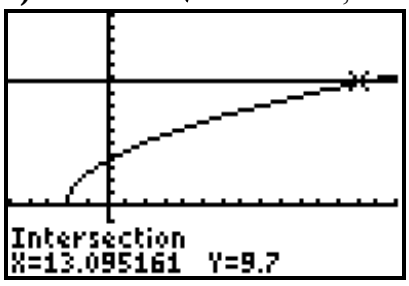
$$x = \sqrt{2x^2-8}+2$$
$$x-2 = \sqrt{2x^2-8}$$
$$x^2-4x+4 = 2x^2-8$$
$$0 = x^2+4x-12$$
$$0 = (x+6)(x-2)$$
$x+6=0$ or $x-2=0$
$x=-6$ $x=2$



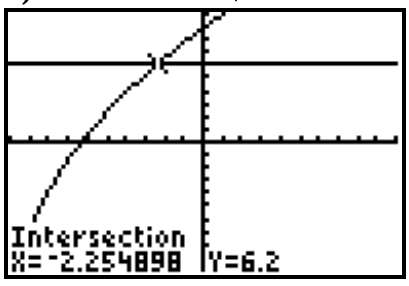
The solution is $x = 2$, as $x = -6$ is an extraneous root.

Section 2.3 Page 97 Question 8

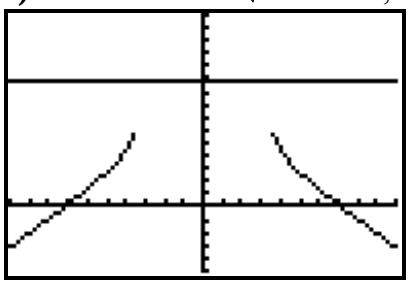
a) For $9.7 = \sqrt{6.2a+12.9}$, $a \approx 13.10$.



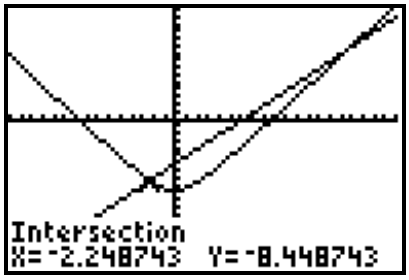
b) For $-12.9+7\sqrt{a+9.7} = 6.2$, $a \approx -2.25$.

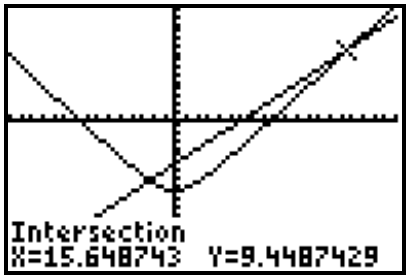


c) For $9.7 = 6.2 - \sqrt{a^2 - 12.9}$, there is no solution.



d) For $\sqrt{2a^2 + 9.7} - 12.9 = a - 6.2$, $a \approx -2.25$ or $a \approx 15.65$.





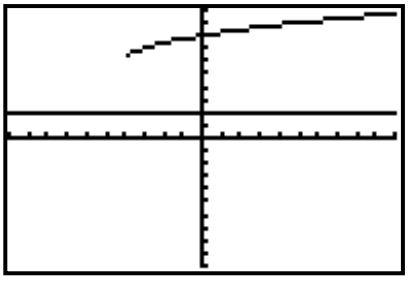
Section 2.3 Page 97 Question 9

a) For $6 + \sqrt{x + 4} = 2$, the restrictions on the variable are $x \geq -4$.

$6 + \sqrt{x + 4} = 2$

$\sqrt{x + 4} = -4$

The square root of the expression cannot be negative. There is no solution.

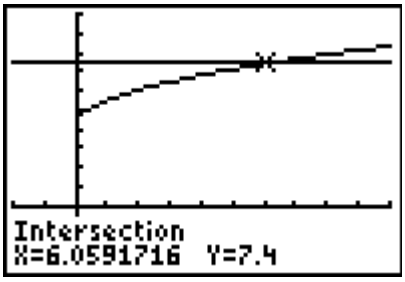


b) Yes, you can tell that this equation has no solutions by examining the equation. As shown in part a), by noticing that isolating the radical expression equates it to a negative value.

Section 2.3 Page 97 Question 10

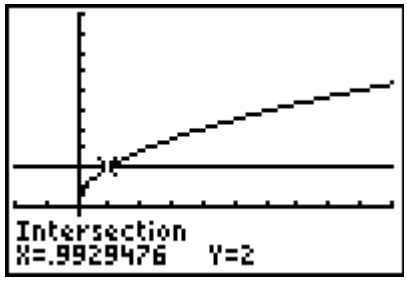
Determine the point of intersection of the graphs of $N(t) = 1.3\sqrt{t} + 4.2$ and $N(t) = 7.4$.

Greg is correct. The entire population would be affected after 6 years.



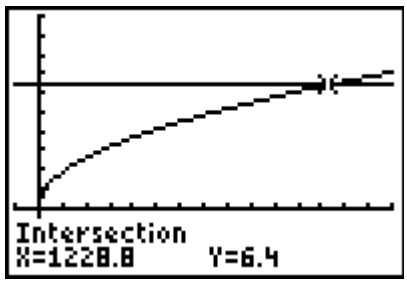
Section 2.3 Page 97 Question 11

Determine the point of intersection of the graphs of $T = 2\pi\sqrt{\frac{L}{9.8}}$ and $T = 2$. The pendulum should be approximately 0.99 m, or 99 cm, in length.

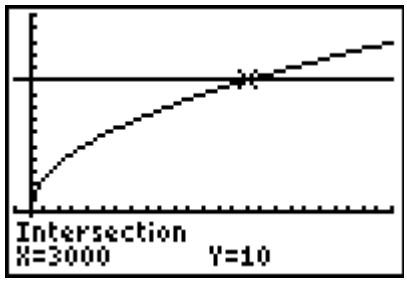


Section 2.3 Page 97 Question 12

a) Determine the point of intersection of the graphs of $d = \sqrt{\frac{b}{30}}$ and $d = 6.4$. Since the value of the b-coordinate is greater than or equal to 1000, a cable with diameter of 6.4 mm will support a mass of 1000 kg.



b) Determine the point of intersection of the graphs of $d = \sqrt{\frac{b}{30}}$ and $d = 10$. Since the value of the b-coordinate is 3000, a safe working load for a cable with diameter of 10 mm is less than or equal to 3000 kg.



Section 2.3 Page 97 Question 13

Hazeem is incorrect. The equation $\sqrt{x^2} = 9$ has two solutions, while the equation $\left(\sqrt{x}\right)^2 = 9$ has only one.

$\sqrt{x^2} = 9$

$x^2 = 81$

$x = \pm 9$

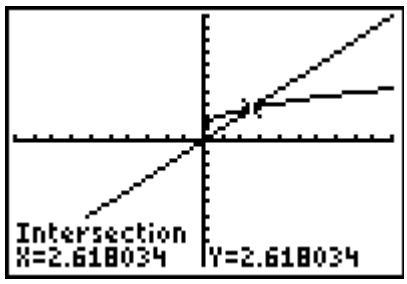
$\left(\sqrt{x}\right)^2 = 9$

$x = 9$

Section 2.3 Page 98 Question 14

Let x represent a real number.

$$x = \sqrt{x} + 1$$
$$x - 1 = \sqrt{x}$$
$$x^2 - 2x + 1 = x$$
$$x^2 - 3x + 1 = 0$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{3 \pm \sqrt{5}}{2}$$



$x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$

$x \approx 2.6$ $x \approx 0.4$

Check:

For $x \approx 2.6$

Left Side	Right Side
x	$\sqrt{x}+1$
≈ 2.6	$\approx \sqrt{2.6}+1$
	≈ 2.6

Left Side = Right Side

For $x \approx 0.4$

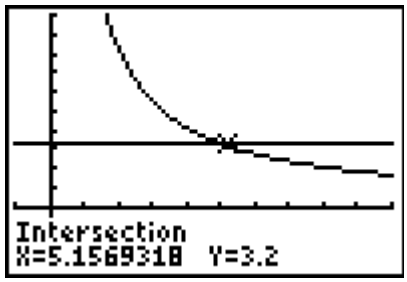
Left Side	Right Side
x	$\sqrt{x}+1$
≈ 0.4	$\approx \sqrt{0.4}+1$
	≈ 1.6

Left Side ≠ Right Side

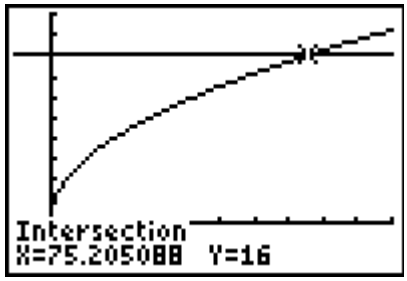
The real number is $\frac{3+\sqrt{5}}{2}$.

Section 2.3 Page 98 Question 15

a) Substitute $m = 20$. Determine the point of intersection of the graphs of $d = 3.69\sqrt{\frac{20}{v^2}}$ and $d = 3.2$. The landing velocity for a 20-kg object using a parachute that is 3.2 m in diameter is approximately 5 m/s.



b) Substitute $v = 2$. Determine the point of intersection of the graphs of $d = 3.69\sqrt{\frac{m}{4}}$ and $d = 16$. The landing velocity for a 2 m/s and a parachute that is 16 m can carry a parachutist with a maximum mass of approximately 75 kg.

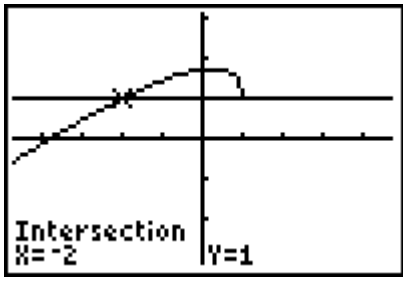


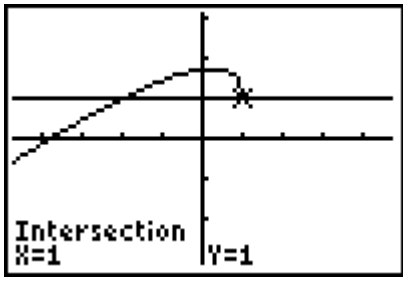
Section 2.3 Page 98 Question 16

Substitute (–1, 1) into $y = \sqrt{-3(x+c)} + c$.

$$1 = \sqrt{-3(-1+c)} + c$$
$$1 - c = \sqrt{3-3c}$$
$$1 - 2c + c^2 = 3 - 3c$$
$$c^2 + c - 2 = 0$$
$$(c+2)(c-1) = 0$$
$$c + 2 = 0 \quad \text{or} \quad c - 1 = 0$$
$$c = -2 \qquad c = 1$$

The solutions are $c = -2$ and $c = 1$.





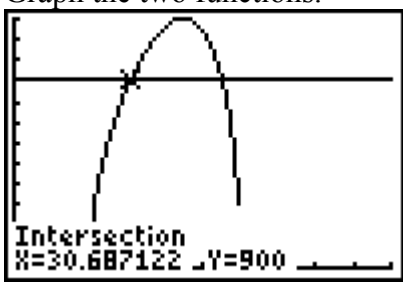
Example: If the function $y = \sqrt{-3(x+c)} + c$ passes through the point (0.25, 0.75), what is the value of c?

Section 2.3 Page 98 Question 17

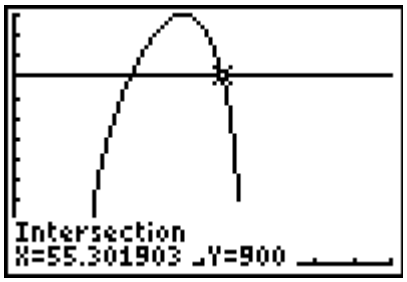
Given $A = 900$ and $a = 60$. Let $c = 2b$. Then, $s = 30 + 1.5b$. Substitute into Heron's formula.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$900 = \sqrt{(30+1.5b)(30+1.5b-60)(30+1.5b-b)(30+1.5b-2b)}$$
$$900 = \sqrt{(30+1.5b)(-30+1.5b)(30+0.5b)(30-0.5b)}$$
$$900 = \sqrt{(2.25b^2-900)(900-0.25b^2)}$$

Graph the two functions.



If $b \approx 30.7$, then $c \approx 61.4$.



If $b \approx 55.3$, then $c \approx 110.6$.

The lengths of the three side of the triangle are 60 cm, 30.7 cm, and 61.4 cm or 60 cm, 55.3 cm, and 110.6 cm.

Section 2.3 Page 98 Question C1

The x-intercepts of the graph of a function are the solutions to the corresponding equation.
Example: A graph of the function $y = \sqrt{x-1} - 2$ would show that the x-intercept is 5. The equation that corresponds to this function is $0 = \sqrt{x-1} - 2$ and its solution is 5.

Section 2.3 Page 98 Question C2

a) Let v represent the speed, in metres per second. Let d represent the depth of the water, in metres. Then, a function for the speed of a tsunami is $v = \sqrt{9.8d}$

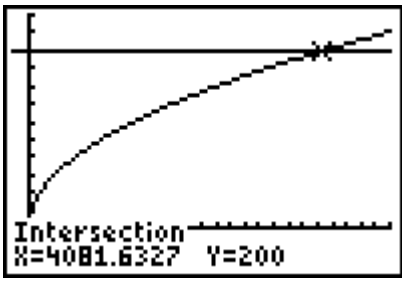
b) Substitute $d = 2500$ into $v = \sqrt{9.8d}$.

$v = \sqrt{(9.8 \text{ m/s}^2)(2500 \text{ m})}$

$v = \sqrt{24\,500 \text{ m}^2/\text{s}^2}$

$v \approx 156.5$ m/s

c) Graphically: Determine the point of intersection of the graphs of
$v = \sqrt{9.8d}$ and $v = 200$.



Algebraically:

$200 = \sqrt{9.8d}$

$40\,000 = 9.8d$

$$d = \frac{40\,000}{9.8}$$

$d \approx 4081.6$

The depth of water that would produce a speed of 200 m/s is approximately 4081.6 m.

d) Example: In this case, I prefer the algebraic method because it is faster than graphing with technology, where I have to adjust window settings to locate the point of intersection.

Section 2.3 Page 98 Question C3

Radical equations only have a solution if the graph of the corresponding function has an x-intercept. For example, $y = \sqrt{x} + 4$ and $y = \sqrt{x+4} + 4$ have no solutions because their graphs have no x-intercepts.

Section 2.3 Page 98 Question C4

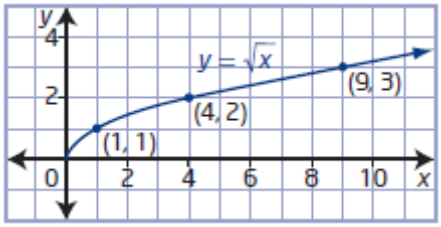
Extraneous roots may occur when solving equations algebraically. For example, a possible solution may not meet the restrictions on the variable in the square root or a check of the solution may yield an extraneous root. An extraneous root can also be identified by graphing.
Extraneous roots of a radical equation may occur anytime an expression is squared. For example, $x^2 = 1$ has two possible solutions, $x = \pm 1$.

Chapter 2 Review

Chapter 2 Review Page 99 Question 1

a)

x	$y = \sqrt{x}$
0	0
1	1
4	2
9	3

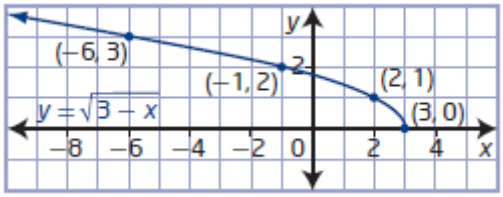


domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The restriction on the domain is because the radicand must be greater than or equal to zero. All x- and y-values in the table are within the domain and range, respectively. The graph exists in quadrant I, as defined by the domain and range.

b)

x	$y = \sqrt{3 - x}$
3	0
2	1
–1	2
–6	3

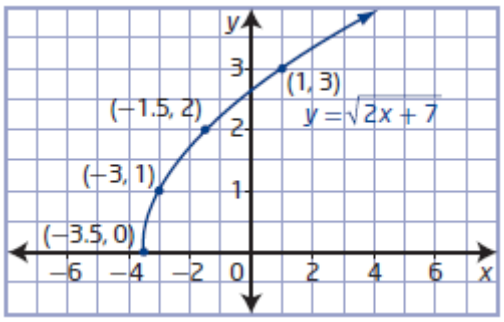


domain
$\{x \mid x \leq 3, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The restriction on the domain is because the radicand must be greater than or equal to zero. All x- and y-values in the table are within the domain and range, respectively. The graph extends from quadrant I to quadrant II, as defined by the domain and range.

c)

x	$y = \sqrt{2x + 7}$
–3.5	0
–3	1
–1.5	2
1	3



domain
$\{x \mid x \geq -3.5, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The restriction on the domain is because the radicand must be greater than or equal to zero. All x- and y-values in the table are within the domain and range, respectively. The graph extends from quadrant II to quadrant I, as defined by the domain and range.

Chapter 2 Review Page 99 Question 2

a) For $y = 5\sqrt{x+20}$, $a = 5$, $b = 1$, $h = -20$, and $k = 0$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 5 and translated 20 units to the left.
domain $\{x \mid x \geq -20, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) For $y = \sqrt{-2x} - 8$, $a = 1$, $b = -2$, $h = 0$, and $k = -8$. The graph of $y = \sqrt{x}$ is horizontally stretched by a factor of 0.5, reflected in the y-axis, and translated 8 units down.
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -8, y \in \mathrm{R}\}$

c) For $y = -\sqrt{\frac{1}{6}(x-11)}$, $a = -1$, $b = \frac{1}{6}$, $h = 11$, and $k = 0$. The graph of $y = \sqrt{x}$ is reflected in the x-axis, horizontally stretched by a factor of 6, and translated 11 units to the right.
domain $\{x \mid x \geq 11, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

Chapter 2 Review Page 99 Question 3

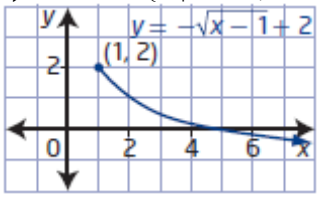
a) For a horizontal stretch by a factor of 10 and a vertical translation of 12 units up, $b = \frac{1}{10}$, $k = 12$, and the equation of the transformed function is $y = \sqrt{\frac{1}{10}x} + 12$.
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 12, y \in \mathrm{R}\}$

b) For a vertical stretch by a factor of 2.5, a reflection in the x-axis, and a horizontal translation of 9 units left, $a = -2.5$, $h = -9$, and the equation of the transformed function is $y = -2.5\sqrt{x+9}$.
domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

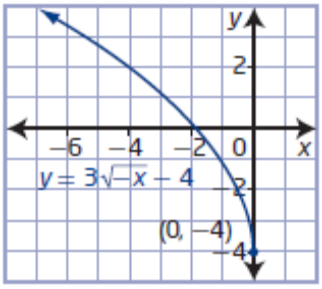
c) For a horizontal stretch by a factor of $\frac{5}{2}$, a vertical stretch by a factor of $\frac{1}{20}$, a reflection in the y-axis, and a translation of 7 units right and 3 units down, $a = \frac{1}{20}$, $b = -\frac{2}{5}$, $h = 7$, $k = -3$, and the equation of the transformed function is $y = \frac{1}{20}\sqrt{-\frac{2}{5}(x-7)} - 3$.
domain $\{x \mid x \leq 7, x \in \mathrm{R}\}$, range $\{y \mid y \geq -3, y \in \mathrm{R}\}$

Chapter 2 Review Page 99 Question 4

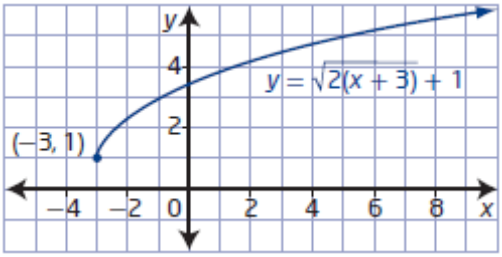
a) domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \leq 2, y \in \mathrm{R}\}$



b) domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$



c) domain $\{x \mid x \geq -3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$



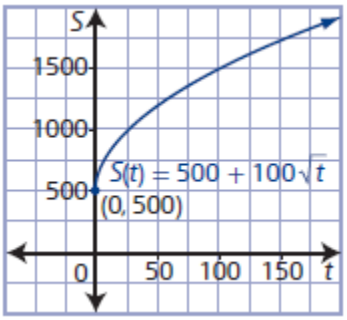
Chapter 2 Review Page 99 Question 5

For $y = -2\sqrt{3(x-4)} + 9$, $a = -2$, $b = 3$, $h = 4$, and $k = 9$. The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 2, reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{3}$, and translated 4 units to the right and 9 units up.
Domain is affected by the values of b and h: $\{x \mid x \geq 4, x \in \mathrm{R}\}$
Range is affected by the values of a and k: $\{y \mid y \leq 9, y \in \mathrm{R}\}$

Chapter 2 Review Page 99 Question 6

a) For $S(t) = 500 + 100\sqrt{t}$, $a = 100$, $b = 1$, $h = 0$, and $k = 500$. The graph of $y = \sqrt{t}$ is vertically stretched by a factor of 100 and translated 500 units up.

b) Since the endpoint of the graph is (0, 500), the minimum sales of the new product is 500 units. Sales will continue to increase with each day.



c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{S \mid S \geq 500, S \in \mathrm{W}\}$
The domain represent time, so it is non-negative. The range represents number of units sold, which is a minimum of 500 units.

d) Substitute $t = 60$ into $S(t) = 500 + 100\sqrt{t}$.
$S(60) = 500 + 100\sqrt{60}$
$S(60) = 1274.596\ldots$
After 60 days, approximately 1274 units will be sold.

Chapter 2 Review Page 99 Question 7

a) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (-3, 2)$
$(1, 1) \rightarrow (1, 3)$
$(4, 2) \rightarrow (13, 4)$
The overall width has changed, so the graph has been horizontally stretched by a factor of 4. From the endpoint, the graph has been translated 3 units to the left and 2 units up.
So, $a = 1$, $b = \frac{1}{4}$, $h = -3$, $k = 2$, and the equation of the transformed graph is $y = \sqrt{\frac{1}{4}(x+3)} + 2$.

b) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (-4, 3)$
$(1, 1) \rightarrow (-3, 1)$
$(4, 2) \rightarrow (0, -1)$
$(9, 3) \rightarrow (5, -3)$
The overall height has changed, so the graph has been vertically stretched by a factor of 2. From the general shape, the graph has been reflected in the x-axis. From the endpoint, the graph has been translated 4 units to the left and 3 units up. So, $a = -2$, $b = 1$, $h = -4$, $k = 3$, and the equation of the transformed graph is $y = -2\sqrt{x+4} + 3$.

c) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (6, -4)$
$(1, 1) \rightarrow (2, 4)$
$(4, 2) \rightarrow (-3, 8)$
The overall height has changed, so the graph has been vertically stretched by a factor of 4. From the general shape, the graph has been reflected in the y-axis. From the endpoint, the graph has been translated 6 units to the right and 4 units down. So, $a = 4$, $b = -1$, $h = 6$, $k = -4$, and the equation of the transformed graph is $y = 4\sqrt{-(x-6)} - 4$.

Chapter 2 Review Page 100 Question 8

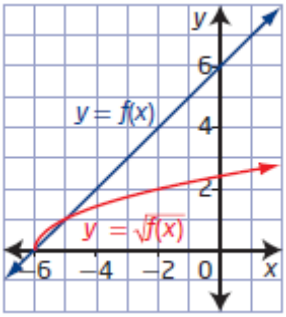
a) For $y = x - 2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{x-2}$, the domain is $\{x \mid x \geq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x-2}$ is undefined for $x < 2$.
The ranges differ because $y = \sqrt{x-2}$ is undefined for $y < 0$.

b) For $y = 10 - x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{10-x}$, the domain is $\{x \mid x \leq 10, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{10-x}$ is undefined for $x > 10$. The ranges differ because $y = \sqrt{10-x}$ is undefined for $y < 0$.

c) For $y = 4x + 11$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{4x+11}$, the domain is $\{x \mid x \geq -\frac{11}{4}, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{4x+11}$ is undefined for $x < -\frac{11}{4}$. The ranges differ because $y = \sqrt{4x+11}$ is undefined for $y < 0$.

Chapter 2 Review Page 100 Question 9

a) I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.



b) Since the graph of $y = f(x)$ is on or above the x-axis for $x \geq -6$, the graph of $y = \sqrt{f(x)}$ exists only for these values.

c) For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \geq -6, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{f(x)}$ is undefined for $x < -6$.
The ranges differ because $y = \sqrt{f(x)}$ is undefined for $y < 0$.

Chapter 2 Review Page 100 Question 10

a) For $y = 4 - x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 4, y \in \mathrm{R}\}$.
For $y = \sqrt{4 - x^2}$, the domain is $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq 2, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{4 - x^2}$ is undefined for $x < -2$ and $x > 2$. The ranges differ because $y = \sqrt{4 - x^2}$ is undefined for $y < 0$ and has a maximum value of 2.

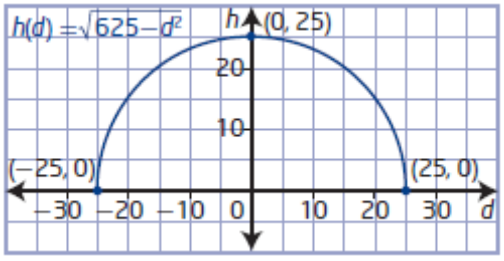
b) For $y = 2x^2 + 24$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 24, y \in \mathrm{R}\}$.
For $y = \sqrt{2x^2 + 24}$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq \sqrt{24}, y \in \mathrm{R}\}$.
The domains are the same because the entire graph of $y = 2x^2 + 24$ is above the x-axis.
The ranges differ because $y = \sqrt{2x^2 + 24}$ has a minimum value of $\sqrt{24}$.

c) For $y = x^2 - 6x$ (or $y = (x - 3)^2 - 9$), the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$.
For $y = \sqrt{x^2 - 6x}$, the domain is $\{x \mid x \leq 0 \text{ and } x \geq 6, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{x^2 - 6x}$ is undefined for $0 < x < 6$. The ranges differ because $y = \sqrt{x^2 - 6x}$ is undefined for $y < 0$.

Chapter 2 Review Page 100 Question 11

a) Use the Pythagorean theorem because the ladder forms a right triangle with the ground and the wall. An equation to represent h as a function of d is $h = \sqrt{625 - d^2}$.

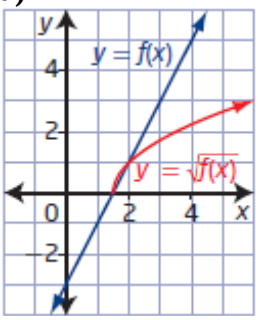
b) The domain is $\{d \mid -25 \leq d \leq 25, d \in \mathrm{R}\}$ and the range is $\{h \mid 0 \leq h \leq 25, h \in \mathrm{R}\}$.



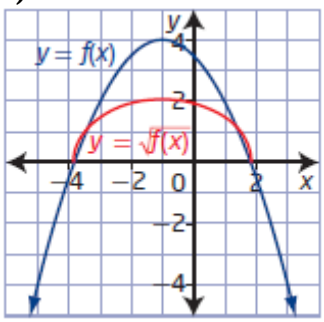
c) Since h and d represent distances, the values must be non-negative. So, the domain becomes $\{d \mid 0 \le d \le 25, d \in \mathrm{R}\}$ to represent the distances of the ladder from the base of the wall. The range represents scenarios from the ladder lying on the ground to flat against the wall.

Chapter 2 Review Page 100 Question 12

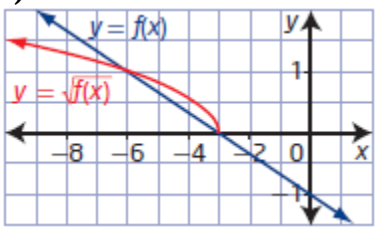
a)



b)



c)



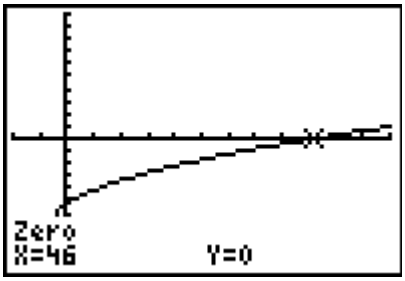
Chapter 2 Review Page 100 Question 13

a) $\sqrt{x+3} - 7 = 0$

$\sqrt{x+3} = 7$

$x + 3 = 49$

$x = 46$

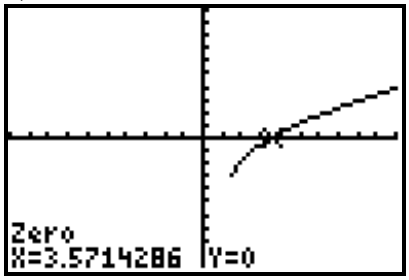


b) The x-intercept is 46.

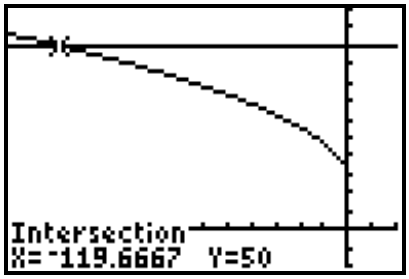
c) The solutions or roots of a radical equation are equivalent to the x-intercepts of the graph of the corresponding radical function.

Chapter 2 Review Page 101 Question 14

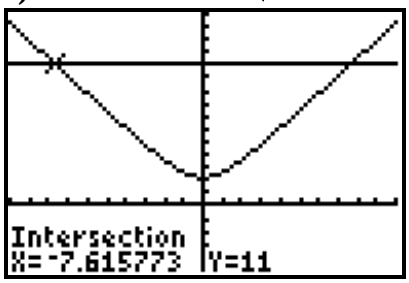
a) The solution to $\sqrt{7x-9} - 4 = 0$ is $x \approx 3.571$.

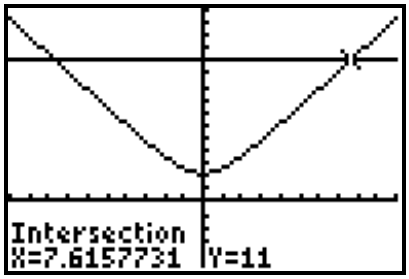


b) The solution to $50 = 12 + \sqrt{8 - 12x}$ is $x \approx -119.667$.



c) The solution to $\sqrt{2x^2 + 5} = 11$ is $x \approx -7.616$ and $x \approx 7.616$.





Chapter 2 Review Page 101 Question 15

Solve $9 = \sqrt{2(9.8)h}$.

$$9 = \sqrt{2(9.8)h}$$
$$81 = 19.6h$$
$$h = \frac{81}{19.6}$$
$$h = 4.132...$$

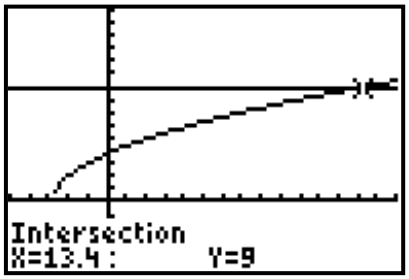
At a height of approximately 4.13 m, the water is flowing out at 9 m/s.

Chapter 2 Review Page 101 Question 16

a) For $\sqrt{5x + 14} = 9$, the restrictions on the variable are $x \geq -\frac{14}{5}$.

$$\sqrt{5x + 14} = 9$$
$$5x + 14 = 81$$
$$5x = 67$$
$$x = \frac{67}{5}$$
$$x = 13.4$$

The solution is $x = 13.4$.



b) For $7 + \sqrt{8 - x} = 12$, the restrictions on the variable are $x \leq 8$.

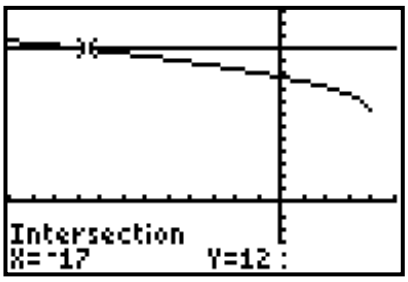
$7+\sqrt{8-x}=12$

$\sqrt{8-x}=5$

$8-x=25$

$x=-17$

The solution is $x=-17$.



c) For $23-4\sqrt{2x-10}=12$, the restrictions on the variable are $x \geq 5$.

$23-4\sqrt{2x-10}=12$

$-4\sqrt{2x-10}=-11$

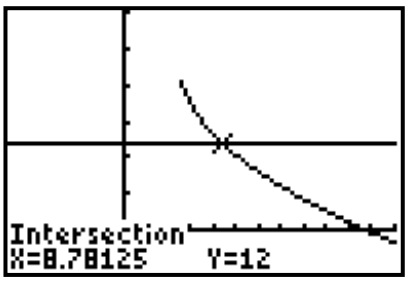
$\sqrt{2x-10}=\frac{11}{4}$

$2x-10=\frac{121}{16}$

$2x=\frac{281}{16}$

$x=\frac{281}{32}$

The solution is $x=\frac{281}{32}$, or $x \approx 8.781$.



d) For $x+3=\sqrt{18-2x^2}$, the restrictions on the variable are $-3 \leq x \leq 3$.

$x+3=\sqrt{18-2x^2}$

$x^2+6x+9=18-2x^2$

$3x^2+6x-9=0$

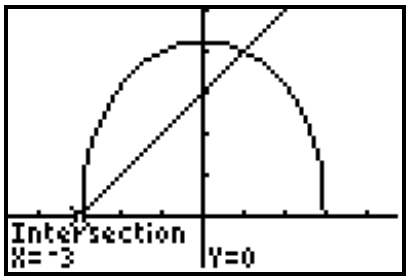
$x^2+2x-3=0$

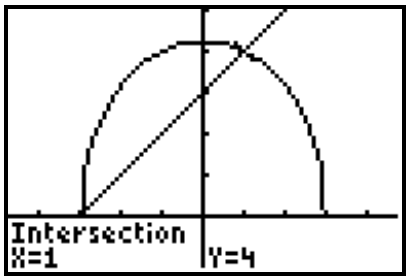
$(x+3)(x-1)=0$

$x+3=0$ or $x-1=0$

$x=-3$ $x=1$

The solution is $x=-3$ and $x=1$.





Chapter 2 Review Page 101 Question 17

a) For $3+\sqrt{x-1}=x$, the restrictions on the variable are $x \geq 1$.

$3+\sqrt{x-1}=x$

$\sqrt{x-1}=x-3$

$x-1=x^2-6x+9$

$0=x^2-7x+10$

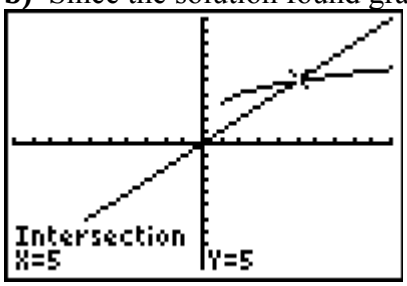
$0=(x-2)(x-5)$

$x-2=0$ or $x-5=0$

$x=2$ $x=5$

Since the solution found algebraically is $x = 2$ and $x = 5$, Jaime most likely used this approach.

b) Since the solution found graphically is $x = 5$, Carly most likely used this approach.



c) Atid most likely made an error, since $x = 2$ is an extraneous root.

Chapter 2 Review Page 101 Question 18

a) Substitute $r = 5.2$ into $S(r) = \pi r\sqrt{36+r^2}$.

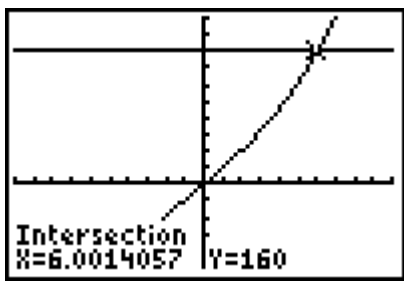
$S(5.2) = \pi(5.2)\sqrt{36+5.2^2}$

$S(5.2) = 5.2\pi\sqrt{63.04}$

$S(5.2) = 129.706\ldots$

For a tipi with radius 5.2 m, the minimum area of canvas required for the walls is 130 m^2, to the nearest square metre.

b) Solve $160 = \pi r\sqrt{36+r^2}$ graphically. If 160 m^2 of canvas form the wall of a tipi, then the radius will be approximately 6.0 m.



Chapter 2 Practice Test

Chapter 2 Practice Test Page 102 Question 1

The points on the graph of $y = \sqrt{f(x)}$, where it exits, have the same x-coordinates as points on the graph of $y = f(x)$ but the y-coordinates are $\sqrt{y}$. If $x = 0$, $f(x) = 1$ and $\sqrt{f(x)} = 1$. The point (0, 1) is on the graph of $y = \sqrt{f(x)}$: choice **B**.

Chapter 2 Practice Test Page 102 Question 2

To solve $\sqrt{2x-5} = 4$ graphically, find determine the x-intercepts of the corresponding graph $y = \sqrt{2x-5} - 4$: choice **A**.

Chapter 2 Practice Test Page 102 Question 3

For a radical function with domain $\{x \mid x \geq 5, x \in \mathrm{R}\}$, $a = 1$ and $h = 5$.
For a radical function with range $\{y \mid y \geq 0, y \in \mathrm{R}\}$, $b = 1$ and $k = 0$.
The equation of this function is $f(x) = \sqrt{x-5}$: choice **A**.

Chapter 2 Practice Test Page 102 Question 4

For a horizontal stretch by a factor of 6, $b = \frac{1}{6}$ and the resulting function is $y = \sqrt{\frac{1}{6}x}$: choice **C**.

Chapter 2 Practice Test Page 102 Question 5

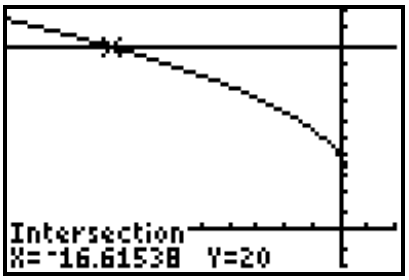
Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
(0, 0) → (0, –2)
(1, 1) → (–1, –1)
(4, 2) → (–4, 0)
The overall width and height have not changed, so the graph has not been stretched horizontally or vertically. From the orientation of the graph, it has been reflected in the y-axis. From the endpoint, the graph has been translated 2 units down. So, $a = 1$, $b = –1$, $h = 0$, $k = –2$, and the equation of the transformed graph is $y = \sqrt{-x} - 2$: choice **D**.

Chapter 2 Practice Test Page 102 Question 6

For $y = \sqrt{x}$, the domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
For $y = \sqrt{5x} + 8$, $a = 1$, $b = 5$, $h = 0$, and $k = 8$. The graph of $y = \sqrt{x}$ is horizontally stretched by a factor of 0.2 and translated 8 units up. The domain remains as $\{x \mid x \geq 0, x \in \mathrm{R}\}$ but the range becomes $\{y \mid y \geq 8, y \in \mathrm{R}\}$: choice **B**.

Chapter 2 Practice Test Page 102 Question 7

The solution is $x \approx -16.62$.



Chapter 2 Practice Test Page 102 Question 8

Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (0, 0)$
$(1, 1) \rightarrow (1, 4)$
$(4, 2) \rightarrow (4, 8)$
$(9, 3) \rightarrow (9, 12)$
The overall height has changed, so the graph has been vertically stretched by a factor of 4. From the general shape, the graph has not been reflected in an axis. From the endpoint, the graph has not been translated. So, $a = 4$, $b = 1$, $h = 0$, $k = 0$, and an equation of the transformed graph is $y = 4\sqrt{x}$. The equation of the function can also be represented as $y = \sqrt{16x}$, a horizontal stretch by a factor of $\frac{1}{16}$.

Chapter 2 Practice Test Page 102 Question 9

For $y = 7 - x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{7 - x}$, the domain is $\{x \mid x \leq 7, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
The domains differ because $y = \sqrt{7 - x}$ is undefined for $x > 7$. The ranges differ because $y = \sqrt{7 - x}$ is undefined for $y < 0$.

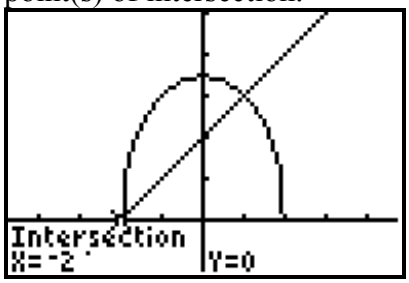
Chapter 2 Practice Test Page 102 Question 10

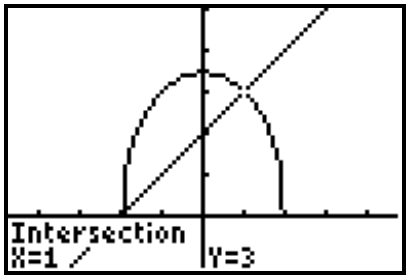
For $f(x) = 8 - 2x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 8, y \in \mathrm{R}\}$.
For $y = \sqrt{8 - 2x^2}$, the domain is $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \leq y \leq \sqrt{8}, y \in \mathrm{R}\}$.

Chapter 2 Practice Test Page 102 Question 11

Solve $\sqrt{12-3x^2} = x+2$ using two graphical methods.

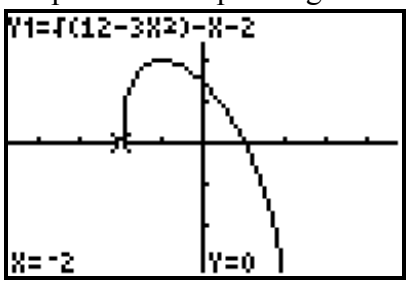
Graph the corresponding functions $y = \sqrt{12-3x^2}$ and $y = x + 2$ and determine the point(s) of intersection.

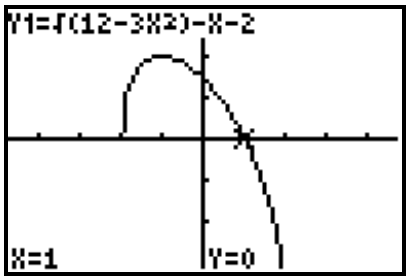




The solution is $x = -2$ and $x = 1$.

Graph the corresponding function $y = \sqrt{12-3x^2} - x - 2$ and determine the x-intercept(s).



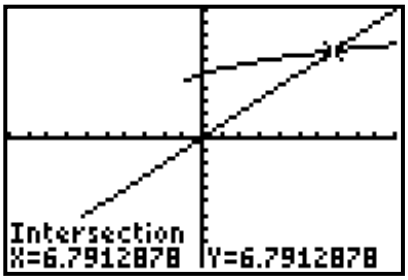


The solution is $x = -2$ and $x = 1$.

Chapter 2 Practice Test Page 102 Question 12

For $4+\sqrt{x+1} = x$, the restrictions on the variable are $x \geq -1$.

$$4+\sqrt{x+1} = x$$
$$\sqrt{x+1} = x-4$$
$$x+1 = x^2-8x+16$$
$$0 = x^2-9x+15$$
$$x = \frac{-(-9) \pm \sqrt{(-9)^2-4(1)(15)}}{2(1)}$$
$$x = \frac{9 \pm \sqrt{21}}{2}$$



$x = \frac{9+\sqrt{21}}{2}$ or $x = \frac{9-\sqrt{21}}{2}$

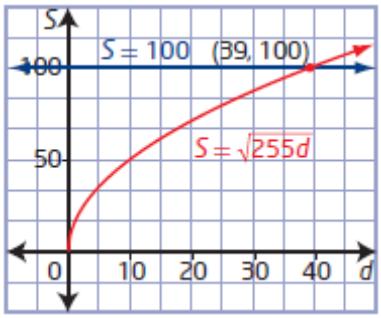
$x \approx 6.8$ $x \approx 2.2$

The solution is $x \approx 6.8$, as $x \approx 2.2$ is an extraneous root.

Chapter 2 Practice Test Page 103 Question 13

a) For $S = \sqrt{255d}$, $a = 1$, $b = 255$, $h = 0$, and $k = 0$. The graph of $S = \sqrt{d}$ is horizontally stretched by a factor of $\frac{1}{255}$.

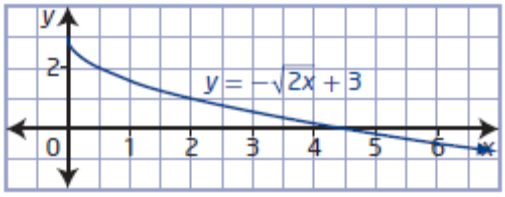
b) The length of skid mark expected from a vehicle travelling at 100 km/h is approximately 39 m.



Chapter 2 Practice Test Page 103 Question 14

a) For $y = -\sqrt{2x} + 3$, $a = -1$, $b = 2$, $h = 0$, and $k = 3$. The graph of $y = \sqrt{x}$ is reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{2}$, and translated 3 units up.

b)



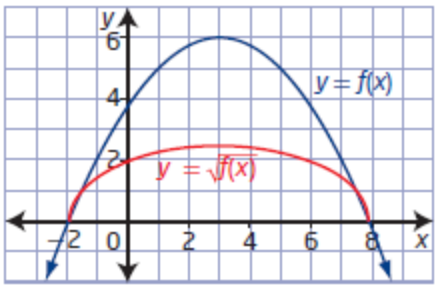
c) domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 3, y \in \mathrm{R}\}$

d) The domain remains the same as that for $y = \sqrt{x}$, since $b > 0$ and $h = 0$. The range differs from that for $y = \sqrt{x}$, since $a < 0$ and $k = 3$.

e) The equation $5 + \sqrt{2x} = 8$ can be solved by graphing the corresponding function $y = -\sqrt{2x} + 3$ and determining its x-intercept.

Chapter 2 Practice Test Page 103 Question 15

I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.



Chapter 2 Practice Test Page 103 Question 16

a) Consider the right half of the roof. The endpoint is (5, 0), so $h = 5$ and $k = 0$. The function is reflected in the y-axis, so $b = -1$. To determine the value of a, substitute the coordinates of the maximum point of the roof, (0, 5), into $y = a\sqrt{-(x-5)}$.

$5 = a\sqrt{-(0-5)}$

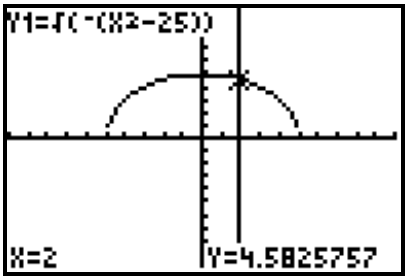
$5 = a\sqrt{5}$

$a = \sqrt{5}$

A function where y represents the distance from the base to the roof and x represents the horizontal distance from the centre is $y = \left(\sqrt{5}\right)\sqrt{-(x-5)}$.

b) For this situation, the domain is $\{x \mid 0 \le x \le 5, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \le y \le 5, y \in \mathrm{R}\}$. The domain cannot be negative nor greater than the radius of the base, or 5. The range cannot be negative nor greater than the height of the roof, or 5.

c) The approximate height of the roof at a point 2 m horizontally from the centre of the roof is 4.58 m.



Chapter 3 Polynomial Functions

Section 3.1 Characteristics of Polynomial Functions

Section 3.1 Page 114 Question 1

A polynomial function has the form
$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$,
where a_n is the leading coefficient; a_0 is the constant; and the degree of the polynomial, n, is the exponent of the greatest power of the variable, x.

a) The function $h(x) = 2 - \sqrt{x}$ is a radical function, not a polynomial function.
$\sqrt{x}$ is the same as $x^{\frac{1}{2}}$, which has an exponent that is not a whole number.

b) The function $y = 3x + 1$ is of the form $y = a_1x + a_0$.
It is a polynomial of degree 1. The leading coefficient is 3 and the constant term is 1.

c) The function $f(x) = 3^x$ is not a polynomial function.
The variable x is the exponent.

d) The function $g(x) = 3x^4 - 7$ is of the form $g(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 4. The leading coefficient is 3 and the constant term is –7.

e) The function $p(x) = x^{-3} + x^2 + 3x$ is not a polynomial function.
The term x^{-3} has an exponent that is not a whole number.

f) The function $y = -4x^3 + 2x + 5$ is of the form $g(x) = a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 3. The leading coefficient is –4 and the constant term is 5.

Section 3.1 Page 114 Question 2

a) The function $f(x) = -x + 3$ has degree 1; it is a linear function with a leading coefficient of –1, and a constant term of 3.

b) The function $y = 9x^2$ has degree 2; it is a quadratic function with a leading coefficient of 9, and a constant term of 0.

c) The function $g(x) = 3x^4 + 3x^3 - 2x + 1$ has degree 4; it is a quartic function with a leading coefficient of 3, and a constant term of 1.

d) First rewrite $k(x) = 4 - 3x^3$ in descending powers of x: $k(x) = -3x^3 + 4$.
The function $k(x) = -3x^3 + 4$ has degree 3; it is a cubic function with a leading coefficient of –3, and a constant term of 4.

e) The function $y = -2x^5 - 2x^3 + 9$ has degree 5; it is a quintic function with a leading coefficient of –2, and a constant term of 9.

f) The function $h(x) = -6$ has degree 0; it is a constant function with a leading coefficient of 0, and a constant term of –6.

Section 3.1 Page 114 Question 3

a) Since the graph of the function extends down into quadrant III and up into quadrant I, it is an odd-degree polynomial function with a positive leading coefficient.
The graph has three x-intercepts. Its domain is $\{x \mid x \in \mathrm{R}\}$ and its range is $\{y \mid y \in \mathrm{R}\}$.

b) Since the graph of the function extends down into quadrant III and up into quadrant I, it is an odd-degree polynomial function with a positive leading coefficient.
The graph has five x-intercepts. Its domain is $\{x \mid x \in \mathrm{R}\}$ and its range is $\{y \mid y \in \mathrm{R}\}$.

c) Since the graph of the function opens downward, extending down into quadrant III and down into quadrant IV, it is an even-degree polynomial function with a negative leading coefficient. The graph has three x-intercepts. Its domain is $\{x \mid x \in \mathrm{R}\}$ and its range is $\{y \mid y \leq 16.9, y \in \mathrm{R}\}$.

d) Since the graph of the function opens downward, extending down into quadrant III and down into quadrant IV, it is an even-degree polynomial function with a negative leading coefficient. The graph has no x-intercepts. Its domain is $\{x \mid x \in \mathrm{R}\}$ and its range is $\{y \mid y \leq -3, y \in \mathrm{R}\}$.

Section 3.1 Page 114 Question 4

a) The function $f(x) = x^2 + 3x - 1$ is a quadratic (degree 2), which is an even-degree polynomial function. Its graph has a maximum of two x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of –1.

b) The function $g(x) = -4x^3 + 2x^2 - x + 5$ is a cubic (degree 3), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most three x-intercepts. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV. The graph has no maximum or minimum values. The graph has a y-intercept of 5.

c) The function $h(x) = -7x^4 + 2x^3 - 3x^2 + 6x + 4$ is a quartic (degree 4), which is an even-degree polynomial function. Its graph has a maximum of four x-intercepts. Since the leading coefficient is negative, the graph of the function opens downward, extending down into quadrant III and down into quadrant IV, and has a maximum value. The graph has a y-intercept of 4.

d) The function $q(x) = x^5 - 3x^2 + 9x$ is a quintic (degree 5), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most five x-intercepts.

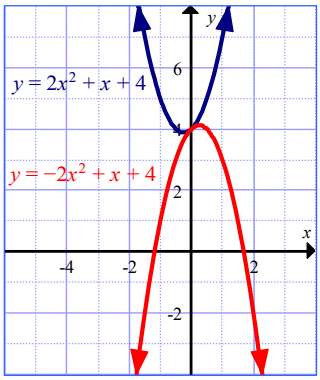
Since the leading coefficient is positive, the graph of the function extends down into quadrant III and up into quadrant I. The graph has no maximum or minimum values. The graph has a y-intercept of 0.

e) First rewrite $p(x) = 4 - 2x$ in descending powers of x: $p(x) = -2x + 4$.
The function $p(x) = -2x + 4$ is linear (degree 1), which is an odd-degree polynomial function. Its graph has one x-intercept. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV. The graph has no maximum or minimum values. The graph has a y-intercept of 4.

e) First rewrite $v(x) = -x^3 + 2x^4 - 4x^2$ in descending powers of x: $v(x) = 2x^4 - x^3 - 4x^2$.
The function $v(x) = 2x^4 - x^3 - 4x^2$ is a quartic (degree 4), which is an even-degree polynomial function. Its graph has a maximum of four x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of 0.

Section 3.1 Page 115 Question 5

Example: I disagree with Jake. Graphs of polynomial functions of the form $y = ax^n + x + b$, where a, b, and n are even integers, will extend from quadrant II to quadrant I only if a and n are positive even integers.
For example, the graph of $y = 2x^2 + x + 4$ has this shape, while the graph of $y = -2x^2 + x + 4$ does not. Also, if n is negative, then the function is no longer a polynomial.



Section 3.1 Page 115 Question 6

Rewrite $P(x) = 1000x + x^4 - 3000$ in descending powers of x: $P(x) = x^4 + 1000x - 3000$.
a) The function $P(x) = x^4 + 1000x - 3000$ has degree 4.

b) The leading coefficient is 1 and the constant is –3000. The constant represents a loss of $3000 if no snowboards are sold.

c) The function is an even-degree polynomial function. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I.

d) The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$. Since x represents the number of snowboards sold, it must be non-negative.

e) The x-intercepts of the graph represent when profit is zero.

f) For 1500 snowboards, substitute $x = 15$.

$P(x) = 1000x + x^4 - 3000$

$P(15) = 1000(15) + (15)^4 - 3000$

$P(15) = 62\ 625$

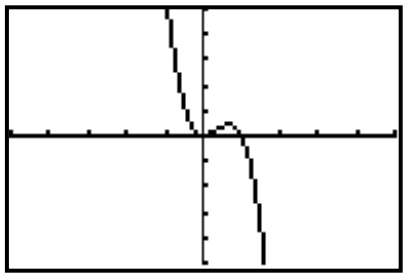
The profit from the sale of 1500 snowboards is \$62 625.

Section 3.1 Page 115 Question 7

a) The function $r(d) = -3d^3 + 3d^2$ is a cubic (degree 3).

b) The leading coefficient is –3 and the constant is 0.

c) Its graph has at least one x-intercept and at most three x-intercepts. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV (similar to the line $y = -x$).

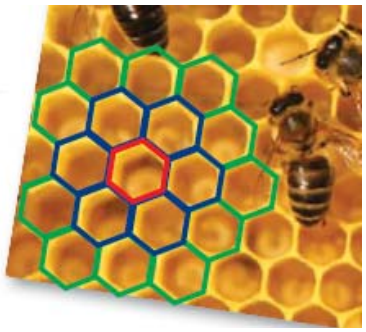


d) The domain is $\{d \mid 0 \leq d \leq 1, d \in \mathrm{R}\}$. Since d represents the amount of drug absorbed into the patient's bloodstream, it must be non-negative and the reaction time must be greater than or equal to zero.

Section 3.1 Page 115 Question 8

a)

Number of Rings r	**Total Number of Hexagons** $f(r) = 3r^2 - 3r + 1$
1	1
2	7
3	19



b) Substitute $r = 12$ into $f(r) = 3r^2 - 3r + 1$.

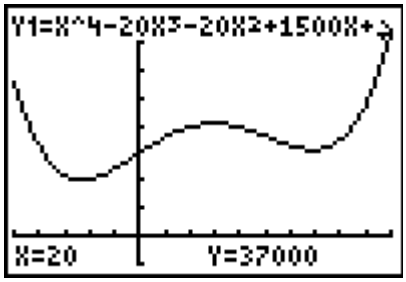
$f(12) = 3(12)^2 - 3(12) + 1$

$f(12) = 397$

The total number of hexagons in a honeycomb with 12 rings is 397.

Section 3.1 Page 116 Question 9

a) The function $P(t) = t^4 - 20t^3 - 20t^2 + 1500t + 15\,000$ is a quartic (degree 4), which is an even-degree polynomial function. This graph has no x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of 15 000. For this situation, the domain is $\{t \mid 0 \le t \le 20, t \in \mathrm{R}\}$ and its range is $\{P \mid 15\,000 \le P \le 37\,000, P \in \mathrm{R}\}$.



b) Currently, at $t = 0$, the population is 15 000 people.

c) Substitute $t = 10$.

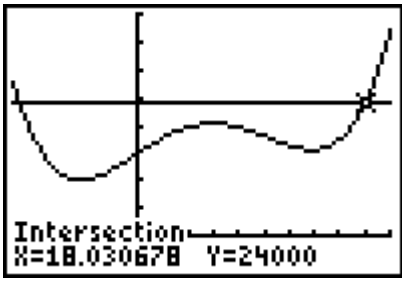
$P(t) = t^4 - 20t^3 - 20t^2 + 1500t + 15\,000$

$P(10) = (10)^4 - 20(10)^3 - 20(10)^2 + 1500(10) + 15\,000$

$P(10) = 18\,000$

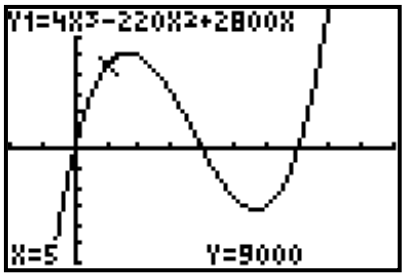
The population of the town 10 years from now will be 18 000 people.

d) The population of the town be approximately 24 000 in 18 years from now.



Section 3.1 Page 116 Question 10

a) For this situation, the domain is $\{x \mid 0 \le x \le 20 \text{ and } x \ge 35, x \in \mathrm{R}\}$ since x, the height of each box, cannot be negative.



b) $V(x) = 4x^3 - 220x^2 + 2800x$

$V(x) = 4x(x^2 - 55x + 700)$

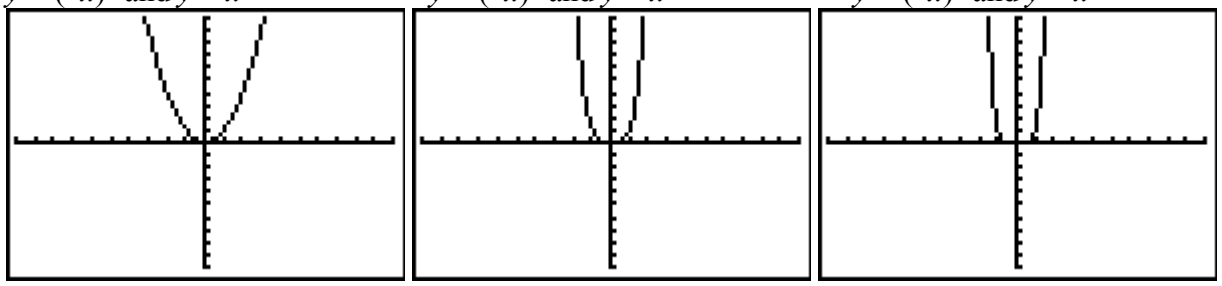
$V(x) = 4x(x - 20)(x - 35)$

The factored form of the function shows that the zeros of the function are the x-intercepts of its graph.

Section 3.1 Page 116 Question 11

a) In each case, the pair of graphs appears to be the same.

$y = (-x)^2$ and $y = x^2$ $y = (-x)^4$ and $y = x^4$ $y = (-x)^6$ and $y = x^6$



Let n represent a whole number, then $2n$ represents an even whole number.

$y = (-x)^{2n}$

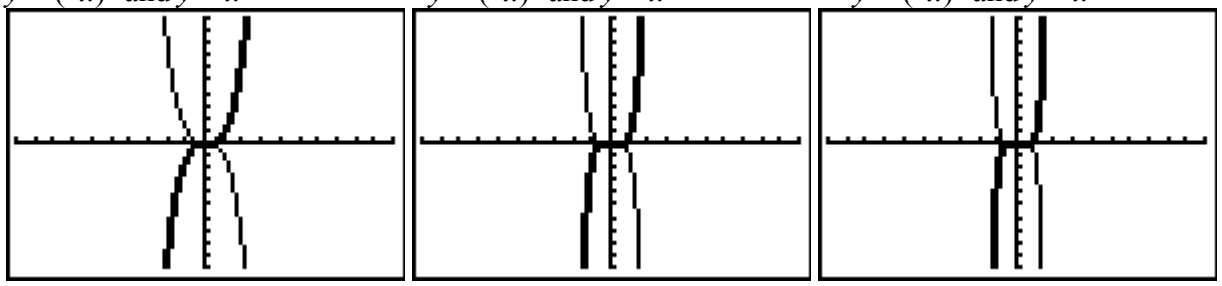
$y = (-1)^{2n}x^{2n}$

$y = 1^n x^{2n}$

$y = x^{2n}$

b) In each case, the graphs appear to be reflections of each other in the y-axis (or x-axis).

$y = (-x)^3$ and $y = x^3$ $y = (-x)^5$ and $y = x^5$ $y = (-x)^7$ and $y = x^7$



Let n represent a whole number, then $2n + 1$ represents an odd whole number.

$y = (-x)^{2n + 1}$

$y = (-1)^{2n + 1}x^{2n + 1}$

$y = (-1)^{2n}(-1)^1 x^{2n + 1}$

$y = -(1)^n x^{2n + 1}$

$y = -x^{2n + 1}$

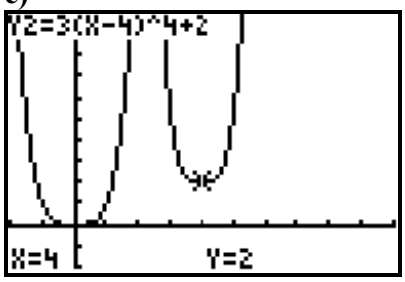
c) For even whole numbers, the graph of the functions are unchanged. For odd whole numbers, the graph of the functions are reflected in the y-axis (or x-axis).

Section 3.1 Page 116 Question 12

a) To obtain the graph of $y = 3(x - 4)^2 + 2$, the graph of $y = x^2$ is stretched vertically by a factor of 3 and translated 4 units to the right and 2 units up.

b) I predict that to obtain the graph of $y = 3(x - 4)^4 + 2$, the graph of $y = x^4$ is stretched vertically by a factor of 3 and translated 4 units to the right and 2 units up.

c)



Section 3.1 Page 116 Question 13

Let a represent the root. If a polynomial equation of degree n has exactly one real root, the corresponding polynomial function is of the form $y = (x - a)^n$. If this is an even-degree function, then its graph will only touch the x-axis. If this is an odd-degree function, then its graph will cross the x-axis once.

Section 3.1 Page 116 Question C1

Let n represent the degree of the polynomial function. Let a represent the maximum or minimum value.

Characteristic	Odd Degree	Even Degree
Number of x-intercepts	minimum of 1 and maximum of n	maximum of n
Number of maximum and minimum points	none	minimum point when $a_n > 0$, maximum point when $a_n < 0$
Domain	$\{x \mid x \in \mathrm{R}\}$	$\{x \mid x \in \mathrm{R}\}$
Range	$\{y \mid y \in \mathrm{R}\}$	$\{y \mid y \geq a, y \in \mathrm{R}\}$, $\{y \mid y \leq a, y \in \mathrm{R}\}$

Section 3.1 Page 116 Question C2

a) i) A graph that extends from quadrant III to I is an odd-degree function with a positive leading coefficient. For example, $y = x^3$ or $y = 3x^3 + 5$.

ii) A graph that extends from quadrant II to I is an even-degree function with a positive leading coefficient. For example, $y = x^4$ or $y = 2x^4 + 1$.

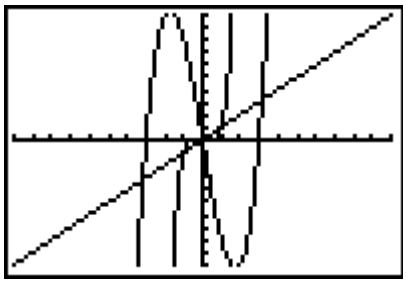
iii) A graph that extends from quadrant II to IV is an odd-degree function with a negative leading coefficient. For example, $y = -x^3$ or $y = -3x^3 + 5$.

iv) A graph that extends from quadrant III to IV is an even-degree function with a negative leading coefficient. For example, $y = -x^4$ or $y = -2x^4 + 1$.

b) Answers should agree on even or odd degree and on the sign of the leading coefficient.

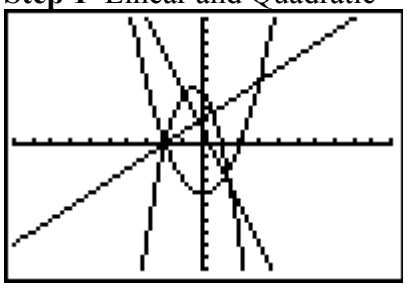
Section 3.1 Page 116 Question C3

Example: The line $y = x$ and polynomial functions with odd degree greater than one and positive leading coefficient extend from quadrant III to quadrant I. Both have no maximum or minimum value. Both have the same domain and range. While the graph of $y = x$ has only one x-intercept, the graph of an odd-degree polynomial function can have one, two, or three x-intercepts.

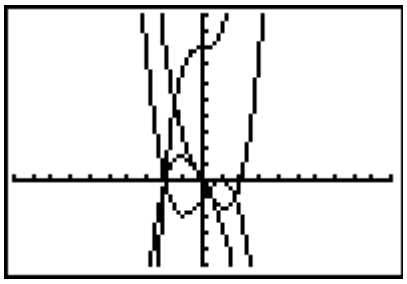


Section 3.1 Page 117 Question C4

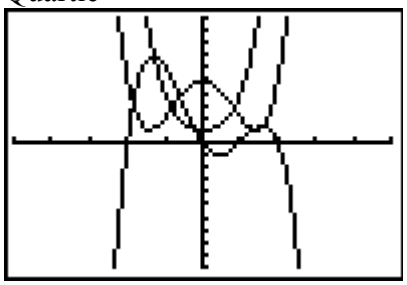
Step 1 Linear and Quadratic



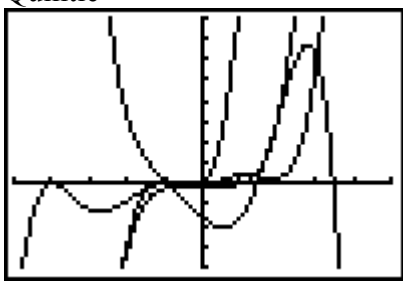
Cubic



Quartic



Quintic



Function	Degree	End Behaviour
$y = x + 2$	1	extends from quadrant III to I
$y = -3x + 1$	1	extends from quadrant II to IV
$y = x^2 - 4$	2	opens upward
$y = -2x^2 - 2x + 4$	2	opens downward
$y = x^3 - 4x$	3	extends from quadrant III to I
$y = -x^3 + 3x - 2$	3	extends from quadrant II to IV
$y = 2x^3 + 16$	3	extends from quadrant III to I
$y = -x^3 - 4x$	3	extends from quadrant II to IV
$y = x^4 - 4x^2 + 5$	4	opens upward
$y = -x^4 + x^3 + 4x^2 - 4x$	4	opens downward
$y = x^4 + 2x^2 + 1$	4	opens upward
$y = x^5 - 2x^4 - 3x^3 + 5x^2 + 4x - 1$	5	extends from quadrant III to I
$y = x^5 - 1$	5	extends from quadrant III to I

$y = -x^5 + x^4 + 8x^3 + 8x^2 - 16x - 16$	5	extends from quadrant II to IV
$y = x(x+1)^2(x+4)^2$	5	extends from quadrant III to I

Step 2 Even-degree functions: a positive leading coefficient means the graph opens upward and a negative leading coefficient means the graph opens downward. Odd-degree functions: a positive leading coefficient means the graph extends from quadrant III to I and a negative leading coefficient means the graph extends from quadrant II to IV.

Step 3 The end behaviours of even-degree functions are always headed in the same direction, either opening upward or downward.

Step 4. The end behaviours of odd-degree functions are always headed away from each other, either extending from quadrant III to I or from quadrant II to IV.

Section 3.2 The Remainder Theorem

Section 3.2 Page 124 Question 1

a)

$$\begin{array}{r} x+12 \\ x-2\overline{)\,x^2+10x-24} \\ \underline{x^2-2x} \\ 12x-24 \\ \underline{12x-24} \\ 0 \end{array}$$

$$\frac{x^2+10x-24}{x-2} = x+12+\frac{0}{x-2}$$

b) The restriction is $x \neq 2$.

c) The corresponding statement that can be used to check the division is $(x-2)(x+12)+0 = x^2+10x-24$.

d) $(x-2)(x+12)+0$
$= x^2+10x-24+0$
$= x^2+10x-24$

Section 3.2 Page 124 Question 2

a)

$$x+1\overline{)3x^4-4x^3-6x^2+17x-8}$$

$$\begin{array}{r} 3x^3-7x^2+x+16 \\ \hline 3x^4+3x^3 \\ -7x^3-6x^2 \\ \underline{-7x^3-7x^2} \\ x^2+17x \\ \underline{x^2+x} \\ 16x-8 \\ \underline{16x+16} \\ -24 \end{array}$$

$$\frac{3x^4-4x^3-6x^2+17x-8}{x+1} = 3x^3-7x^2+x+16+\left(\frac{-24}{x+1}\right)$$

b) The restriction is $x \neq -1$.

c) The corresponding statement that can be used to check the division is $(x + 1)(3x^3 - 7x^2 + x + 16) - 24 = 3x^4 - 4x^3 - 6x^2 + 17x - 8$.

d) $(x + 1)(3x^3 - 7x^2 + x + 16) - 24$
$= 3x^4 - 7x^3 + x^2 + 16x + 3x^3 - 7x^2 + x + 16 - 24$
$= 3x^4 - 4x^3 - 6x^2 + 17x - 8$

Section 3.2 Page 124 Question 3

a) For $(x^3 + 3x^2 - 3x - 2) \div (x - 1)$, the quotient, Q, is $x^2 + 4x + 1$.

$$x-1\overline{)x^3+3x^2-3x-2}$$

$$\begin{array}{r} x^2+4x+1 \\ \hline x^3-x^2 \\ 4x^2-3x \\ \underline{4x^2-4x} \\ x-2 \\ \underline{x-1} \\ -1 \end{array}$$

b) For $\frac{x^3+2x^2-7x-2}{x-2}$, the quotient, Q, is $x^2 + 4x + 1$.

$$x-2\overline{)x^3+2x^2-7x-2}$$

$$\begin{array}{r} x^2+4x+1 \\ \hline x^3-2x^2 \\ 4x^2-7x \\ \underline{4x^2-8x} \\ x-2 \\ \underline{x-2} \\ 0 \end{array}$$

c) For $(2w^3 + 3w^2 - 5w + 2) \div (w + 3)$, the quotient, Q, is $2w^2 - 3w - 4$.

$$
\begin{array}{r}
2w^2 - 3w - 4 \\
w+3 \overline{)\,2w^3 + 3w^2 - 5w + 2} \\
\underline{2w^3 + 6w^2} \\
-3w^2 - 5w \\
\underline{-3w^2 - 9w} \\
-4w + 2 \\
\underline{-4w - 12} \\
14
\end{array}
$$

d) For $(9m^3 - 6m^2 + 3m + 2) \div (m - 1)$, the quotient, Q, is $9m^2 + 3m + 6$.

$$
\begin{array}{r}
9m^2 + 3m + 6 \\
m-1 \overline{)\,9m^3 - 6m^2 + 3m + 2} \\
\underline{9m^3 - 9m^2} \\
3m^2 + 3m \\
\underline{3m^2 - 3m} \\
6m + 2 \\
\underline{6m - 6} \\
8
\end{array}
$$

e) For $\dfrac{t^4 + 6t^3 - 3t^2 - t + 8}{t + 1}$, the quotient, Q, is $t^3 + 5t^2 - 8t + 7$.

$$
\begin{array}{r}
t^3 + 5t^2 - 8t + 7 \\
t+1 \overline{)\,t^4 + 6t^3 - 3t^2 - t + 8} \\
\underline{t^4 + t^3} \\
5t^3 - 3t^2 \\
\underline{5t^3 + 5t^2} \\
-8t^2 - t \\
\underline{-8t^2 - 8t} \\
7t + 8 \\
\underline{7t + 7} \\
1
\end{array}
$$

f) For $(2y^4 - 3y^2 + 1) \div (y - 3)$, the quotient, Q, is $2y^3 + 6y^2 + 15y + 45$.

$$
\begin{array}{r}
2y^3 + 6y^2 + 15y + 45 \\
y-3 \overline{)\,2y^4 + 0y^3 - 3y^2 + 0y + 1} \\
\underline{2y^4 - 6y^3} \\
6y^3 - 3y^2 \\
\underline{6y^3 - 18y^2} \\
15y^2 + 0y \\
\underline{15y^2 - 45y} \\
45y + 1 \\
\underline{45y - 135} \\
136
\end{array}
$$

Section 3.2 Page 124 Question 4

a) For $(x^3 + x^2 + 3) \div (x + 4)$, the quotient, Q, is $x^2 - 3x + 12$.

+4	1	1	0	1
−		4	−12	48
×	1	−3	12	−47

b) For $\dfrac{m^4 - 2m^3 + m^2 + 12m - 6}{m - 2}$, the quotient, Q, is $m^3 + m + 14$.

−2	1	−2	1	12	−6
−		−2	0	−2	−28
×	1	0	1	14	22

c) For $(2 - x + x^2 - x^3 - x^4) \div (x + 2)$, the quotient, Q, is $-x^3 + x^2 - x + 1$.

+2	–1	–1	1	–1	2
–		–2	2	–2	2
×	–1	1	–1	1	0

d) For $(2s^3 + 3s^2 - 9s - 10) \div (s - 2)$, the quotient, Q, is $2s^2 + 7s + 5$.

–2	2	3	–9	–10
–		–4	–14	–10
×	2	7	5	0

e) For $\dfrac{h^3 + 2h^2 - 3h + 9}{h + 3}$, the quotient, Q, is $h^2 - h$.

+3	1	2	–3	9
–		3	–3	0
×	1	–1	0	9

f) For $(2x^3 + 7x^2 - x + 1) \div (x + 2)$, the quotient, Q, is $2x^2 + 3x - 7$.

+2	2	7	–1	1
–		4	6	–14
×	2	3	–7	15

Section 3.2 Page 124 Question 5

a) For $(x^3 + 7x^2 - 3x + 4) \div (x + 2)$, $x \neq -2$.

+2	1	7	–3	4
–		2	10	–26
×	1	5	–13	30

$$\frac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \frac{30}{x + 2}$$

b) For $\dfrac{11t - 4t^4 - 7}{t - 3}$, $t \neq 3$.

–3	–4	0	0	11	–7
–		12	36	108	291
×	–4	–12	–36	–97	–298

$$\frac{11t - 4t^4 - 7}{t - 3} = -4t^3 - 12t^2 - 36t - 97 - \frac{298}{t - 3}$$

c) For $(x^3 + 3x^2 - 2x + 5) \div (x + 1)$, $x \neq -1$.

+1	1	3	–2	5
–		1	2	–4
×	1	2	–4	9

$$\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$$

d) For $(4n^2 + 7n - 5) \div (n + 3)$, $n \neq -3$.

+3	4	7	–5
–		12	–15
×	4	–5	10

$$\frac{4n^2 - 7n - 5}{n + 3} = 4n - 5 + \frac{10}{n + 3}$$

e) For $\dfrac{4n^3 - 15n + 2}{n - 3}$, $n \neq 3$.

–3	4	0	–15	2
–		–12	–36	–63
×	4	12	21	65

$$\frac{4n^3 - 15n + 2}{n - 3} = 4n^2 + 12n + 21 + \frac{65}{n - 3}$$

f) For $(x^3 + 6x^2 - 4x + 1) \div (x + 2)$, $x \neq -2$.

+2	1	6	–4	1
–		2	8	–24
×	1	4	–12	25

$$\frac{x^3 + 6x^2 - 4x + 1}{x + 2} = x^2 + 4x - 12 + \frac{25}{x + 2}$$

Section 3.2 Page 124 Question 6

Evaluate $P(-2)$.

a) $P(x) = x^3 + 3x^2 - 5x + 2$

$P(-2) = (-2)^3 + 3(-2)^2 - 5(-2) + 2$

$P(-2) = -8 + 12 + 10 + 2$

$P(-2) = 16$

The remainder when $x^3 + 3x^2 - 5x + 2$ is divided by $x + 2$ is 16.

b) $P(x) = 2x^4 - 2x^3 + 5x$

$P(-2) = 2(-2)^4 - 2(-2)^3 + 5(-2)$

$P(-2) = 32 + 16 - 10$

$P(-2) = 38$

The remainder when $2x^4 - 2x^3 + 5x$ is divided by $x + 2$ is 38.

c) $P(x) = x^4 + x^3 - 5x^2 + 2x - 7$

$P(-2) = (-2)^4 + (-2)^3 - 5(-2)^2 + 2(-2) - 7$

$P(-2) = 16 - 8 - 20 - 4 - 7$

$P(-2) = -23$

The remainder when $x^4 + x^3 - 5x^2 + 2x - 7$ is divided by $x + 2$ is -23.

d) $P(x) = 8x^3 + 4x^2 - 19$

$P(-2) = 8(-2)^3 + 4(-2)^2 - 19$

$P(-2) = -64 + 16 - 19$

$P(-2) = -67$

The remainder when $8x^3 + 4x^2 - 19$ is divided by $x + 2$ is -67.

e) $P(x) = 3x^3 - 12x - 2$

$P(-2) = 3(-2)^3 - 12(-2) - 2$

$P(-2) = -24 + 24 - 2$

$P(-2) = -2$

The remainder when $3x^3 - 12x - 2$ is divided by $x + 2$ is -2.

f) $P(x) = 2x^3 + 3x^2 - 5x + 2$

$P(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) + 2$

$P(-2) = -16 + 12 + 10 + 2$

$P(-2) = 8$

The remainder when $2x^3 + 3x^2 - 5x + 2$ is divided by $x + 2$ is 8.

Section 3.2 Page 124 Question 7

a) Evaluate $P(-3)$.

$P(x) = x^3 + 2x^2 - 3x + 9$

$P(-3) = (-3)^3 + 2(-3)^2 - 3(-3) + 9$

$P(-3) = -27 + 18 + 9 + 9$

$P(-3) = 9$

The remainder when $x^3 + 2x^2 - 3x + 9$ is divided by $x + 3$ is 9.

b) Evaluate $P(2)$.
$P(t) = 2t - 4t^3 - 3t^2$
$P(2) = 2(2) - 4(2)^3 - 3(2)^2$
$P(2) = 4 - 32 - 12$
$P(2) = -40$
The remainder when $2t - 4t^3 - 3t^2$ is divided by $t - 2$ is -40.

c) Evaluate $P(3)$.
$P(x) = x^3 + 2x^2 - 3x + 5$
$P(3) = (3)^3 + 2(3)^2 - 3(3) + 5$
$P(3) = 27 + 18 - 9 + 5$
$P(3) = 41$
The remainder when $x^3 + 2x^2 - 3x + 5$ is divided by $x - 3$ is 41.

d) Evaluate $P(2)$.
$P(n) = n^4 - 3n^2 - 5n + 2$
$P(2) = (2)^4 - 3(2)^2 - 5(2) + 2$
$P(3) = 16 - 12 - 10 + 2$
$P(3) = -4$
The remainder when $n^4 - 3n^2 - 5n + 2$ is divided by $n - 2$ is -4.

Section 3.2 Page 124 Question 8

a) Solve $P(1) = 3$ to determine the value of k.
$P(x) = x^3 + 4x^2 - x + k$
$3 = 1^3 + 4(1)^2 - 1 + k$
$3 = 1 + 4 - 1 + k$
$k = -1$

b) Solve $P(2) = 3$ to determine the value of k.
$P(x) = x^3 + x^2 + kx - 15$
$3 = 2^3 + 2^2 + k(2) - 15$
$3 = 8 + 4 + 2k - 15$
$k = 3$

c) Solve $P(-2) = 3$ to determine the value of k.
$P(x) = x^3 + kx^2 + x + 5$
$3 = (-2)^3 + k(-2)^2 + (-2) + 5$
$3 = -8 + 4k - 2 + 5$
$k = 2$

d) Solve $P(-2) = 3$ to determine the value of k.

$P(x) = kx^3 + 3x + 1$

$3 = k(-2)^3 + 3(-2) + 1$

$3 = -8k - 6 + 1$

$k = -1$

Section 3.2 Page 124 Question 9

First, determine an expression for $P(2)$ and $P(-1)$.

$P(x) = -2x^3 + cx^2 - 5x + 2$ $\qquad$ $P(x) = -2x^3 + cx^2 - 5x + 2$

$P(2) = -2(2)^3 + c(2)^2 - 5(2) + 2$ $\qquad$ $P(-1) = -2(-1)^3 + c(-1)^2 - 5(-1) + 2$

$P(2) = -16 + 4c - 10 + 2$ $\qquad$ $P(-1) = 2 + c + 5 + 2$

$P(2) = 4c - 24$ $\qquad$ $P(-1) = c + 9$

Then, solve $P(2) = P(-1)$ to determine the value of c.

$4c - 24 = c + 9$

$3c = 33$

$c = 11$

Section 3.2 Page 125 Question 10

Solve $P(-k) = 14$ to determine the value of k.

$P(x) = 3x^2 + 6x - 10$

$14 = 3(-k)^2 + 6(-k) - 10$

$14 = 3k^2 - 6k - 10$

$0 = 3k^2 - 6k - 24$

$0 = k^2 - 2k - 8$

$0 = (k - 4)(k + 2)$

$k - 4 = 0$ or $k + 2 = 0$

$k = 4$ $\qquad$ $k = -2$

Section 3.2 Page 125 Question 11

a) $A(x) = 2x^2 - x - 6$

$A(x) = (2x + 3)(x - 2)$

If the height of the rectangle is $x - 2$, then the width is $2x + 3$.

b) Evaluate $A(3)$.

$A(x) = 2x^2 - x - 6$

$A(3) = 2(3)^2 - (3) - 6$

$A(3) = 18 - 3 - 6$

$A(3) = 9$

The remainder is 9. This represents a portion of the width: $\frac{2x^2 - x - 6}{x - 3} = 2x + 5 + \frac{9}{x - 3}$.

Section 3.2 Page 125 Question 12

a) Determine $2n^2 - 4n + 3 \div n - 3$.

–3	2	–4	3
–		–6	–6
×	2	2	9

An expression for the other real number is $2n + 2 + \frac{9}{n-3}$.

b) For $n = 1$:

$$n - 3 = 1 - 3 = -2$$

$$2n + 2 + \frac{9}{n-3} = 2(1) + 2 + \frac{9}{1-3} = 2 + 2 - 4.5 = -0.5$$

For $n = 1$, the two numbers are –2 and –0.5.

Section 3.2 Page 125 Question 13

a) Use synthetic division.

+3	9π	51π	88π	48π
–		27π	72π	48π
×	9π	24π	16π	0

The expression $9\pi x^2 + 24\pi x + 16\pi$ represents the area of the base of the cylindrical containers.

b) $V(x) = (9\pi x^2 + 24\pi x + 16\pi)(x + 3)$
$V(x) = \pi(9x^2 + 24x + 16)(x + 3)$
$V(x) = \pi(3x + 4)^2(x + 3)$
$V = \pi r^2 h$
In this factored form, $3x + 4$ represents the radius, r, and $x + 3$ represents the height, h.

c) If $2 \le x \le 8$, then $10 \le r \le 28$ and $5 \le h \le 11$.
For the given values of x, the radius can be from 10 cm to 28 cm and the height can be from 5 cm to 11 cm.

Section 3.2 Page 125 Question 14

Solve a system of equations represented by $P(-3) = -1$ and $P(2) = -4$.

$P(x) = mx^3 - 3x^2 + nx + 2$
$-1 = m(-3)^3 - 3(-3)^2 + n(-3) + 2$
$-1 = -27m - 27 - 3n + 2$
$24 = -27m - 3n$
$8 = -9m - n$ ①

$P(x) = mx^3 - 3x^2 + nx + 2$
$-4 = m(2)^3 - 3(2)^2 + n(2) + 2$
$-4 = 8m - 12 + 2n + 2$
$6 = 8m + 2n$
$3 = 4m + n$ ②

Add equations ① and ②: $11 = -5m$

$$m = -\frac{11}{5}$$

Substitute $m = -\frac{11}{5}$ into equation ②: $3 = 4\left(-\frac{11}{5}\right) + n$

$$n = \frac{59}{5}$$

Section 3.2 Page 125 Question 15

Solve a system of equations represented by $P(2) = -5$ and $P(-1) = -16$.

$P(x) = 3x^3 + ax^2 + bx - 9$

$-5 = 3(2)^3 + a(2)^2 + b(2) - 9$

$-5 = 24 + 4a + 2b - 9$

$-20 = 4a + 2b$

$-10 = 2a + b$ ①

$P(x) = 3x^3 + ax^2 + bx - 9$

$-16 = 3(-1)^3 + a(-1)^2 + b(-1) - 9$

$-16 = -3 + a - b - 9$

$-4 = a - b$ ②

Add equations ① and ②: $-14 = 3a$

$$a = -\frac{14}{3}$$

Substitute $a = -\frac{14}{3}$ into equation ②: $-4 = -\frac{14}{3} - b$

$$b = -\frac{2}{3}$$

Section 3.2 Page 125 Question 16

To determine the remainder when $10x^4 - 11x^3 - 8x^2 + 7x + 9$ is divided by $2x - 3$, using synthetic division, rewrite the binomial as $x - \frac{3}{2}$, or $x - 1.5$.

–1.5	10	–11	–8	7	9
–		–15	–6	3	–6
×	10	4	–2	4	15

The remainder is 15.

Section 3.2 Page 125 Question 17

Examples:

a) A quadratic polynomial that gives $P(3) = -4$ is $P(x) = (x - 1)(x - 3) - 4$, or $P(x) = x^2 - 4x - 1$.

b) A cubic polynomial that gives $P(-2) = 4$ is $P(x) = (x - 1)^2(x + 2) + 4$, or $P(x) = x^3 - 3x + 6$.

c) A quartic polynomial that gives $P(0.5) = 1$ is $P(x) = (x - 1)^3(2x - 1) + 1$, or $P(x) = 2x^4 - 7x^3 + 9x^2 - 5x + 2$.

Section 3.2 Page 125 Question C1

Example: The process is basically the same. However in long division of polynomials, additional terms are brought down and added to the previous results. Polynomial division also results in a restriction.

Section 3.2 Page 125 Question C2

a) If the remainder is 0, then $x - a$ is a factor of $bx^2 + cx + d$.

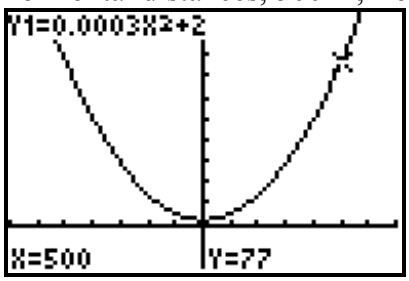
b) Evaluate $P(a)$.
$P(x) = bx^2 + cx + d$
$P(a) = ba^2 + ca + d$
$P(a) = a^2b + ac + d$
The remainder is $a^2b + ac + d$.

Section 3.2 Page 125 Question C3

a) Evaluate $h(500)$.
$h(d) = 0.0003d^2 + 2$
$h(500) = 0.0003(500)^2 + 2$
$h(500) = 77$
The remainder is 77.

b) Evaluate $h(-500)$.
$h(d) = 0.0003d^2 + 2$
$h(-500) = 0.0003(-500)^2 + 2$
$h(-500) = 752$
The remainder is 77.

c) The remainders are the same. They represent the height of the cable at the given horizontal distances, 500 m, from the lowest point.



Section 3.3 The Factor Theorem

Section 3.3 Page 133 Question 1

a) If $P(1) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x - 1$.

b) If $P(-3) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x + 3$.

c) If $P(4) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x - 4$.

d) If $P(a) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x - a$.

Section 3.3 Page 133 Question 2

a) Evaluate $P(1)$.
$P(x) = x^3 - 3x^2 + 4x - 2$
$P(1) = (1)^3 - 3(1)^2 + 4(1) - 2$
$P(1) = 1 - 3 + 4 - 2$
$P(1) = 0$
Since the remainder is zero, $x - 1$ is a factor of $P(x)$.

b) Evaluate $P(1)$.
$P(x) = 2x^3 - x^2 - 3x - 2$
$P(1) = 2(1)^3 - (1)^2 - 3(1) - 2$
$P(1) = 2 - 1 - 3 - 2$
$P(1) = -4$
Since the remainder is not zero, $x - 1$ is not a factor of $P(x)$.

c) Evaluate $P(1)$.
$P(x) = 3x^3 - x - 3$
$P(1) = 3(1)^3 - (1)^2 - 3$
$P(1) = 3 - 1 - 3$
$P(1) = -1$
Since the remainder is not zero, $x - 1$ is not a factor of $P(x)$.

d) Evaluate $P(1)$.
$P(x) = 2x^3 + 4x^2 - 5x - 1$
$P(1) = 2(1)^3 + 4(1)^2 - 5(1) - 1$
$P(1) = 2 + 4 - 5 - 1$
$P(1) = 0$
Since the remainder is zero, $x - 1$ is a factor of $P(x)$.

e) Evaluate $P(1)$.
$P(x) = x^4 - 3x^3 + 2x^2 - x + 1$
$P(1) = (1)^4 - 3(1)^3 + 2(1)^2 - 1 + 1$
$P(1) = 1 - 3 + 2 - 1 + 1$
$P(1) = 0$

Since the remainder is zero, $x - 1$ is a factor of $P(x)$.

f) Evaluate $P(1)$.
$P(x) = 4x^4 - 2x^3 + 3x^2 - 2x + 1$
$P(1) = 4(1)^4 - 2(1)^3 + 3(1)^2 - 2(1) + 1$
$P(1) = 4 - 2 + 3 - 2 + 1$
$P(1) = 4$
Since the remainder is not zero, $x - 1$ is not a factor of $P(x)$.

Section 3.3 Page 133 Question 3

a) Evaluate $P(-2)$.
$P(x) = 5x^2 + 2x + 6$
$P(-2) = 5(-2)^2 + 2(-2) + 6$
$P(-2) = 20 - 4 + 6$
$P(-2) = 22$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

b) Evaluate $P(-2)$.
$P(x) = 2x^3 - x^2 - 5x - 8$
$P(-2) = 2(-2)^3 - (-2)^2 - 5(-2) - 8$
$P(-2) = -16 - 4 + 10 - 8$
$P(-2) = -18$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

c) Evaluate $P(-2)$.
$P(x) = 2x^3 + 2x^2 - x - 6$
$P(-2) = 2(-2)^3 + 2(-2)^2 - (-2) - 6$
$P(-2) = -16 + 8 + 2 - 6$
$P(-2) = -12$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

d) Evaluate $P(-2)$.
$P(x) = x^4 - 2x^2 + 3x - 4$
$P(-2) = (-2)^4 - 2(-2)^2 + 3(-2) - 4$
$P(-2) = 16 - 8 - 6 - 4$
$P(-2) = -2$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

e) Evaluate $P(-2)$.
$P(x) = x^4 + 3x^3 - x^2 - 3x + 6$
$P(-2) = (-2)^4 + 3(-2)^3 - (-2)^2 - 3(-2) + 6$
$P(-2) = 16 - 24 - 4 + 6 + 6$
$P(-2) = 0$
Since the remainder is zero, $x + 2$ is a factor of $P(x)$.

f) Evaluate $P(-2)$.
$P(x) = 3x^4 + 5x^3 + x - 2$
$P(-2) = 3(-2)^4 + 5(-2)^2 + (-2) - 2$
$P(-2) = 48 + 20 - 2 - 2$
$P(-2) = 64$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

Section 3.3 Page 133 Question 4

a) Let $P(x) = x^3 + 3x^2 - 6x - 8$. The possible integral zeros of the polynomial are the factors of the constant term, –8: ±1, ±2, ±4, and ±8.

b) Let $P(s) = s^3 + 4s^2 - 15s - 18$. The possible integral zeros of the polynomial are the factors of the constant term, –18: ±1, ±2, ±3, ±6, ±9, and ±18.

c) Let $P(n) = n^3 - 3n^2 - 10n + 24$. The possible integral zeros of the polynomial are the factors of the constant term, 24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24.

d) Let $P(p) = p^4 - 2p^3 - 8p^2 + 3p - 4$. The possible integral zeros of the polynomial are the factors of the constant term, –4: ±1, ±2, and ±4.

e) Let $P(z) = z^4 + 5z^3 + 2z^2 + 7z - 15$. The possible integral zeros of the polynomial are the factors of the constant term, –15: ±1, ±3, ±5, and ±15.

f) Let $P(y) = y^4 - 5y^3 - 7y^2 + 21y + 4$. The possible integral zeros of the polynomial are the factors of the constant term, 4: ±1, ±2, and ±4.

Section 3.3 Page 134 Question 5

a) For $P(x) = x^3 - 6x^2 + 11x - 6$, the possible integral zeros of the polynomial are the factors of the constant term, –6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	–6	11	–6
–		–1	5	–6
×	1	–5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $P(x) = (x - 1)(x - 2)(x - 3)$.

b) For $P(x) = x^3 + 2x^2 - x - 2$, the possible integral zeros of the polynomial are the factors of the constant term, –2: ±1 and ±2. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	2	–1	–2
–		–1	–3	–2
×	1	3	2	0

Then, the remaining factor $x^2 + 3x + 2$ can be factored as $(x + 1)(x + 2)$.
So, $P(x) = (x - 1)(x + 1)(x + 2)$.

c) For $P(v) = v^3 + v^2 - 16v - 16$, the possible integral zeros of the polynomial are the factors of the constant term, –16: ±1, ±2, ±4, ±8, and ±16. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	1	–16	–16
–		1	0	–16
×	1	0	–16	0

Then, the remaining factor $v^2 - 16$ can be factored as $(v + 4)(v - 4)$.
So, $P(v) = (v + 1)(v + 4)(v - 4)$.

d) For $P(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$, the possible integral zeros of the polynomial are the factors of the constant term, –24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	4	–7	–34	–24
–		1	3	–10	–24
×	1	3	–10	–24	0

Repeat the process with the remaining factor $x^3 + 3x^2 - 10x - 24$. Test factors of the constant term, –24: $P(-2) = 0$.

+2	1	3	–10	–24
–		2	2	–24
×	1	1	–12	0

Then, the remaining factor $x^2 + x - 12$ can be factored as $(x + 4)(x - 3)$.
So, $P(x) = (x + 1)(x + 2)(x + 4)(x - 3)$.

e) For $P(k) = k^5 + 3k^4 - 5k^3 - 15k^2 + 4k + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	3	–5	–15	4	12
–		–1	–4	1	16	12
×	1	4	–1	–16	–12	0

Repeat the process with the remaining factor $k^4 + 4k^3 - k^2 - 16k - 12$. Test factors of the constant term, –12: $P(-1) = 0$.

+1	1	4	–1	–16	–12
–		1	3	–4	–12
×	1	3	–4	–12	0

Then, the remaining factor $k^3 + 3k^2 - 4k - 12$ can be factored by grouping as $(k + 3)(k^2 - 4)$.
So, $P(k) = (k - 1)(k + 1)(k + 3)(k + 2)(k - 2)$.

Section 3.3 Page 134 Question 6

a) For $P(x) = x^3 - 2x^2 - 9x + 18$, the possible integral zeros of the polynomial are the factors of the constant term, 18: ±1, ±2, ±3, ±6, ±9, and ±18. Test these values to find a first factor: $P(2) = 0$. Use synthetic division to find the other factors.

–2	1	–2	–9	18
–		–2	0	18
×	1	0	–9	0

Then, the remaining factor $x^2 - 9$ can be factored as $(x + 3)(x - 3)$.
So, $P(x) = (x - 2)(x + 3)(x - 3)$.

b) For $P(t) = t^3 + t^2 - 22t - 40$, the possible integral zeros of the polynomial are the factors of the constant term, –40: ±1, ±2, ±4, ±5, ±8, ±10, ±20, and ±40. Test these values to find a first factor: $P(-2) = 0$. Use synthetic division to find the other factors.

+2	1	1	–22	–40
–		2	–2	–40
×	1	–1	–20	0

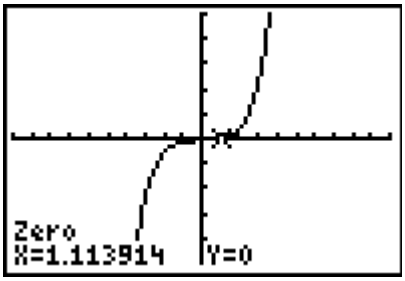
Then, the remaining factor $t^2 - t - 20$ can be factored as $(t + 4)(t - 5)$.
So, $P(t) = (t + 2)(t + 4)(t - 5)$.

c) For $P(h) = h^3 - 27h + 10$, the possible integral zeros of the polynomial are the factors of the constant term, 10: ±1, ±2, ±5, and ±10. Test these values to find a first factor: $P(5) = 0$. Use synthetic division to find the other factors.

–5	1	0	–27	10
–		–5	–25	10
×	1	5	–2	0

Then, the remaining factor $h^2 + 5h - 2$ cannot be factored.
So, $P(h) = (h - 5)(h^2 + 5h - 2)$.

d) For $P(x) = x^5 + 8x^3 + 2x - 15$, the possible integral zeros of the polynomial are the factors of the constant term, –15: ±1, ±3, ±5, and ±15. Test these values to find that none of them is a factor.
Graphing the corresponding function shows one non-integral zero at $x \approx 1.1$.



e) For $P(q) = q^4 + 2q^3 + 2q^2 - 2q - 3$, the possible integral zeros of the polynomial are the factors of the constant term, –3: ±1 and ±3. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	2	2	–2	–3
–		–1	–3	–5	–3
×	1	3	5	3	0

Repeat the process with the remaining factor $q^3 + 3q^2 + 5q + 3$. Test factors of the constant term, 3: $P(–1) = 0$.

+1	1	3	5	3
–		1	2	3
×	1	2	3	0

Then, the remaining factor $q^2 + 2q + 3$ cannot be factored.
So, $P(q) = (q – 1)(q + 1)(q^2 + 2q + 3)$.

Section 3.3 Page 134 Question 7

a) Solve $P(2) = 0$ to determine the value(s) of k.

$P(x) = x^2 – x + k$
$0 = 2^2 – 2 + k$
$0 = 2 + k$
$k = –2$

b) Solve $P(–k) = 0$ to determine the value(s) of k.

$P(x) = x^2 – 6x – 7$
$0 = (–k)^2 – 6(–k) – 7$
$0 = k^2 + 6k – 7$
$0 = (k + 7)(k – 1)$
$k + 7 = 0$ or $k – 1 = 0$
$k = –7$ $k = 1$

c) Solve $P(–2) = 0$ to determine the value(s) of k.

$P(x) = x^3 + 4x^2 + x + k$
$0 = (–2)^3 + 4(–2)^2 + (–2) + k$
$0 = 6 + k$
$k = –6$

d) Solve $P(2) = 0$ to determine the value(s) of k.

$P(x) = x^2 + kx – 16$
$0 = 2^2 + k(2) – 16$
$0 = 2k – 12$
$k = 6$

Section 3.3 Page 134 Question 8

Factor $V(h) = h^3 - 2h^2 + h$.
$V(h) = h^3 - 2h^2 + h$
$V(h) = h(h^2 - 2h + 1)$
$V(h) = h(h - 1)(h - 1)$
The possible dimensions of the bookcase are h, $h - 1$, and $h - 1$.

Section 3.3 Page 134 Question 9

Factor $V(\ell) = \ell^3 - 2\ell^2 - 15\ell$.
$V(\ell) = \ell^3 - 2\ell^2 - 15\ell$
$V(\ell) = \ell(\ell^2 - 2\ell - 15)$
$V(\ell) = \ell(\ell - 5)(\ell + 3)$
The possible width and height of the racquetball court are $\ell - 5$ and $\ell + 3$.

Section 3.3 Page 134 Question 10

For $V(x) = x^3 + 5x^2 - 2x - 24$, the possible integral zeros of the polynomial are the factors of the constant term, –24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24. Test these values to find a first factor: $P(2) = 0$. Use synthetic division to find the other factors.

–2	1	5	–2	–24
–		–2	–14	–24
×	1	7	12	0

Then, the remaining factor $x^2 + 7x + 12$ can be factored as $(x + 3)(x + 4)$.
So, $V(x) = (x - 2)(x + 3)(x + 4)$.
The possible dimensions of the block, in centimetres, are $(x - 2)$ cm by $(x + 3)$ cm by $(x + 4)$ cm.

Section 3.3 Page 134 Question 11

For $V(x) = x^3 + 14x^2 + 63x + 90$, the given factor is $x + 6$. Use synthetic division to find the other factors.

+6	1	14	63	90
–		6	48	90
×	1	8	15	0

Then, the remaining factor $x^2 + 8x + 15$ can be factored as $(x + 3)(x + 5)$.
So, $V(x) = (x + 6)(x + 3)(x + 5)$.
The polynomials that represent the possible length and width of the fish tank are $x + 3$ and $x + 5$.

Section 3.3 Page 135 Question 12

a) $x - 5$ is a possible factor because it is the corresponding factor for $x = 5$.
Evaluate $f(5)$.
$f(x) = x^4 - 14x^3 + 69x^2 - 140x + 100$
$f(5) = (5)^4 - 14(5)^3 + 69(5)^2 - 140(5) + 100$
$f(5) = 625 - 1750 + 1725 - 700 + 100$
$f(5) = 0$
Since $f(5) = 0$, $x - 5$ is a factor of the polynomial function.

b) Use synthetic division to find the other factors.

–5	1	–14	69	–140	100
–		–5	45	–120	100
×	1	–9	24	–20	0

Repeat the process with the remaining factor $x^3 - 9x^2 + 24x - 20$. Test factors of the constant term, –20: $P(2) = 0$.

–2	1	–9	24	–20
–		–2	14	–20
×	1	–7	10	0

Then, the remaining factor $x^2 - 7x + 10$ can be factored as $(x - 5)(x - 2)$.
So, $f(x) = (x - 5)^2(x - 2)^2$.
Since $x - 2$ is also a factor, another length of plastic that is extremely weak is a 2-ft section.

Section 3.3 Page 135 Question 13

$x^4 + 6x^3 + 11x^2 + 6x = x(x^3 + 6x^2 + 11x + 6)$
Factor $x^3 + 6x^2 + 11x + 6$ to find expressions for the other three integers.
The possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(–1) = 0$. Use synthetic division to find the other factors.

+1	1	6	11	6
–		1	5	6
×	1	5	6	0

Then, the remaining factor $x^2 + 5x + 6$ can be factored as $(x + 2)(x + 3)$.
So, $x^4 + 6x^3 + 11x^2 + 6x = x(x + 1)(x + 2)(x + 3)$.
Possible expressions for the other three integers are $x + 1$, $x + 2$, and $x + 3$.

Section 3.3 Page 135 Question 14

Evaluate $f(1)$.
$f(x) = ax^4 + bx^3 + cx^2 - dx + e$
$f(1) = a(1)^4 + b(1)^3 + c(1)^2 - d(1) + e$
$f(1) = a + b + c + d + e$
Since $a + b + c + d + e = 0$, this polynomial is divisible by $x - 1$.

Section 3.3 Page 135 Question 15

Let $P(x) = 2x^3 + mx^2 + nx - 3$ and $Q(x) = x^3 - 3mx^2 + 2nx + 4$.
Solve a system of equations represented by $P(2) = 0$ and $Q(2) = 0$.

$P(x) = 2x^3 + mx^2 + nx - 3$
$0 = 2(2)^3 + m(2)^2 + n(2) - 3$
$0 = 16 + 4m + 2n - 3$
$-13 = 4m + 2n$ ①

$Q(x) = x^3 - 3mx^2 + 2nx + 4$
$0 = 2^3 - 3m(2)^2 + 2n(2) + 4$
$0 = 8 - 12m + 4n + 4$
$-12 = -12m + 4n$
$-6 = -6m + 2n$ ②

Subtract equation ② from ①: $-7 = 10m$

$$m = -\frac{7}{10}$$

Substitute $m = -\frac{7}{10}$ into equation ②: $-6 = -6\left(-\frac{7}{10}\right) + 2n$

$$n = -\frac{102}{20}$$

$$n = -\frac{51}{10}$$

Section 3.3 Page 135 Question 16

a) i) For $P(x) = x^3 - 1$, the possible integral zeros of the polynomial are the factors of the constant term, -1. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

-1	1	0	0	-1
$-$		-1	-1	-1
$\times$	1	1	1	0

Then, the remaining factor $x^2 + x + 1$ cannot be factored.
So, $P(x) = (x - 1)(x^2 + x + 1)$.

ii) For $P(x) = x^3 - 27$, the possible integral zeros of the polynomial are the factors of the constant term, -27. Test these values to find a first factor: $P(3) = 0$. Use synthetic division to find the other factors.

–3	1	0	0	–27
–		–3	–9	–27
×	1	3	9	0

Then, the remaining factor $x^2 + 3x + 9$ cannot be factored.
So, $P(x) = (x - 3)(x^2 + 3x + 9)$.

iii) For $P(x) = x^3 + 1$, the possible integral zeros of the polynomial are the factors of the constant term, 1. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	0	0	1
–		1	–1	1
×	1	–1	1	0

Then, the remaining factor $x^2 - x + 1$ cannot be factored.
So, $P(x) = (x + 1)(x^2 - x + 1)$.

iv) For $P(x) = x^3 + 64$, the possible integral zeros of the polynomial are the factors of the constant term, 64. Test these values to find a first factor: $P(-4) = 0$. Use synthetic division to find the other factors.

+4	1	0	0	64
–		4	–16	64
×	1	–4	16	0

Then, the remaining factor $x^2 - 4x + 16$ cannot be factored.
So, $P(x) = (x + 4)(x^2 - 4x + 16)$.

b) From the results in part a), I would expect $x + y$ to be a factor of $x^3 + y^3$. Then, the remaining factor will be $x^2 - xy + y^2$.

c) From the results in part a), I would expect $x - y$ to be a factor of $x^3 - y^3$. Then, the remaining factor will be $x^2 + xy + y^2$.

d) Rewrite $x^6 + y^6$ as $(x^2)^3 + (y^2)^3$. From the results in part b),
$x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.

Section 3.3 Page 135 Question C1

The x-intercepts of the graph of $f(x) = x^4 - 3x^2 - 4$ give the zeros of the function. In turn, these provide the binomial factors of the polynomial. Since the x-intercepts are –2 and 2, $x + 2$ and $x - 2$ are factors. Use division by one of these factors to determine the other factors.

+2	1	0	–3	0	–4
–		2	–4	2	–4
×	1	–2	1	–2	0

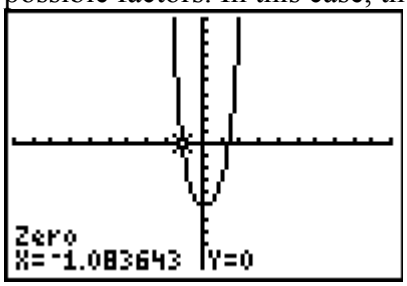
Repeat the process with $P(x) = x^3 - 2x^2 + x - 2$ with the other known factor.

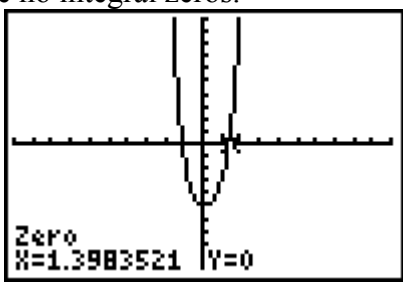
–2	1	–2	1	–2
–		–2	0	–2
×	1	0	1	0

Then, the remaining factor $x^2 + 1$ cannot be factored.
So, $f(x) = (x + 2)(x - 2)(x^2 + 1)$.

Section 3.3 Page 135 Question C2

For $P(x) = x^4 - x^3 + 2x^2 - 5$, the possible integral zeros of the polynomial are the factors of the constant term, –5: ±1 and ±5.
Graphing the polynomial and determining the x-intercepts is another method of finding possible factors. In this case, there are no integral zeros.





Section 3.3 Page 135 Question C3

You can use the factor theorem, the integral zero theorem, the quadratic formula, and synthetic division to factor a polynomial of degree greater than or equal to three.
• Use the integral zero theorem to list possible integer values for the zeros.
• Next, apply the factor theorem to determine one factor.
• Then, use synthetic division to determine the remaining factor.
• Repeat the above steps until all factors are found.
• When the remaining factor is a quadratic that cannot be factored, the exact roots may be found using the quadratic formula.

Section 3.4 Equations and Graphs of Polynomial Functions

Section 3.4 Page 147 Question 1

a) $x(x + 3)(x - 4) = 0$
$x = 0$ or $x + 3 = 0$ or $x - 4 = 0$
$x = -3$ $x = 4$

b) $(x - 3)(x - 5)(x + 1) = 0$
$x - 3 = 0$ or $x - 5 = 0$ or $x + 1 = 0$
$x = 3$ $x = 5$ $x = -1$

c) $(2x + 4)(x - 3) = 0$

$2x + 4 = 0$ or $x - 3 = 0$

$x = -2$ $x = 3$

Section 3.4 Page 147 Question 2

a) $(x + 1)^2(x + 2) = 0$

$x + 1 = 0$ or $x + 2 = 0$

$x = -1$ $x = -2$

b) $x^3 - 1 = 0$

$x^3 = 1$

$x = 1$

c) $(x + 4)^3(x + 2)^2 = 0$

$x + 4 = 0$ or $x + 2 = 0$

$x = -4$ $x = -2$

Section 3.4 Page 148 Question 3

a) Since the graph of the polynomial function crosses the x-axis at all three x-intercepts, they are of odd multiplicity. The least possible multiplicity of each x-intercept is 1, so the least possible degree is 3. The graph extends down into quadrant III and up into quadrant I, so the leading coefficient is positive. Since the x-intercepts, or roots, are -3, -2, and 1, the factors are $x + 3$, $x + 2$, and $x - 1$. The corresponding polynomial possible equation is $(x + 3)(x + 2)(x - 1) = 0$.

b) Since the graph of the polynomial function crosses the x-axis at all three x-intercepts, they are of odd multiplicity. The least possible multiplicity of each x-intercept is 1, so the least possible degree is 3. The graph extends up into quadrant II and down into quadrant IV, so the leading coefficient is negative. Since the x-intercepts, or roots, are -4, 1, and 3, the factors are $x + 4$, $x - 1$, and $x - 3$. The corresponding polynomial possible equation is $-(x + 4)(x - 1)(x - 3) = 0$.

c) Since the graph of the polynomial function crosses the x-axis at two of the x-intercepts and touches the x-axis at one of the x-intercepts, the least possible multiplicities of these x-intercepts are, respectively, 1 and 2, so the least possible degree is 4. The graph extends down into quadrant III and down into quadrant IV, so the leading coefficient is negative. Since the x-intercepts, or roots, are -4 (multiplicity 2), 1, and 3, the factors are $(x + 4)^2$, $x - 1$, and $x - 3$. The corresponding polynomial possible equation is $-(x + 4)^2(x - 1)(x - 3) = 0$.

Section 3.4 Page 148 Question 4

a) i) The x-intercepts are -4, -1, and 1.

ii) The function is positive for values of x in the intervals $-4 < x < -1$ and $x > 1$. The function is negative for values of x in the intervals $x < -4$ and $-1 < x < 1$.

iii) Since the graph of the polynomial function crosses the x-axis at all three x-intercepts, the least possible multiplicity of each zero is 1.

b) i) The x-intercepts are -1 and 4.

ii) The function is negative for values of x in the intervals $x < -1$, $-1 < x < 4$, and $x > 4$.

iii) Since the graph of the polynomial function touches the x-axis at both x-intercepts, the least possible multiplicity of each zero is 2.

c) i) The x-intercepts are -3 and 1.

ii) The function is positive for values of x in the intervals $x < -3$ and $x > 1$. The function is negative for values of x in the interval $-3 < x < 1$.

iii) Since the graph of the polynomial function crosses the x-axis at both x-intercepts, the least possible multiplicity of each zero is 1. However, the shape of the graph close to the x-intercept of 1 is similar to the shape of the cubic curve $y = (x - 1)^3$, so that zero has multiplicity of 3.

d) i) The x-intercepts are -1 and 3.

ii) The function is positive for values of x in the interval $x < -1$. The function is negative for values of x in the intervals $-1 < x < 3$ and $x > 3$.

iii) Since the graph of the polynomial function crosses the x-axis at one of the x-intercepts and touches the x-axis at one of the x-intercepts, the least possible multiplicities of these x-intercepts are, respectively, 1 and 2. However, the shape of the graph close to the x-intercept of -1 is similar to the shape of the cubic curve $y = (x + 1)^3$, so that zero has multiplicity of 3.

Section 3.4 Page 148 Question 5

a) Compare to the graph of the base function $y = x^3$. This graph has been translated 2 units to the right and 2 units down: $h = 2$ and $k = -2$. So, the equation of the function is $y = (x - 2)^3 - 2$: choice **B**.

b) Compare to the graph of the base function $y = x^3$. This graph has been reflected in the y-axis and translated 1 unit up: $b = -1$ and $k = 1$. So, the equation of the function is $y = (-x)^3 + 1$: choice **D**.

c) Compare to the graph of the base function $y = x^4$. This graph has been stretched vertically and translated 3 units up: $a > 0$ and $k = 3$. So, the equation of the function must be $y = 0.5x^4 + 3$: choice **C**.

d) Compare to the graph of the base function $y = x^4$. This graph has been stretched horizontally and translated 1 unit to the right and 2 units down: $b > 0$, $h = 1$, and $k = -2$. So, the equation of the function must be $y = (2(x - 1))^4 - 2$: choice **A**.

Section 3.4 Page 149 Question 6

a) Compare the functions $y = 0.5(-3(x - 1))^3 + 4$ and $y = a(b(x - h))^n + k$ to determine the values of the parameters.

$b = -3$ corresponds to a horizontal stretch of factor $\frac{1}{3}$ and a reflection in the y-axis.

Multiply the x-coordinates of the points in column 1 by $-\frac{1}{3}$.

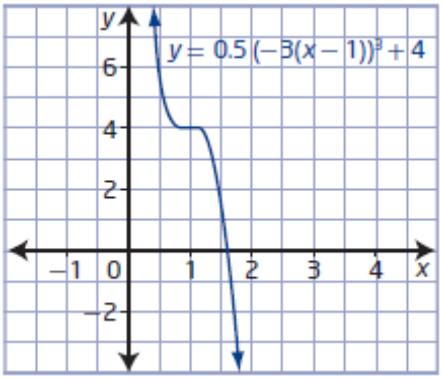
$a = 0.5$ corresponds to a vertical stretch of factor 0.5. Multiply the y-coordinates of the points in column 2 by 0.5.

$h = 1$ corresponds to a translation of 1 unit to the right and $k = 4$ corresponds to a translation of 4 units up. Add 1 to the x-coordinates and 4 to the y-coordinates of the points in column 3.

b)

$y = x^3$	$y = (-3x)^3$	$y = 0.5(-3x)^3$	$y = 0.5(-3(x - 1))^3 + 4$
$(-2, -8)$	$\left(\frac{2}{3}, -8\right)$	$\left(\frac{2}{3}, -4\right)$	$\left(\frac{5}{3}, 0\right)$
$(-1, -1)$	$\left(\frac{1}{3}, -1\right)$	$\left(\frac{1}{3}, -\frac{1}{2}\right)$	$\left(\frac{4}{3}, \frac{7}{2}\right)$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(1, 4)$
$(1, 1)$	$\left(-\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, \frac{1}{2}\right)$	$\left(\frac{2}{3}, \frac{9}{2}\right)$
$(2, 8)$	$\left(-\frac{2}{3}, 8\right)$	$\left(-\frac{2}{3}, 4\right)$	$\left(\frac{1}{3}, 8\right)$

c)



Section 3.4 Page 149 Question 7

a) i) Factor the polynomial expression $x^3 - 4x^2 - 45x$.
$y = x^3 - 4x^2 - 45x$
$y = x(x^2 - 4x - 45)$
$y = x(x - 9)(x + 5)$
So, the x-intercepts of the graph are 0, 9, and –5.

ii) The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.

iii) The zeros are 0, 9, and –5, each of multiplicity 1.

iv) The y-intercept is 0.

v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = -5$ to above the x-axis, down through the origin to under the x-axis, and passes through $x = 9$ upward into quadrant I. The function is positive for values of x in the intervals $-5 < x < 0$ and $x > 9$. The function is negative for values of x in the intervals $x < -5$ and $0 < x < 9$.

b) i) Factor the polynomial expression $x^4 - 81x^2$.
$f(x) = x^4 - 81x^2$
$f(x) = x^2(x^2 - 81)$
$f(x) = x^2(x - 9)(x + 9)$
So, the x-intercepts of the graph are 0, 9, and –9.

ii) The function is of degree 4 and the leading coefficient is positive. The graph of the function extends up into quadrant II and up into quadrant I.

iii) The zeros are 0 (multiplicity 2) and 9 and –9, each of multiplicity 1.

iv) The y-intercept is 0.

v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant II, passes through $x = -9$ to below the x-axis, touches the x-axis at $x = 0$, passes through $x = 9$ upward into quadrant I. The function is positive for values of x in the intervals $x < -9$ and $x > 9$. The function is negative for values of x in the intervals $-9 < x < 0$ and $0 < x < 9$.

c) i) Factor the polynomial expression $x^3 + 3x^2 - x - 3$.
$h(x) = x^3 + 3x^2 - x - 3$
$h(x) = x^2(x + 3) - (x + 3)$
$h(x) = (x + 3)(x^2 - 1)$
$h(x) = (x + 3)(x - 1)(x + 1)$

So, the x-intercepts of the graph are -3, 1, and -1.

ii) The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.

iii) The zeros are -3, 1, and -1, each of multiplicity 1.

iv) The y-intercept is -3.

v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = -3$ to above the x-axis, passes through $x = -1$ to below the x-axis, passes through the y-intercept and the x-axis at $x = 1$ upward into quadrant I. The function is positive for values of x in the intervals $-3 < x < -1$ and $x > 1$. The function is negative for values of x in the intervals $x < -3$ and $-1 < x < 1$.

d) i) For $k(x) = -x^4 - 2x^3 + 7x^2 + 8x - 12$, the possible integral zeros of the polynomial are the factors of the constant term, -12: ± 1, ± 2, ± 3, ± 4, ± 6, and ± 12. Test these values to find a first factor: $k(1) = 0$. Use synthetic division to find the other factors.

-1 |	-1	-2	7	8	-12
$-$ |		1	3	-4	-12
$\times$ |	-1	-3	4	12	0

The remaining factor $-x^3 - 3x^2 + 4x + 12$ can be factored.
$-x^3 - 3x^2 + 4x + 12$
$= -x^2(x + 3) + 4(x + 3)$
$= (x + 3)(-x^2 + 4)$
$= -(x + 3)(x^2 - 4)$
$= -(x + 3)(x + 2)(x - 2)$
So, $k(x) = -(x - 1)(x + 3)(x + 2)(x - 2)$.
The x-intercepts of the graph are 1, -3, -2, and 2.

ii) The function is of degree 4 and the leading coefficient is negative. The graph of the function extends down into quadrant III and down into quadrant IV.

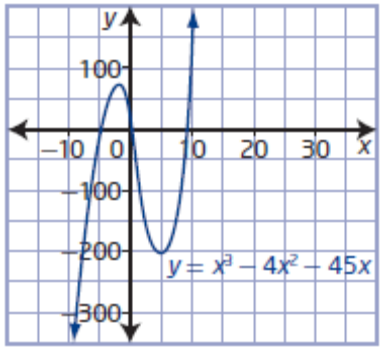
iii) The zeros are 1, –3, –2, and 2, each of multiplicity 1.

iv) The y-intercept is –12.

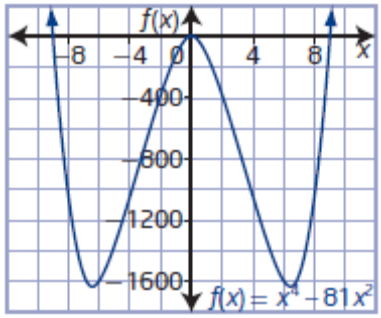
v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = -3$ to above the x-axis, passes down through $x = -2$ to below the x-axis, passes through the y-intercept and up through the $x = 1$, and back down through $x = 2$ into quadrant IV. The function is positive for values of x in the intervals $-3 < x < -2$ and $1 < x < 2$. The function is negative for values of x in the intervals $x < -3$, $-2 < x < 1$, and $x > 2$.

Section 3.4 Page 149 Question 8

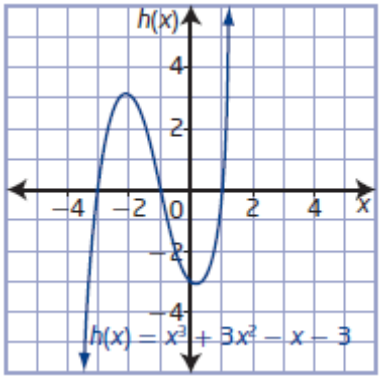
a)



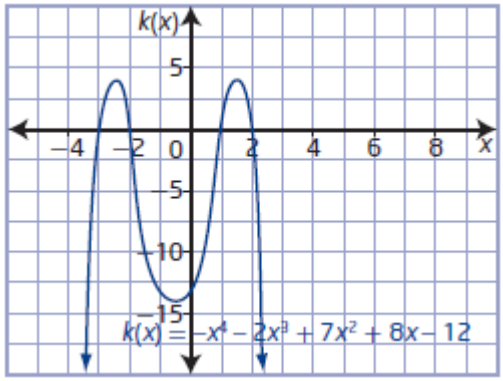
b)



c)



d)



Section 3.4 Page 149 Question 9

a) First factor out the common factor.

$f(x) = x^4 - 4x^3 + x^2 + 6x$

$f(x) = x(x^3 - 4x^2 + x + 6)$

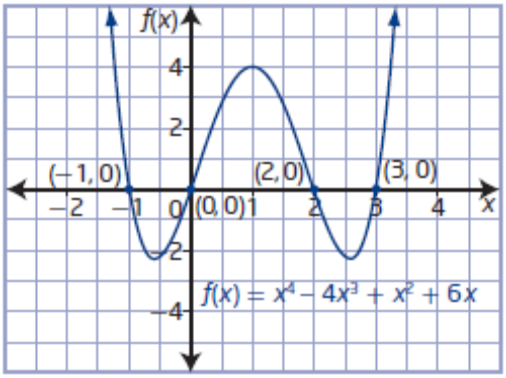
For $P(x) = x^3 - 4x^2 + x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–4	1	6
–		1	–5	6
×	1	–5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $f(x) = x(x + 1)(x - 2)(x - 3)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	4
Leading Coefficient	1
End Behaviour	extends up into quadrant II and up into quadrant I
Zeros/*x*-Intercepts	−1, 0, 2, and 3
***y*-Intercept**	0
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $x < -1$, $0 < x < 2$, and $x > 3$ negative values of $f(x)$ in the intervals $-1 < x < 0$ and $2 < x < 3$



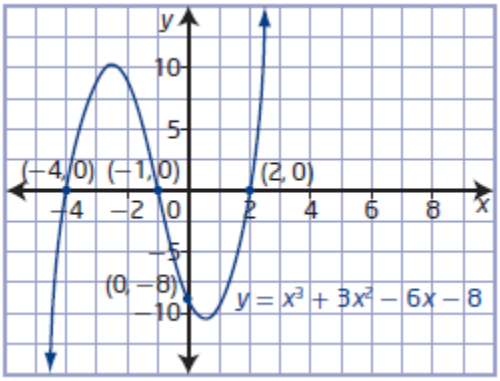
b) For $P(x) = x^3 + 3x^2 - 6x - 8$, the possible integral zeros of the polynomial are the factors of the constant term, –8: ±1, ±2, ±4, and ±8. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	3	–6	–8
–		1	2	–8
×	1	2	–8	0

Then, the remaining factor $x^2 + 2x - 8$ can be factored as $(x + 4)(x - 2)$.
So, $y = (x + 1)(x + 4)(x - 2)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	1
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	−4, –1, and 2
***y*-Intercept**	–8
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $-4 < x < -1$ and $x > 2$ negative values of $f(x)$ in the intervals $x < -4$ and $-1 < x < 2$



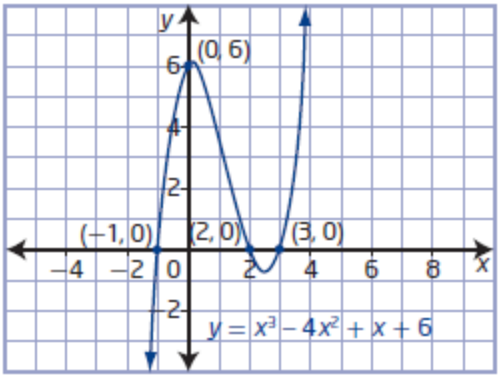
c) For $P(x) = x^3 - 4x^2 + x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–4	1	6
–		1	–5	6
×	1	–5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $y = (x + 1)(x - 2)(x - 3)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	1
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	−1, 2, and 3
***y*-Intercept**	6
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $-1 < x < 2$ and $x > 3$ negative values of $f(x)$ in the intervals $x < -1$ and $2 < x < 3$



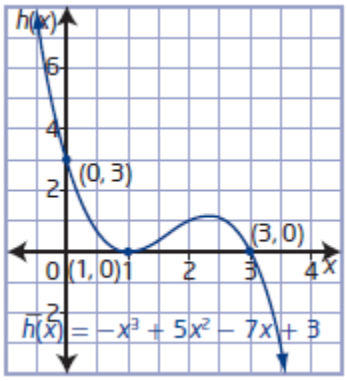
d) For $P(x) = -x^3 + 5x^2 - 7x + 3$, the possible integral zeros of the polynomial are the factors of the constant term, 3: ±1 and ±3. Test the values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	–1	5	–7	3
–		1	–4	3
×	–1	4	–3	0

Then, the remaining factor $-x^2 + 4x - 3$ can be factored as $-(x - 1)(x - 3)$.
So, $h(x) = -(x - 1)^2(x - 3)$.

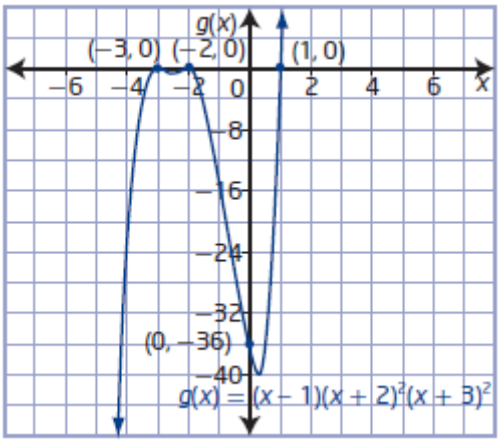
Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	–1
End Behaviour	extends up into quadrant II and down into quadrant IV
Zeros/*x*-Intercepts	1 (multiplicity 2) and 3
***y*-Intercept**	3
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $x < 1$ and $1 < x < 3$ negative values of $f(x)$ in the interval $x > 3$



e) The function $g(x) = (x - 1)(x + 2)^2(x + 3)^2$ is in factored form.
Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	When the function is expanded, the exponent of the highest-degree term is 5. The function is of degree 5.
Leading Coefficient	When the function is expanded, the leading coefficient is $1(1^2)(1^2)$ or 1.
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	–3 (multiplicity 2), –2 (multiplicity 2), and 1
***y*-Intercept**	$(0 - 1)(0 + 2)^2(0 + 3)^2 = -36$
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the interval $x > 1$ negative values of $f(x)$ in the intervals $x < -3$, $-3 < x < -2$, and $-2 < x < 1$



f) For $P(x) = -x^4 - 2x^3 + 3x^2 + 4x - 4$, the possible integral zeros of the polynomial are the factors of the constant term, –4: ±1, ±2, and ±4. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

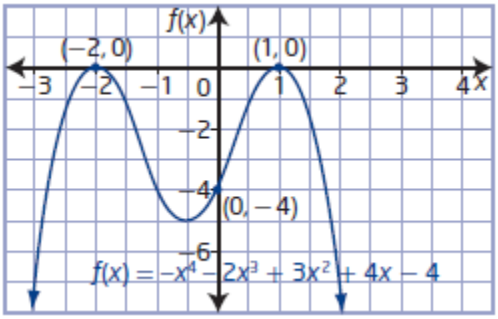
–1	–1	–2	3	4	–4
–		1	3	0	–4
×	–1	–3	0	4	0

Repeat the process with the remaining factor $-x^3 - 3x^2 + 4$. Test factors of the constant term, 4: $P(1) = 0$.

–1	–1	–3	0	4
–		1	4	4
×	–1	–4	–4	0

Then, the remaining factor $-x^2 - 4x - 4$ can be factored as $-(x + 2)^2$.
So, $f(x) = -(x - 1)^2(x + 2)^2$.
Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	4
Leading Coefficient	–1
End Behaviour	extends down into quadrant III and down into quadrant IV
Zeros/*x*-Intercepts	–2 (multiplicity 2) and 1 (multiplicity 2)
***y*-Intercept**	–4
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in no interval negative values of $f(x)$ in the intervals $x < -2$, $-2 < x < 1$, and $x > 1$



Section 3.4 Page 149 Question 10

a) Since the graph extends down into quadrant III and up into quadrant I, the sign of the leading coefficient is positive. The x-intercepts are –2 (multiplicity 3) and 3 (multiplicity 2). The function is positive for values of x in the intervals $-2 < x < 3$ and $x > 3$. The function is negative for values of x in the interval $x < -2$. The equation for the polynomial function is $y = (x + 2)^3(x - 3)^2$.

b) Since the graph extends up into quadrant II and down into quadrant IV, the sign of the leading coefficient is negative. The x-intercepts are –4, –1, and 3. The function is positive for values of x in the intervals $x < -4$ and $-1 < x < 3$. The function is negative for values of x in the intervals $-4 < x < -1$ and $x > 3$. The equation for the polynomial function is $y = -(x + 4)(x + 1)(x - 3)$.

c) Since the graph extends down into quadrant III and down into quadrant IV, the sign of the leading coefficient is negative. The x-intercepts are –2, –1, 2, and 3. The function is positive for values of x in the intervals $-2 < x < -1$ and $2 < x < 3$. The function is negative for values of x in the intervals $x < -2$, $-1 < x < 2$, and $x > 3$. The equation for the polynomial function is $y = -(x + 2)(x + 1)(x - 2)(x - 3)$.

d) Since the graph extends up into quadrant II and up into quadrant I, the sign of the leading coefficient is positive. The x-intercepts are –1, 1, and 3 (multiplicity 2). The function is positive for values of x in the intervals $x < -1$, $1 < x < 3$, and $x > 3$. The function is negative for values of x in the interval $-1 < x < 1$. The equation for the polynomial function is $y = (x + 1)(x - 1)(x - 3)^2$.

Section 3.4 Page 150 Question 11

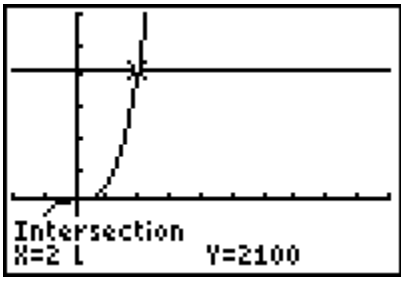
a) For $y = \left(\frac{1}{2}(x-2)\right)^3 - 3$, $a = 1$, $b = \frac{1}{2}$, $h = 2$, and $k = -3$.

b) $a = 1$: no vertical stretch
$b = \frac{1}{2}$: horizontal stretch by a factor of 2
$h = 2$: horizontal translation of 2 units to the right
$k = -3$: vertical translation of 3 units down

c) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

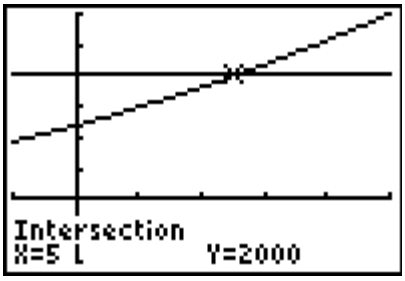
Section 3.4 Page 150 Question 12

Solve $2100 = x(25x)(10x + 1)$ by graphing.
The x-coordinate of the point of intersection is 2.
So, the dimensions of the pool are 2 m by 50 m by 21 m.



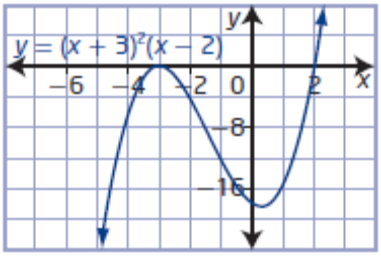
Section 3.4 Page 150 Question 13

The combined surface area of the pond and the boardwalk are given by $SA = (30 + 2x)(40 + 2x)$.
Solve $2000 = (30 + 2x)(40 + 2x)$ graphically.
At the point of intersection, $x = 5$.
The width of the boardwalk is 5 ft.

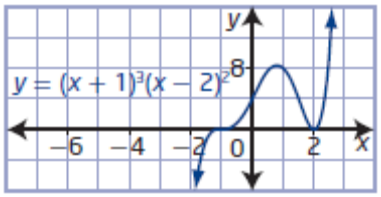


Section 3.4 Page 150 Question 14

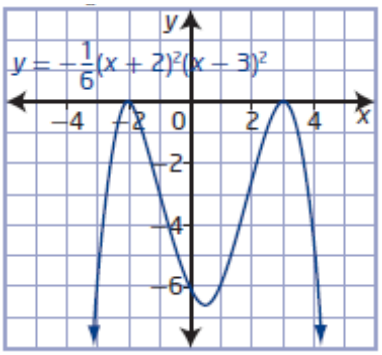
a) For a cubic function with zeros –3 (multiplicity 2) and 2 and y-intercept –18, the corresponding factors are $(x + 3)^2$ and $(x - 2)$ and $a_0 = -18$.
Try an equation with just the factors: $y = (x + 3)^2(x - 2)$.
Check the y-intercept.
$y = (0 + 3)^2(0 - 2)$
$y = -18$
The equation with least degree is $y = (x + 3)^2(x - 2)$.



b) For a quintic function with zeros –1 (multiplicity 3) and 2 (multiplicity 2) and y-intercept 4, the corresponding factors are $(x + 1)^3$ and $(x – 2)^2$ and $a_0 = 4$.
Try an equation with just the factors: $y = (x + 1)^3(x – 2)^2$.
Check the y-intercept.
$y = (0 + 1)^3(0 – 2)^2$
$y = 4$
The equation with least degree is $y = (x + 1)^3(x – 2)^2$.



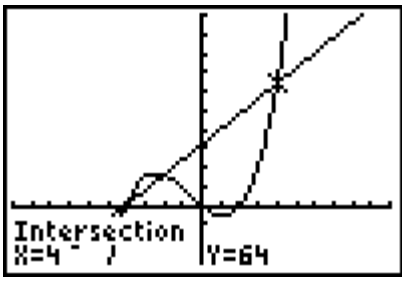
c) For a quartic function with a negative leading coefficient, zeros –2 (multiplicity 2) and 3 (multiplicity 2) and a constant term of –6, the corresponding factors are $(x + 2)^2$ and $(x – 3)^2$ and $a_0 = –6$.
Try an equation with just the factors: $y = –(x + 2)^2(x – 3)^2$.
Check the constant term.
$y = –(0 + 2)^2(0 – 3)^2$
$y = –36$
To adjust the constant term, use $a = \frac{1}{6}$ then $a_0 = –6$.
The equation with least degree is $y = -\frac{1}{6}(x + 2)^2(x – 3)^2$.



Section 3.4 Page 150 Question 15

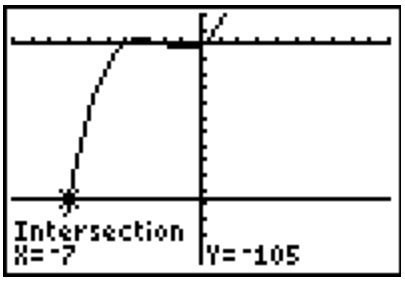
The width of a rectangular prism is w. Then, the height is $w – 2$, the length is $w + 4$, and $V = 8(w + 4)$.
Solve $8(w + 4) = w(w – 2)(w + 4)$ graphically.

The dimensions of the prism are 4 cm by 2 cm by 8 cm.



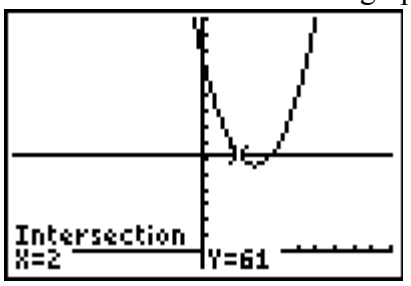
Section 3.4 Page 150 Question 16

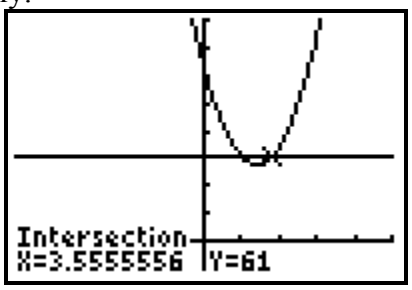
Let three consecutive odd numbers be represented by x, $x + 2$, and $x + 4$.
Solve $–105 = x(x + 2)(x + 4)$ graphically.
The three integers are –7, –5, and –3.



Section 3.4 Page 150 Question 17

Let x represent the side length of the smaller cube. Then $5 - x$ represents the side length of the larger cube.
Determine an expression for the exposed surface area.
$SA_E = SA_S + SA_L$
$SA_E = 5x^2 + 5(5 - x)^2 - x^2$
$SA_E = 4x^2 + 125 - 50x + 5x^2$
$SA_E = 9x^2 - 50x + 125$
Solve $61 = 9x^2 - 50x + 125$ graphically.



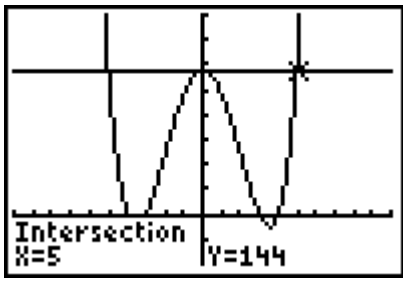


Since x represents the smaller cube, $x = 2$ is the only acceptable answer.
The side length of the smaller cube is 2 m and the side length of the larger cube is 3 m.

Section 3.4 Page 151 Question 18

a) The area of the border can be modelled by the polynomial expression $(x^2 - 12)^2 - x^2$.

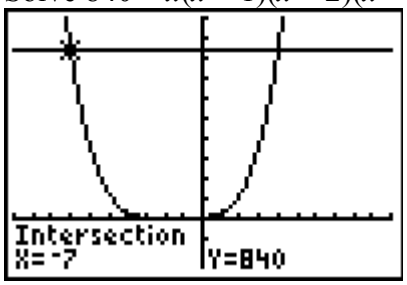
b) Solve $144 = (x^2 - 12)^2 - x^2$ graphically.
The dimensions of Olutie's wall hanging are 5 in. by 5 in.

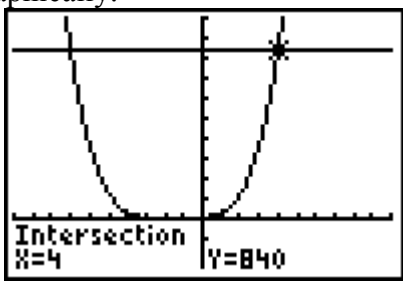


c) The dimensions of the border are 13 in. by 13 in.

Section 3.4 Page 151 Question 19

Let four consecutive integers be represented by x, $x + 1$, $x + 2$, and $x + 3$.
Solve $840 = x(x + 1)(x + 2)(x + 3)$ graphically.





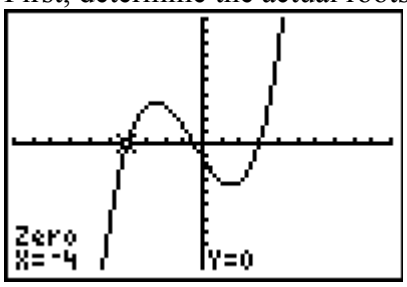
The four consecutive integers are –7, –6, –5, and –4 or 4, 5, 6, and 7.

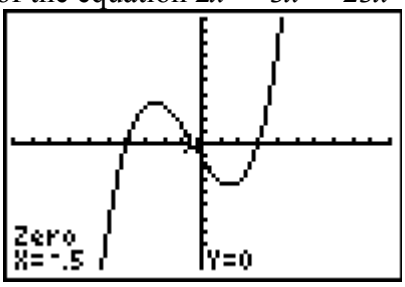
Section 3.4 Page 151 Question 20

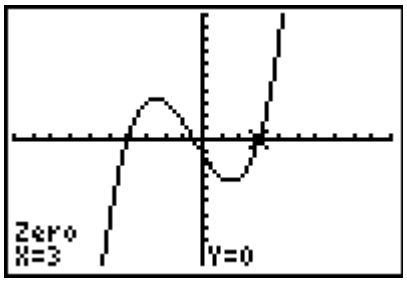
For a cubic function with x-intercepts of $\sqrt{3}$, $-\sqrt{3}$, and 1 and a y-intercept of –1, the corresponding factors are $x - \sqrt{3}$, $x + \sqrt{3}$, and $x - 1$ and $a_0 = -1$.
Try an equation with just the factors: $y = (x - \sqrt{3})(x + \sqrt{3})(x - 1)$.
Check the y-intercept.
$y = (0 - \sqrt{3})(0 + \sqrt{3})(0 - 1)$
$y = 3$
The equation of the cubic function is $y = -\frac{1}{3}(x - \sqrt{3})(x + \sqrt{3})(x - 1)$.

Section 3.4 Page 151 Question 21

First, determine the actual roots of the equation $2x^3 + 3x^2 - 23x - 12 = 0$.





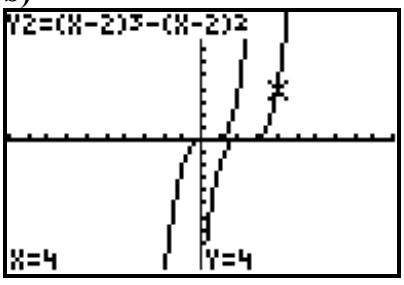


So, $a = -4$, $b = -0.5$, and $c = 3$.
Then, $a + b = -4.5$, $\frac{a}{b} = 8$, and $ab = 2$.
An equation with these roots is $(x + 4.5)(x - 8)(x - 2) = 0$.

Section 3.4 Page 151 Question 22

a) I predict that the graph of $y = (x - 2)^3 - (x - 2)^2$ represents a horizontal translation by 2 units to the right of the graph of $y = x^3 - x^2$.

b)



c) $y = x^3 - x^2$

$y = x^2(x - 1)$

$0 = x^2(x - 1)$

$x^2 = 0$ or $x - 1 = 0$

$x = 0$ $x = 1$

The x-intercepts are 0 and 1.

$y = (x - 2)^3 - (x - 2)^2$

$y = (x - 2)^2(x - 2 - 1)$

$y = (x - 2)^2(x - 3)$

$0 = (x - 2)^2(x - 3)$

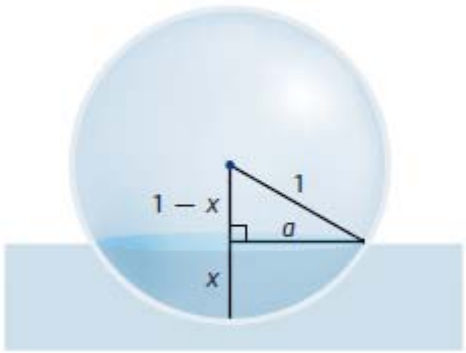
$(x - 2)^2 = 0$ or $x - 3 = 0$

$x = 2$ $x = 3$

The x-intercepts are 2 and 3.

Section 3.4 Page 151 Question 23

Use the information in the diagram and the Pythagorean theorem to determine an expression for a^2 in terms of x: $a^2 = 2x - x^2$. Substitute the expression for a^2 into



$$V_{cap} = \frac{\pi x}{6}(3a^2 + x^2).$$

$$V_{cap} = \frac{\pi x}{6}(3a^2 + x^2)$$

$$V_{cap} = \frac{\pi x}{6}(3(2x - x^2) + x^2)$$

$$V_{cap} = \frac{\pi x}{6}(6x - 2x^2)$$

$$V_{cap} = \frac{\pi x^2}{3}(3 - x)$$

$$V_{cap} = V_{disp} = \frac{\pi x^2}{3}(3 - x)$$

Then, substitute $V_{disp} = \frac{\pi x^2}{3}(3 - x)$, $\rho_{buoy} = \frac{1}{4}\rho_{fluid}$, where ρ represents density, and $V_{buoy} = \frac{4}{3}\pi(1)^2$ into $\rho_{fluid}V_{disp} = \rho_{buoy}V_{buoy}$ and solve for x.

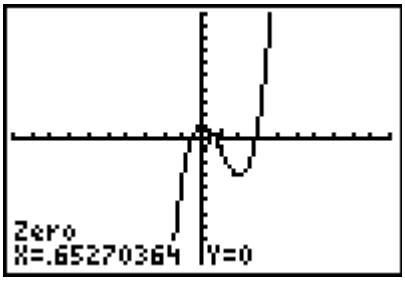
$$\rho_{fluid}V_{disp} = \rho_{buoy}V_{buoy}$$

$$\rho_{fluid}\left(\frac{\pi x^2}{3}(3 - x)\right) = \frac{1}{4}\rho_{fluid}\left(\frac{4\pi}{3}(1)^2\right)$$

$$\pi x^2(3 - x) = \pi$$

$$3x^2 - x^3 = 1$$

$$x^3 - 3x^2 + 1 = 0$$



Graph the corresponding function and determine the value of the zero that makes sense in this case. So, $x \approx 0.65$.

The buoy sinks to a depth of approximately 0.65 m.

Section 3.4 Page 152 Question C1

Example: It is easier to identify the zeros when a polynomial is in factored form. For example,
$y = (x - 1)(x + 2)(x + 4)$ instead of $y = x^3 + 5x^2 + 2x - 8$.

Section 3.4 Page 152 Question C2

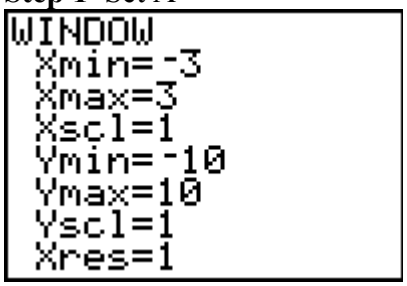
A root of an equation is a solution of the equation. A zero of a function is a value of x for which $f(x) = 0$. An x-intercept of a graph is the x-coordinate of the point where a line or curve crosses or touches the x-axis. They all represent the same value for x.

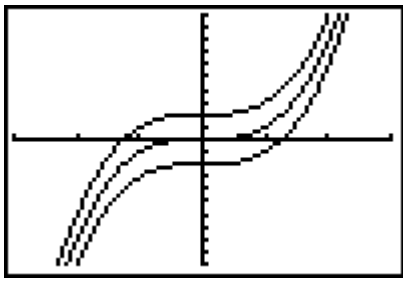
Section 3.4 Page 152 Question C3

Example: If the multiplicity of a zero is 1, the graph crosses the x-axis at that value. If the multiplicity of a zero is even, the graph only touches the x-axis at that value. The shape of a graph close to a zero of $x = a$ (multiplicity n, where n is an odd number greater than 1) is similar to the shape of the graph of a function with degree equal to n of the form $y = (x - a)^n$.

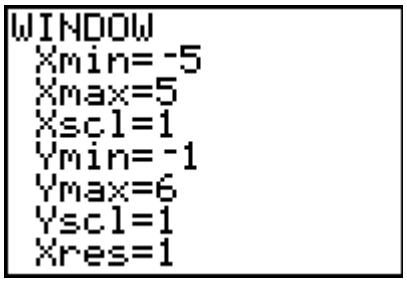
Section 3.4 Page 152 Question C4

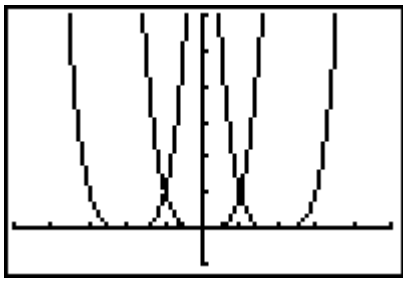
Step 1 Set A





Set B



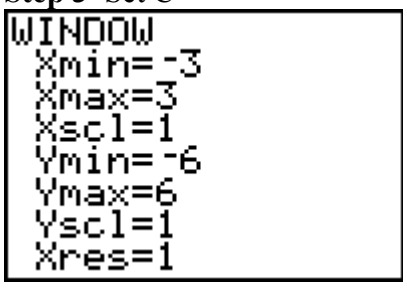


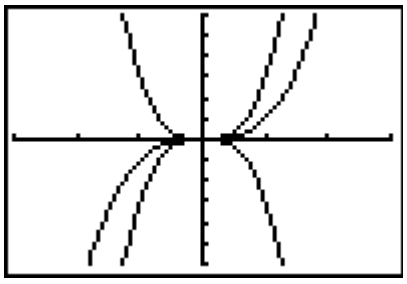
a) The graph of $y = x^3 + k$ is translated vertically k units compared to the graph of $y = x^3$. For example, $y = x^3 + 3$ is translated vertically 3 units compared to the graph of $y = x^3$.

b) The graph of $y = (x - h)^4$ is translated horizontally h units compared to the graph of $y = x^4$. For example, $y = (x + 2)^4$ is translated horizontally 2 units to the left compared to the graph of $y = x^4$.

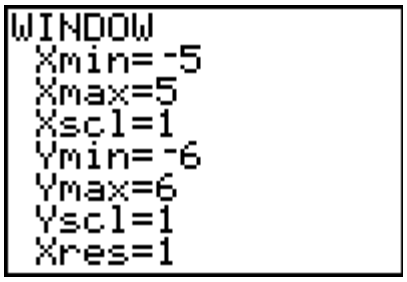
Step 2 In functions of the form $y = a(b(x - h))^n + k$, parameter h represents any horizontal translation while parameter k represents any vertical translation.

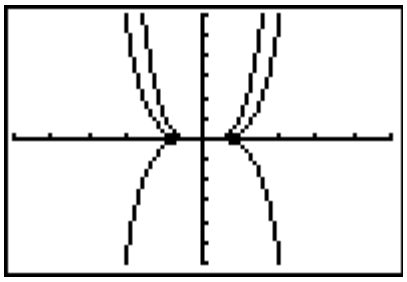
Step 3 Set C





Set D

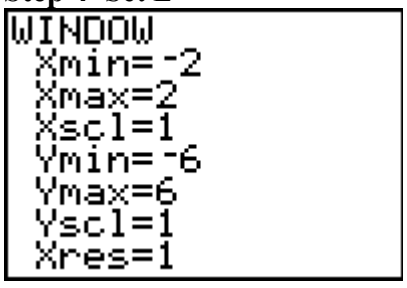


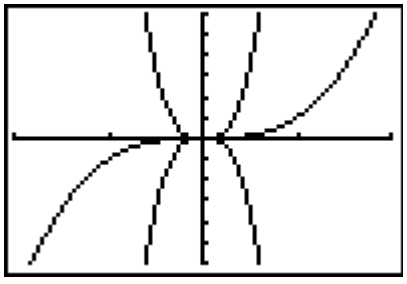


a) The graph of $y = ax^3$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^3$. When a is negative, the graph is reflected in the x-axis.

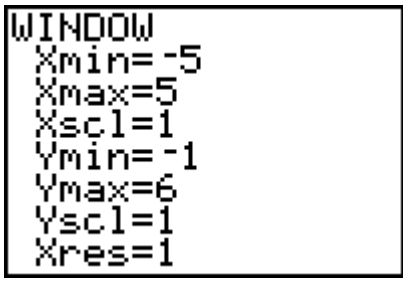
b) When the value of a is $-1 < a < 0$ or $0 < a < 1$, the graph of $y = ax^4$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^4$. When a is negative, the graph is reflected in the x-axis.

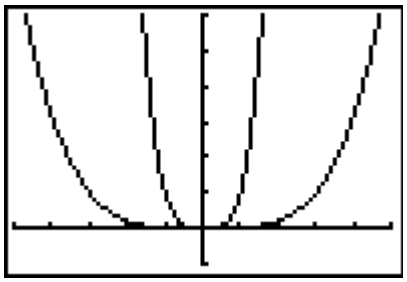
Step 4 Set E





Set F





a) The graph of $y = (bx)^3$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^3$. When b is negative, the graph is reflected in the y-axis.

b) When the value of b is $-1 < b < 0$ or $0 < b < 1$, the graph of $y = (bx)^4$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^4$. When b is negative, the graph is reflected in the y-axis.

Step 5 In functions of the form $y = a(b(x - h))^n + k$, parameter a represents any vertical stretch and/or reflection in the x-axis while parameter b represents any horizontal stretch and/or reflection in the y-axis.

Chapter 3 Review

Chapter 3 Review Page 153 Question 1

a) The function $y = \sqrt{x+1}$ is a radical function, not a polynomial function.

$\sqrt{x+1}$ is the same as $(x+1)^{\frac{1}{2}}$, which has an exponent that is not a whole number.

b) The function $f(x) = 3x^4$ is of the form $g(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 4. The leading coefficient is 3 and the constant term is 0.

c) The function $g(x) = -3x^3 - 2x^2 + x$ is of the form $g(x) = a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 3. The leading coefficient is –3 and the constant term is 0.

d) The function $y = \frac{1}{2}x + 7$ is of the form $g(x) = a_1x + a_0$.

It is a polynomial of degree 1. The leading coefficient is $\frac{1}{2}$ and the constant term is 7.

Chapter 3 Review Page 153 Question 2

a) The function $s(x) = x^4 - 3x^2 + 5x$ is a quartic (degree 4), which is an even-degree polynomial function. Its graph has a maximum of four x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of 0.

b) The function $p(x) = -x^3 + 5x^2 - x + 4$ is a cubic (degree 3), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most three x-intercepts. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV. The graph has no maximum or minimum values. The graph has a y-intercept of 4.

c) The function $y = 3x - 2$ is linear (degree 1), which is an odd-degree polynomial function. Its graph has one x-intercept. Since the leading coefficient is positive, the graph of the function extends down into quadrant III and up into quadrant I, and has a no maximum or minimum value. The graph has a y-intercept of -2.

d) The function $y = 2x^2 - 4$ is a quadratic (degree 2), which is an even-degree polynomial function. Its graph has a maximum of two x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of -4.

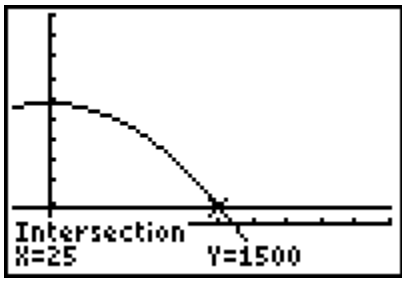
e) The function $y = 2x^5 - 3x^3 + 1$ is a quintic (degree 5), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most five x-intercepts. Since the leading coefficient is positive, the graph of the function extends down into quadrant III and up into quadrant I. The graph has no maximum or minimum values. The graph has a y-intercept of 1.

Chapter 3 Review Page 153 Question 3

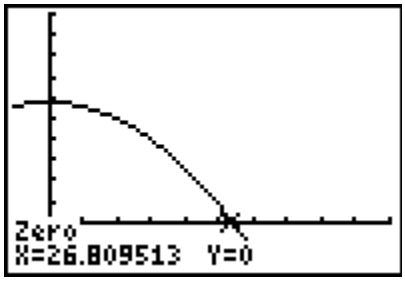
a) The function $h(t) = 11\,500 - 16t^2$ is a quadratic function (degree 2).

b) Substitute $t = 12$.
$h(12) = 11\,500 - 16(12)^2$
$h(12) = 9196$
The parachutist's height above the ground after 12 s will be 9196 ft.

c) Solve $1500 = 11\,500 - 16t^2$ graphically.
The parachutist will be 1500 ft above the ground after 25 s.



d) Solve $0 = 11\,500 - 16t^2$ graphically.
The parachutist will reach the ground after approximately 26.81 s.



Chapter 3 Review Page 153 Question 4

a) $P(x) = x^3 + 9x^2 - 5x + 3$
$P(2) = 2^3 + 9(2)^2 - 5(2) + 3$
$P(2) = 8 + 36 - 10 + 3$
$P(2) = 37$
The remainder when $x^3 + 9x^2 - 5x + 3$ is divided by $x - 2$ is 37.

For $(x^3 + 9x^2 - 5x + 3) \div (x - 2)$, $x \neq 2$.

$$\begin{array}{r} x^2 + 11x + 17 \\ x-2\overline{)\,x^3 + 9x^2 - 5x + 3} \\ \underline{x^3 - 2x^2} \\ 11x^2 - 5x \\ \underline{11x^2 - 22x} \\ 17x + 3 \\ \underline{17x - 34} \\ 37 \end{array}$$

$$\frac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \frac{37}{x - 2}$$

b) $P(x) = 2x^3 + x^2 - 2x + 1$
$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) + 1$
$P(-1) = -2 + 1 + 2 + 1$
$P(-1) = 2$
The remainder when $2x^3 + x^2 - 2x + 1$ is divided by $x + 1$ is 2.

For $(2x^3 + x^2 - 2x + 1) \div (x + 1)$, $x \neq -1$.

$$
\begin{array}{r}
2x^2 - x - 1 \\
x+1\overline{)2x^3 + x^2 - 2x + 1} \\
\underline{2x^3 + 2x^2} \\
-x^2 - 2x \\
\underline{-x^2 - x} \\
-x + 1 \\
\underline{-x - 1} \\
2
\end{array}
$$

$$\frac{2x^3 + x^2 - 2x + 1}{x+1} = 2x^2 - x - 1 + \frac{2}{x+1}$$

c) $P(x) = 12x^3 + 13x^2 - 23x + 7$
$P(1) = 12(1)^3 + 13(1)^2 - 23(1) + 7$
$P(1) = 12 + 13 - 23 + 7$
$P(1) = 9$
The remainder when $12x^3 + 13x^2 - 23x + 7$ is divided by $x - 1$ is 9.

For $(12x^3 + 13x^2 - 23x + 7) \div (x - 1)$, $x \neq 1$.

1	12	13	−23	7
−		−12	−25	−2
×	12	25	2	9

$$\frac{12x^3 + 13x^2 - 23x + 7}{x-1} = 12x^2 + 25x + 2 + \frac{9}{x-1}$$

d) Ensure the polynomial is written in descending powers of x: $-8x^4 + 10x^3 - 4x + 15$.
$P(x) = -8x^4 + 10x^3 - 4x + 15$
$P(-1) = -8(-1)^4 + 10(-1)^3 - 4(-1) + 15$
$P(-1) = -8 - 10 + 4 + 15$
$P(-1) = 1$
The remainder when $-8x^4 + 10x^3 - 4x + 15$ is divided by $x + 1$ is 1.

For $(-8x^4 + 10x^3 - 4x + 15) \div (x + 1)$, $x \neq -1$.

+1	−8	10	0	−4	15
−		−8	18	−18	14
×	−8	18	−18	14	1

$$\frac{-8x^4 - 4x + 10x^3 + 15}{x+1} = -8x^3 + 18x^2 - 18x + 14 + \frac{1}{x+1}$$

Chapter 3 Review Page 153 Question 5

a) Solve $P(3) = -14$ to determine the value of k.

$f(x) = x^4 + kx^3 - 3x - 5$

$-14 = 3^4 + k(3)^3 - 3(3) - 5$

$-14 = 81 + 27k - 9 - 5$

$-81 = 27k$

$k = -3$

b) Evaluate $f(-3)$.

$f(x) = x^4 - 3x^3 - 3x - 5$

$f(-3) = (-3)^4 - 3(-3)^3 - 3(-3) - 5$

$f(-3) = 81 + 81 + 9 - 5$

$f(-3) = 166$

The remainder when $x^4 + kx^3 - 3x - 5$ is divided by $x + 3$ is 166.

Chapter 3 Review Page 153 Question 6

First, determine an expression for $P(1)$ and $P(-3)$.

$P(x) = 4x^3 - 3x^2 + bx + 6$

$P(1) = 4(1)^3 - 3(1)^2 + b(1) + 6$

$P(1) = 4 - 3 + b + 6$

$P(1) = b + 7$

$P(x) = 4x^3 - 3x^2 + bx + 6$

$P(-3) = 4(-3)^3 - 3(-3)^2 + b(-3) + 6$

$P(-3) = -108 - 27 - 3b + 6$

$P(-3) = -3b - 129$

Then, solve $P(1) = P(-3)$ to determine the value of b.

$b + 7 = -3b - 129$

$4b = -136$

$b = -34$

Chapter 3 Review Page 153 Question 7

a) Evaluate $P(1)$.

$P(x) = x^3 - x^2 - 16x + 16$

$P(1) = 1^3 - 1^2 - 16(1) + 16$

$P(1) = 1 - 1 - 16 + 16$

$P(1) = 0$

Since the remainder is zero, $x - 1$ is a factor of $P(x)$.

b) Evaluate $P(-1)$.

$P(x) = x^3 - x^2 - 16x + 16$

$P(-1) = (-1)^3 - (-1)^2 - 16(-1) + 16$

$P(-1) = -1 - 1 + 16 + 16$

$P(-1) = 30$

Since the remainder is not zero, $x + 1$ is not a factor of $P(x)$.

c) Evaluate $P(-4)$.
$P(x) = x^3 - x^2 - 16x + 16$
$P(-4) = (-4)^3 - (-4)^2 - 16(-4) + 16$
$P(-4) = -64 - 16 + 64 + 16$
$P(-4) = 0$
Since the remainder is zero, $x + 4$ is a factor of $P(x)$.

d) Evaluate $P(4)$.
$P(x) = x^3 - x^2 - 16x + 16$
$P(4) = 4^3 - 4^2 - 16(4) + 16$
$P(1) = 64 - 16 - 64 + 16$
$P(1) = 0$
Since the remainder is zero, $x - 4$ is a factor of $P(x)$.

Chapter 3 Review Page 153 Question 8

a) For $P(x) = x^3 - 4x^2 + x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, -6: ± 1, ± 2, ± 3, and ± 6. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

$+1$ |	1	-4	1	6
$-$ |		1	-5	6
$\times$ |	1	-5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $P(x) = (x + 1)(x - 2)(x - 3)$.

b) $P(x) = -4x^3 - 4x^2 + 16x + 16$ can be factored by grouping.
$P(x) = -4x^3 - 4x^2 + 16x + 16$
$P(x) = -4x^2(x + 1) + 16(x + 1)$
$P(x) = (x + 1)(-4x^2 + 16)$
$P(x) = -4(x + 1)(x^2 - 4)$
$P(x) = -4(x + 1)(x - 2)(x + 2)$

c) For $P(x) = x^4 - 4x^3 - x^2 + 16x - 12$, the possible integral zeros of the polynomial are the factors of the constant term, -12: ± 1, ± 2, ± 3, ± 4, ± 6, and ± 12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

-1 |	1	-4	-1	16	-12
$-$ |		-1	3	4	-12
$\times$ |	1	-3	-4	12	0

Then, the remaining factor $x^3 - 3x^2 - 4x + 12$ can be factored by grouping as $(x - 3)(x^2 - 4)$.
So, $P(x) = (x - 1)(x - 3)(x - 2)(x + 2)$.

d) For $P(x) = x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	–3	–5	27	–32	12
–		–1	2	7	–20	12
×	1	–2	–7	20	–12	0

Repeat the process with the remaining factor $x^4 - 2x^3 - 7x^2 + 20x - 12$. Test factors of the constant term, –12: $P(2) = 0$.

–2	1	–2	–7	20	–12
–		–2	0	14	–12
×	1	0	–7	6	0

Repeat the process with the remaining factor $x^3 - 7x + 6$. Test factors of the constant term, 6: $P(1) = 0$.

–1	1	0	–7	6
–		–1	–1	6
×	1	1	–6	0

Then, the remaining factor $x^2 + x - 6$ can be factored as $(x + 3)(x - 2)$.
So, $P(x) = (x - 1)^2(x - 2)^2(x + 3)$.

Chapter 3 Review Page 154 Question 9

a) For $V(x) = 2x^3 + 7x^2 + 2x - 3$, the possible integral zeros of the polynomial are the factors of the constant term, –3: ±1 and ±3. Test these values to find a first factor: $P(–1) = 0$. Use synthetic division to find the other factors.

+1	2	7	2	–3
–		2	5	–3
×	2	5	–3	0

Then, the remaining factor $2x^2 + 5x - 3$ can be factored as $(2x - 1)(x + 3)$.
So, $V(x) = (x + 1)(2x - 1)(x + 3)$.
The possible dimensions of the block, in metres, are $(x + 1)$ by $(2x - 1)$ by $(x + 3)$.

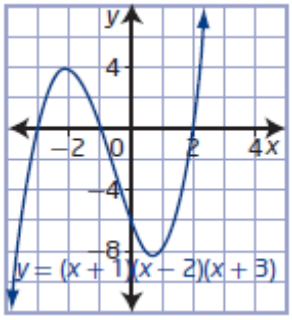
b) When $x = 1$, the possible dimensions are 2 m by 1 m by 4 m.

Chapter 3 Review Page 154 Question 10

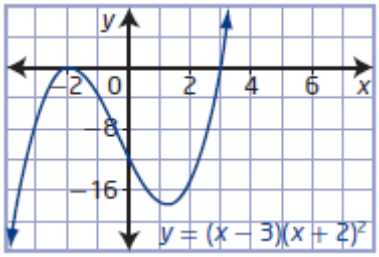
Solve $P(–3) = 0$ to determine the value(s) of k.
$P(x) = x^3 + 4x^2 - 2kx + 3$
$0 = (–3)^3 + 4(–3)^2 - 2k(–3) + 3$
$0 = –27 + 36 + 6k + 3$
$–12 = 6k$
$k = –2$

Chapter 3 Review Page 154 Question 11

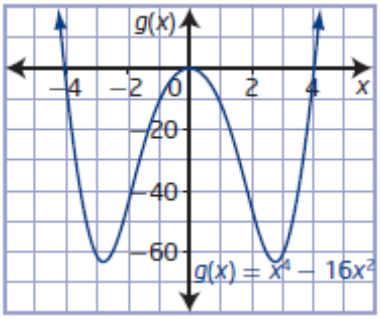
a) The function $y = (x + 1)(x – 2)(x + 3)$ is in factored form.
• So, the x-intercepts of the graph are –1, 2, and –3.
• The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.
• The zeros are –1, 2, and –3, each of multiplicity 1.
• The y-intercept is $(0 + 1)(0 – 2)(0 + 3)$, or –6.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = –3$ to above the x-axis, down through $x = –1$ to under the x-axis, passes through the y-intercept and through $x = 2$ upward into quadrant I. The function is positive for values of x in the intervals $–3 < x < –1$ and $x > 2$. The function is negative for values of x in the intervals $x < –3$ and $–1 < x < 2$.



b) The function $y = (x – 3)(x + 2)^2$ is in factored form.
• So, the x-intercepts of the graph are –2 and 3.
• The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.
• The zeros are –2 (multiplicity 2) and 3 (multiplicity 1).
• The y-intercept is $(0 – 3)(0 + 2)^2$, or –12.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, touches the x-axis at $x = –2$, goes down through the y-intercept and back up passing through $x = 3$ upward into quadrant I. The function is positive for values of x in the interval $x > 3$. The function is negative for values of x in the intervals $x < –2$ and $–2 < x < 3$.



c) Factor the polynomial expression $x^4 – 16x^2$.
$g(x) = x^4 – 16x^2$
$g(x) = x^2(x^2 – 16)$
$gx) = x^2(x – 4)(x + 4)$
• So, the x-intercepts of the graph are 0, 4, and –4.
• The function is of degree 4 and the leading coefficient is positive. The graph of the function extends up into quadrant II and up into quadrant I.
• The zeros are 0 (multiplicity 2) and 4 and –4, each of multiplicity 1.
• The y-intercept is a_0, or 0.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant II, passes through $x = –4$ to below the x-axis, touches the x-axis at $x = 0$, passes through $x = 4$ upward into quadrant I. The function is positive for values of x in the intervals $x < –4$ and $x > 4$. The function is negative for values of x in the intervals $–4 < x < 0$ and $0 < x < 4$.



d) Factor the polynomial expression $-x^5 + 16x$.

$g(x) = -x^5 + 16x$

$g(x) = -x(x^4 - 16)$

$g(x) = -x(x^2 - 4)(x^2 + 4)$

$g(x) = -x(x - 2)(x + 2)(x^2 + 4)$

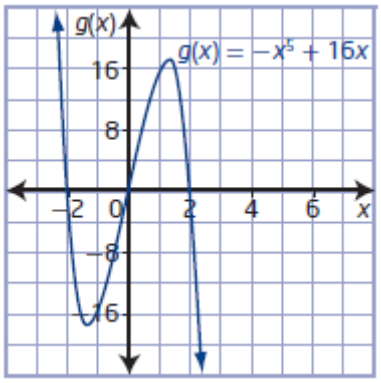
• So, the x-intercepts of the graph are 0, 2, and –2.

• The function is of degree 5 and the leading coefficient is negative. The graph of the function extends up into quadrant II and down into quadrant IV.

• The zeros are 0, 2, and –2, each of multiplicity 1.

• The y-intercept is a_0, or 0.

• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant II, passes through $x = -2$ to below the x-axis, passes through $x = 0$ to above the x-axis, and passes through $x = 2$ downward into quadrant IV. The function is positive for values of x in the intervals $x < -2$ and $0 < x < 2$. The function is negative for values of x in the intervals $-2 < x < 0$ and $x > 2$.



Chapter 3 Review Page 154 Question 12

a) Compare the functions $y = 2(-4(x - 1))^3 + 3$ and $y = a(b(x - h))^n + k$ to determine the values of the parameters.

$b = -4$ corresponds to a horizontal stretch of factor $\frac{1}{4}$ and a reflection in the y-axis.

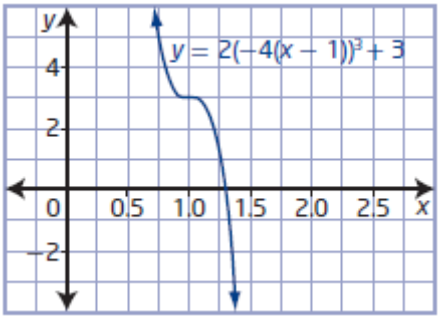
$a = 2$ corresponds to a vertical stretch of factor 2.

$h = 1$ corresponds to a translation of 1 unit to the right and $k = 3$ corresponds to a translation of 3 units up.

b)

Transformation	Parameter Value	Equation
horizontal stretch/ reflection in the y-axis	–4	$y = (-4x)^3$
vertical stretch/ reflection in the x-axis	2	$y = 2(-4x)^3$
translation left/right	1	$y = 2(-4(x - 1))^3$
Translation up/down	3	$y = 2(-4(x - 1))^3 + 3$

c)



Chapter 3 Review Page 154 Question 13

a) Since the graph extends down into quadrant III and up into quadrant I, the sign of the leading coefficient is positive. The x-intercepts are –3 (multiplicity 2) and –1. The function is positive for values of x in the interval $x > -1$. The function is negative for values of x in the intervals $x < -3$ and $-3 < x < -1$. The equation for the polynomial function is $y = (x + 3)^2(x + 1)$.

b) Since the graph extends down into quadrant III and down into quadrant IV, the sign of the leading coefficient is negative. The x-intercepts are –1 and 2 (multiplicity 3). The function is positive for values of x in the interval $-1 < x < 2$. The function is negative for values of x in the intervals $x < -1$ and $x > 2$. The equation for the polynomial function is $y = -(x + 1)(x - 2)^3$.

Chapter 3 Review Page 154 Question 14

a) For a quartic function with zeros –2, –1, and 3 (multiplicity 2), the corresponding factors are $(x + 2)$, $(x + 1)$, and $(x - 3)^2$.
Example: Two equations that satisfy this condition are: $y = (x + 2)(x + 1)(x - 3)^2$ and $y = -(x + 2)(x + 1)(x - 3)^2$.

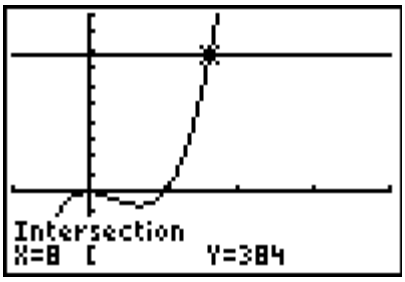
b) Determine the value of a so that the graph of $y = a(x + 2)(x + 1)(x - 3)^2$ passes through the point (2, 24).
$24 = a(2 + 2)(2 + 1)(2 - 3)^2$
$24 = a(4)(3)(1)$
$24 = 12a$
$a = 2$
So, the equation of the function that satisfies these conditions is
$y = 2(x + 2)(x + 1)(x - 3)^2$.

Chapter 3 Review Page 154 Question 15

a) Let x represent the length of the cardboard box. Then, the width is $x - 5$, the height is $2x$, and the equation for the volume is $V(x) = x(x - 5)(2x)$.

b) Solve $384 = x(x - 5)(2x)$ graphically.

The dimensions of the box are 8 cm by 3 cm by 16 cm.



Chapter 3 Practice Test

Chapter 3 Practice Test Page 155 Question 1

Odd-degree polynomials have at least one x-intercept. So, choice **C** is true.

Chapter 3 Practice Test Page 155 Question 2

Evaluate $P(1)$.
$P(x) = 3x^3 + 4x^2 + 2x - 9$
$P(1) = 3(1)^3 + 4(1)^2 + 2(1) - 9$
$P(1) = 3 + 4 + 2 - 9$
$P(1) = 0$
Since the remainder is zero, $x - 1$ is a factor of $P(x)$ and statement **B** is true.

Chapter 3 Practice Test Page 155 Question 3

For $P(x) = x^4 - 2x^3 - 7x^2 - 8x + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$, and ± 12. Choice **D** is correct.

Chapter 3 Practice Test Page 155 Question 4

Consider A: Evaluate $P(1)$.
$P(x) = 2x^3 - 5x^2 - 9x + 18$
$P(1) = 2(1)^3 - 5(1)^2 - 9(1) + 18$
$P(1) = 2 - 5 - 9 + 18$
$P(1) = 6$
Since the remainder is not zero, $x - 1$ is not a factor of $P(x)$.

Consider B: Evaluate $P(-2)$.
$P(x) = 2x^3 - 5x^2 - 9x + 18$
$P(-2) = 2(-2)^3 - 5(-2)^2 - 9(-2) + 18$
$P(-2) = -16 - 20 + 18 + 18$
$P(-2) = 0$
Since the remainder is zero, $x + 2$ is a factor of $P(x)$.
Choice **B** is correct.

Chapter 3 Practice Test Page 155 Question 5

For $y = 3\left(\frac{1}{4}(x-5)\right)^3 - 2$, $a = 3$, $b = \frac{1}{4}$, $h = 5$, and $k = -2$. To obtain this graph, the graph of $y = x^3$ will be vertically stretched by a factor of 3, horizontally stretched by a factor of 4, and translated 5 units to the right and 2 units down: choice **C**.

Chapter 3 Practice Test Page 155 Question 6

a) $(x + 4)^2(x - 3) = 0$
$x + 4 = 0$ or $x - 3 = 0$
$x = -4$ $x = 3$

b) $(x - 3)^3(x + 1)^2 = 0$
$x - 3 = 0$ or $x + 1 = 0$
$x = 3$ $x = -1$

c) $(4x^2 - 16)(x^2 - 3x - 10) = 0$
$4(x - 2)(x + 2)(x - 5)(x + 2) = 0$
$x - 2 = 0$ or $x + 2 = 0$ or $x - 5 = 0$
$x = 2$ $x = -2$ $x = 5$

d) $(9x^2 - 81)(x^2 - 9) = 0$
$9(x - 3)(x + 3)(x - 3)(x + 3) = 0$
$x - 3 = 0$ or $x + 3 = 0$
$x = 3$ $x = -3$

Chapter 3 Practice Test Page 155 Question 7

a) For $P(x) = x^3 + 4x^2 + 5x + 2$, the possible integral zeros of the polynomial are the factors of the constant term, 2: ±1 and ±2. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	4	5	2
–		1	3	2
×	1	3	2	0

Then, the remaining factor $x^2 + 3x + 2$ can be factored as $(x + 2)(x + 1)$.
So, $P(x) = (x + 1)^2(x + 2)$.

b) For $P(x) = x^3 - 13x^2 + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	–13	0	12
–		–1	12	12
×	1	–12	–12	0

Then, the remaining factor $x^2 - 12x - 12$ cannot be factored.
So, $P(x) = (x - 1)(x^2 - 12x - 12)$.

c) $P(x) = -x^3 + 6x^2 - 9x$
$P(x) = -x(x^2 - 6x - 9)$
$P(x) = -x(x - 3)^2$

d) For $P(x) = x^3 - 3x^2 + x + 5$, the possible integral zeros of the polynomial are the factors of the constant term, 5: ±1 and ±5. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–3	1	5
–		1	–4	5
×	1	–4	5	0

Then, the remaining factor $x^2 - 4x + 5$ cannot be factored.
So, $P(x) = (x + 1)(x^2 - 4x + 5)$.

Chapter 3 Practice Test Page 156 Question 8

a) For $P(x) = x^4 + 3x^3 - 3x^2 - 7x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	3	–3	–7	6
–		–1	–4	–1	6
×	1	4	1	–6	0

Repeat the process with the remaining factor $x^3 + 4x^2 + x - 6$. Test factors of the constant term, –6: $P(1) = 0$.

–1	1	4	1	–6
–		–1	–5	–6
×	1	5	6	0

Then, the remaining factor $x^2 + 5x + 6$ can be factored as $(x + 2)(x + 3)$.
So, $y = (x + 1)^2(x + 2)(x + 3)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	4
Leading Coefficient	1
End Behaviour	extends up into quadrant II and up into quadrant I
Zeros/*x*-Intercepts	−1 (multiplicity 2), –2, and –3
***y*-Intercept**	6
Intervals Where the Function Is Positive or Negative	positive values of y in the intervals $x < -3$, $-2 < x < -1$, and $x > -1$ negative values of y in the interval $-3 < x < -2$

This matches graph **B**.

b) $y = x^3 - 4x^2 + 4x$
$y = x(x^2 - 4x + 4)$
$y = x(x - 2)^2$

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	1
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	0 and 2 (multiplicity)
***y*-Intercept**	0
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $0 < x < 2$ and $x > 2$ negative values of $f(x)$ in the interval $x < 0$

This matches graph **C**.

c) $y = -2x^3 + 6x^2 + 2x - 6$
$y = -2x^2(x - 3) + 2(x - 3)$
$y = -2(x - 3)(x^2 - 1)$
$y = -2(x - 3)(x - 1)(x + 1)$

Use a table to organize information about the function. Then, use the information to sketch the graph.

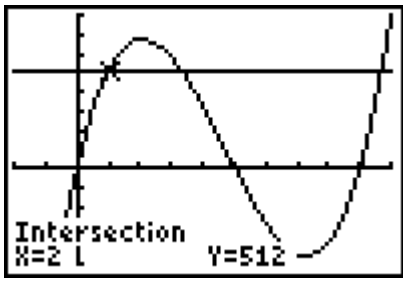
Degree	3
Leading Coefficient	–2
End Behaviour	extends up into quadrant II and down into quadrant IV
Zeros/*x*-Intercepts	–1, 1, and 3
***y*-Intercept**	–6
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $x < -1$ and $1 < x < 3$ negative values of $f(x)$ in the intervals $-1 < x < 1$ and $x > 3$

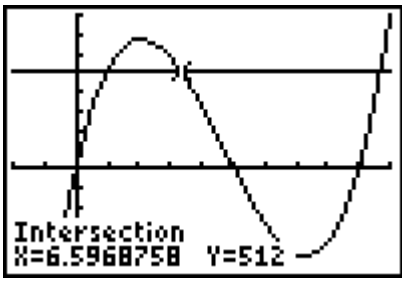
This matches graph **A**.

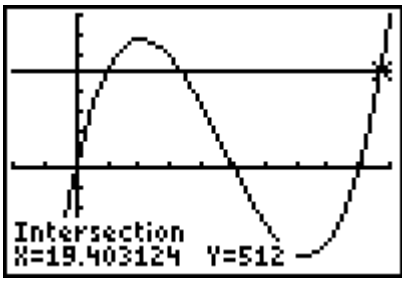
Chapter 3 Practice Test Page 156 Question 9

a) The volume of the box is given by $V(x) = x(20 - 2x)(18 - x)$.

b) Solve $512 = x(20 - 2x)(18 - x)$ graphically.







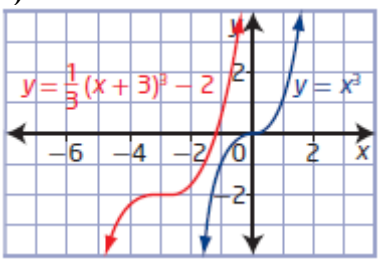
The possible whole-number dimensions of the box are 2 cm by 16 cm by 16 cm.

Chapter 3 Practice Test Page 156 Question 10

a) Compare the functions $y = \frac{1}{3}(x + 3)^3 - 2$ and $y = a(b(x - h))^n + k$ to determine the values of the parameters.
$b = 1$ corresponds to no horizontal stretch.
$a = \frac{1}{3}$ corresponds to a vertical stretch of factor $\frac{1}{3}$.
$h = -3$ corresponds to a translation of 3 units to the left and $k = -2$ corresponds to a translation of 2 units down.

b) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

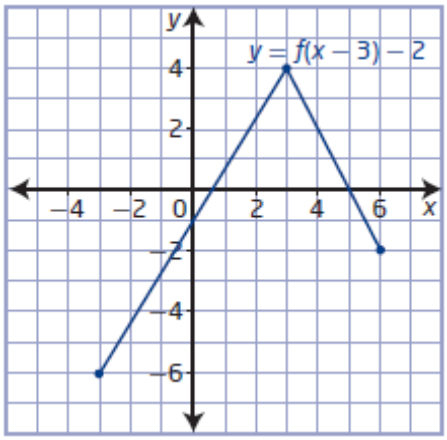
c)



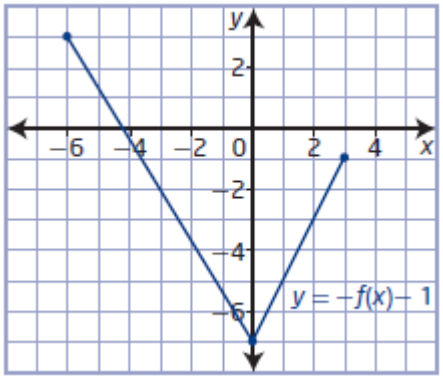
Cumulative Review, Chapters 1–3

Cumulative Review Page 158 Question 1

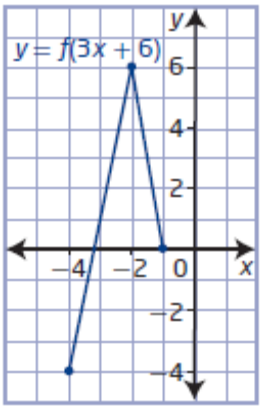
a) For the graph of $y + 2 = f(x - 3)$, translate the graph of $y = f(x)$ right 3 units and down 2 units.



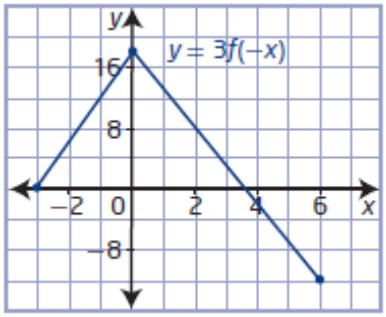
b) For the graph of $y + 1 = -f(x)$, reflect the graph of $y = f(x)$ in the x-axis and translate down 1 unit.



c) Rewrite $y = f(3x + 6)$ as $y = f(3(x + 2))$.
For the graph of $y = f(3(x + 2))$, stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{3}$ and translate left 2 units.



b) For the graph of $y = 3f(-x)$, stretch the graph of $y = f(x)$ vertically by a factor of 3 and reflect in the y-axis.



Cumulative Review Page 158 Question 2

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-2, 4) \rightarrow (1, 0)$
$(0, 0) \rightarrow (3, -4)$
$(2, 2) \rightarrow (5, 0)$
The orientation is unchanged, the graph has not been reflected.
The overall width or height have not changed, so the graph has not been stretched. Since the point (0, 0) is not affected by stretches, the graph has been translated 3 units to the right and 4 units down.
So, $a = 1$, $b = 1$, $h = 3$, $k = -4$, and the equation of the transformed graph is $y + 4 = f(x - 3)$.

Cumulative Review Page 158 Question 3

a) For $g(x) = f(x + 1) - 5$, $a = 1$, $b = 1$, $h = -1$, and $k = -5$. The graph of $y = f(x)$ is translated 1 unit to the left and 5 units down.

b) For $g(x) = -3(x - 2)^2$, $a = -3$, $b = 1$, $h = 2$, and $k = 0$. The graph of $f(x) = x^2$ is vertically stretched about the x-axis by a factor of 3, reflected in the x-axis, and then translated 2 units to the right.

c) Rewrite $g(x) = |-x + 1| + 3$ as $g(x) = |-(x - 1)| + 3$.
For $g(x) = |-(x - 1)| + 3$, $a = 1$, $b = -1$, $h = 1$, and $k = 3$. The graph of $f(x) = |x|$ is reflected in the y-axis and then translated 1 unit to the right and 3 units up.

Cumulative Review Page 158 Question 4

a) For $h(x) = f(x - 3) + 1$, $a = 1$, $b = 1$, $h = 3$, and $k = 1$. Use the mapping $(x, y) \rightarrow (x + 3, y + 1)$. So, the key point (6, 9) becomes the image point (9, 10).

b) For $i(x) = -2f(x)$, $a = -2$, $b = 1$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow (x, -2y)$. So, the key point (6, 9) becomes the image point (6, –18).

c) For $j(x) = f(-3x)$, $a = 1$, $b = -3$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow \left(-\frac{1}{3}x, y\right)$.

So, the key point (6, 9) becomes the image point (–2, 9).

Cumulative Review Page 158 Question 5

a) For $y = f(3x)$, $a = 1$, $b = 3$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow \left(\frac{1}{3}x, y\right)$.

Key Point	Image Point
(–4, 0)	$\left(-\frac{4}{3}, 0\right)$
(6, 0)	(2, 0)
(0, –3)	(0, –3)

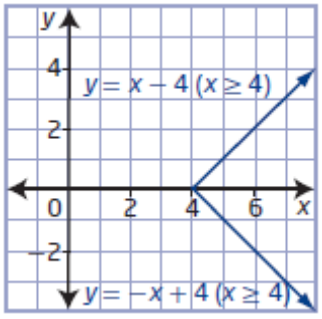
b) For $y = -2f(x)$, $a = -2$, $b = 1$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow (x, -2y)$.

Key Point	Image Point
(–4, 0)	(–4, 0)
(6, 0)	(6, 0)
(0, –3)	(0, 6)

Cumulative Review Page 158 Question 6

a) Since the graph of $y = |x| + 4$ passes the vertical line test, it is a function.

b) Reflect the graph of $y = |x| + 4$ in the line $y = x$.



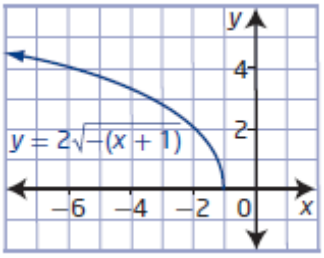
c) The graph of the inverse is not a function, since it fails the vertical line test.
Example: A restricted domain for which the function has an inverse that is also a function is the right side of the V-shape: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.
$f(x) = |x| + 4, x \geq 0$ and $f^{-1}(x) = x - 4, x \geq 4$

Cumulative Review Page 158 Question 7

Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (-2, -3)$
$(4, 2) \rightarrow (0, -1)$
The overall width has changed, so the graph has been horizontally stretched by a factor of $\frac{1}{2}$. From the endpoint, the graph has been translated 2 units to the left and 3 units down.
So, $a = 1$, $b = 2$, $h = -2$, $k = -3$, and the equation of the transformed graph is $y = \sqrt{2(x+2)} - 3$.

Cumulative Review Page 158 Question 8

For a vertical stretch by a factor of 2, horizontal reflection in the y-axis, and a horizontal translation of 1 unit to the left, $a = 2$, $b = -1$, $h = -1$, $k = 0$, and the equation of the transformed function is $y = 2\sqrt{-(x+1)}$.
domain $\{x \mid x \leq -1, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$



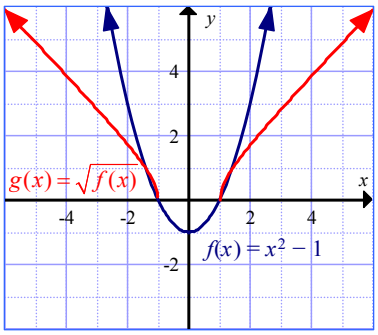
Cumulative Review Page 159 Question 9

a) Compare key points on the graph of $f(x)$ and their image points on the given graph if $g(x)$ is a horizontal stretch of $f(x)$.
$(0, 0) \rightarrow (0, 0)$
$(9, 3) \rightarrow (1, 3)$
The overall width has changed, so the graph has been horizontally stretched by a factor of $\frac{1}{9}$. So, $a = 1$, $b = 9$, $h = 0$, $k = 0$, and the equation of the transformed graph is $g(x) = \sqrt{9x}$.

b) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph if $g(x)$ is a horizontal stretch of $f(x)$.
$(0, 0) \rightarrow (0, 0)$
$(1, 1) \rightarrow (1, 3)$
The overall height has changed, so the graph has been vertically stretched by a factor of 3. So, $a = 3$, $b = 1$, $h = 0$, $k = 0$, and the equation of the transformed graph is $g(x) = 3\sqrt{x}$.

c) The equations from parts a) and b) are equivalent.
$g(x) = \sqrt{9x}$
$g(x) = \sqrt{9}\sqrt{x}$
$g(x) = 3\sqrt{x}$

Cumulative Review Page 159 Question 10



a) The x-intercepts of both graphs are –1 and 1. This is because points where $f(x) = 0$ are invariant.

b) For $f(x) = x^2 - 1$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$. For $g(x) = \sqrt{f(x)}$, the domain is $\{x \mid x \leq -1 \text{ or } x \geq 1, x \in \mathrm{R}\}$, and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$. The domain is restricted for $g(x)$ because it is a radical function.

Cumulative Review Page 159 Question 11

a) For $2x = \sqrt{x+3} - 5$, the restrictions on the variable are $x \geq -3$.

$$2x = \sqrt{x+3} - 5$$
$$2x + 5 = \sqrt{x+3}$$
$$4x^2 + 20x + 25 = x + 3$$
$$4x^2 + 19x + 22 = 0$$
$$(4x + 11)(x + 2) = 0$$

$4x + 11 = 0$ or $x + 2 = 0$

$4x = -11$ $x = -2$

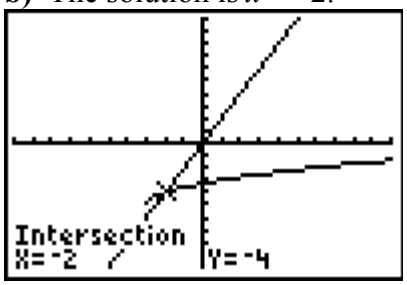
$x = -\frac{11}{4}$ or –2.75

Check. Substitute $x = -2.75$ and $x = -2$ into the original equation to identify any extraneous roots.

Left Side	Right Side	Left Side	Right Side
$2x$	$\sqrt{x+3} - 5$	$2x$	$\sqrt{x+3} - 5$
$= 2(-2.75)$	$= \sqrt{-2.75+3} - 5$	$= 2(-2)$	$= \sqrt{-2+3} - 5$
$= -5.5$	$= -4.5$	$= -4$	$= -4$
Left Side ≠ Right Side		Left Side = Right Side	

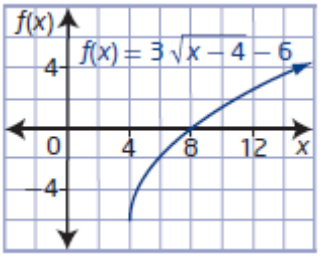
Ron is incorrect, $x = -2.75$ does not check. The solution is $x = -2$.

b) The solution is $x = -2$.



Cumulative Review Page 159 Question 12

a) The x-intercept is 8.



b) $0 = 3\sqrt{x-4} - 6$

$6 = 3\sqrt{x-4}$

$2 = \sqrt{x-4}$

$4 = x - 4$

$x = 8$

c) The solutions or roots of a radical equation are equivalent to the x-intercepts of the graph of the corresponding radical function.

Cumulative Review Page 159 Question 13

a)

$$x+1\overline{)x^4 - 0x^3 - 0x^2 + 3x + 4}$$

$$\begin{array}{r} x^3 - x^2 + x + 2 \\ x+1\overline{)x^4 - 0x^3 - 0x^2 + 3x + 4} \\ \underline{x^4 + x^3} \\ -x^3 - 0x^2 \\ \underline{-x^3 - x^2} \\ x^2 + 3x \\ \underline{x^2 + x} \\ 2x + 4 \\ \underline{2x + 2} \\ 2 \end{array}$$

$$\frac{x^4 + 3x + 4}{x+1} = x^3 - x^2 + x + 2 + \left(\frac{2}{x+1}\right)$$

$P(x) = x^4 + 3x + 4$

$P(-1) = (-1)^4 + 3(-1) + 4$

$P(-1) = 1 - 3 + 4$

$P(-1) = 2$

The remainder when $x^4 + 3x + 4$ is divided by $x + 1$ is 2.

b)

$$
\begin{array}{r}
x^2+2x-5 \\
x+3\overline{)x^3+5x^2+x-9} \\
\underline{x^3+3x^2} \\
2x^2+x \\
\underline{2x^2+6x} \\
-5x-9 \\
\underline{-5x-15} \\
+6
\end{array}
$$

$$\frac{x^3+5x^2+x-9}{x+3} = x^2+2x-5+\frac{6}{x+3}$$

$P(x) = x^3 + 5x^2 + x - 9$
$P(-3) = (-3)^3 + 5(-3)^2 + (-3) - 9$
$= -27 + 45 - 3 - 9$
$= 6$
The remainder when $x^3 + 5x^2 + x - 9$ is divided by $x - 3$ is 6.

Cumulative Review Page 159 Question 14

For $P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$, the possible integral zeros of the polynomial are the factors of the constant term: –6: ±1, ±2, ±3, and ±6.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(1) = 1^4 - 3(1)^3 - 3(1)^2 + 11(1) - 6$
$P(1) = 1 - 3 - 3 + 11 - 6$
$P(1) = 0$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 1$ is 0.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-1) = (-1)^4 - 3(-1)^3 - 3(-1)^2 + 11(-1) - 6$
$P(-1) = 1 + 3 - 3 - 11 - 6$
$P(-1) = -16$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 1$ is –16.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(2) = 2^4 - 3(2)^3 - 3(2)^2 + 11(2) - 6$
$P(2) = 16 - 24 - 12 + 22 - 6$
$P(2) = -4$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 2$ is –4.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-2) = (-2)^4 - 3(-2)^3 - 3(-2)^2 + 11(-2) - 6$
$P(-2) = 16 + 24 - 12 - 22 - 6$
$P(-2) = 0$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 2$ is 0.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(3) = 3^4 - 3(3)^3 - 3(3)^2 + 11(3) - 6$
$P(3) = 81 - 81 - 27 + 33 - 6$
$P(3) = 0$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 3$ is 0.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-3) = (-3)^4 - 3(-3)^3 - 3(-3)^2 + 11(-3) - 6$
$P(-3) = 81 + 81 - 27 - 33 - 6$
$P(-3) = 96$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 3$ is 96.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(6) = 6^4 - 3(6)^3 - 3(6)^2 + 11(6) - 6$
$P(6) = 1296 - 648 - 108 + 66 - 6$
$P(6) = 600$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 6$ is 600.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-6) = (-6)^4 - 3(-6)^3 - 3(-6)^2 + 11(-6) - 6$
$P(-6) = 1296 + 648 - 108 - 66 - 6$
$P(-6) = 1764$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 6$ is 1764.

Cumulative Review Page 159 Question 15

a) For $P(x) = x^3 - 21x + 20$, the possible integral zeros of the polynomial are the factors of the constant term, 20: ±1, ±2, ±4, ±5, ±10, and ±20. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	0	–21	20
–		–1	–1	20
×	1	1	–20	0

Then, the remaining factor $x^2 + x - 20$ can be factored as $(x - 4)(x + 5)$.
So, $P(x) = (x - 1)(x - 4)(x + 5)$.

b) For $P(x) = x^3 + 3x^2 - 10x - 24$, the possible integral zeros of the polynomial are the factors of the constant term, –24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24. Test these values to find a first factor: $P(-2) = 0$. Use synthetic division to find the other factors.

+2	1	3	–10	–24
–		2	2	–24
×	1	1	–12	0

Then, the remaining factor $x^2 + x - 12$ can be factored as $(x + 4)(x - 3)$.
So, $P(x) = (x + 2)(x + 4)(x - 3)$.

c) For $P(x) = -x^4 + 8x^2 - 16$, the possible integral zeros of the polynomial are the factors of the constant term, –16: ±1, ±2, ±4, ±8, and ±16. Test these values to find a first factor: $P(2) = 0$. Use synthetic division to find the other factors.

–2	–1	0	8	0	–16
–		2	4	–8	–16
×	–1	–2	4	8	0

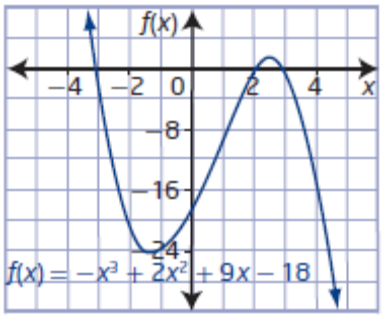
The remaining factor $-x^3 - 2x^2 + 4x + 8$ can be factored.
$-x^3 - 2x^2 + 4x + 8$
$= -x^2(x + 2) + 4(x + 2)$
$= (x + 2)(-x^2 + 4)$
$= -(x + 2)(x^2 - 4)$
$= -(x + 2)^2(x - 2)$
So, $P(x) = -(x - 2)^2(x + 2)^2$.

Cumulative Review Page 159 Question 16

a) Factor $-x^3 + 2x^2 + 9x - 18$.

$-x^3 + 2x^2 + 9x - 18$
$= -x^2(x - 2) + 9(x - 2)$
$= (x - 2)(-x^2 + 9)$
$= -(x - 2)(x^2 - 9)$
$= -(x - 2)(x - 3)(x + 3)$

So, $f(x) = -(x - 2)(x - 3)(x + 3)$.
Then, the x-intercepts are 2, 3, and –3.
The y-intercept is –18.



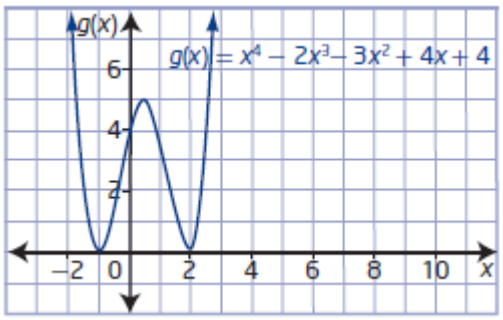
b) For $g(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$, the possible integral zeros of the polynomial are the factors of the constant term, 4: ±1, ±2, and ±4. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–2	–3	4	4
–		1	–3	0	4
×	1	–3	0	4	0

Repeat the process with the remaining factor $x^3 - 3x^2 + 4$. Test factors of the constant term, 4: $P(2) = 0$.

–2	1	–3	0	4
–		–2	2	4
×	1	–1	–2	0

The remaining factor $x^2 - x - 2$ can be factored as $(x + 1)(x - 2)$.
So, $g(x) = (x + 1)^2(x - 2)^2$.
Then, the x-intercepts are –1 and 2.
The y-intercept is 4.



Cumulative Review Page 159 Question 17

a) For $V(x) = x^3 + 2x^2 - 11x - 12$, the given factor is $x + 1$. Use synthetic division to find the other factors.

+1	1	2	–11	–12
–		1	1	–12
×	1	1	–12	0

Then, the remaining factor $x^2 + x - 12$ can be factored as $(x + 4)(x - 3)$.
So, $V(x) = (x + 1)(x + 4)(x - 3)$.
The polynomials that represent the possible length and width of the box are $x + 4$ and $x - 3$.

b) When the height of the box is 4.5 m, $x = 3.5$. So, the possible dimensions are 4.5 m by 7.5 m by 0.5 m.

Cumulative Review Page 159 Question 18

If $f(x) = x^3$ is stretched vertically about the x-axis by a factor of 3, then reflected in the y-axis, and then translated horizontally 5 units to the right: $a = 3$, $b = -1$, $h = 5$, $k = 0$, and the equation of the transformed function is $g(x) = 3(-(x - 5))^3$.

Unit 1 Test

Unit 1 Test Page 160 Question 1

a) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-4, 0) \rightarrow (-5, 1)$
$(-2, 4) \rightarrow (-4, 5)$
$(2, 3) \rightarrow (-2, 4)$
$(4, -2) \rightarrow (-1, -1)$

Since the orientation is unchanged, the graph has not been reflected in either axis.
The overall width has changed, so the graph has been horizontally stretched by a factor of 0.5. Since the x-intercept has changed, the graph has been translated 3 units to the left and 1 unit up. So, $a = 1$, $b = 2$, $h = -3$, $k = 1$, and the equation of the transformed graph is $g(x) = f(2(x + 3)) + 1$.
Choice **D** is correct.

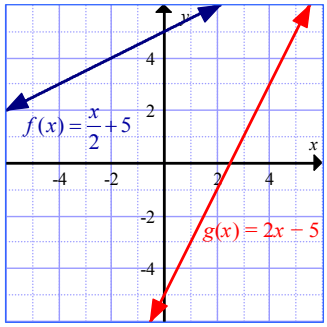
Unit 1 Test Page 160 Question 2

For a reflection in the y-axis and a horizontal stretch about the y-axis by a factor of 3, $a = 1$, $b = -\frac{1}{3}$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow (-3x, y)$. The graph of the transformed function will have a different domain and different x-intercepts.
Choice **C** is correct.

Unit 1 Test Page 160 Question 3

Graph each pair of functions to check whether the graphs are reflections of each other in the line $y = x$.

The graphs of $f(x) = \frac{x}{2} + 5$ and $g(x) = 2x - 5$ are not inverses of each other.
Choice **D** is correct.



Unit 1 Test Page 160 Question 4

The function $y = |x + 4| - 3$ has domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \geq -3, y \in \mathrm{R}\}$.
Choice **A** is correct.

Unit 1 Test Page 160 Question 5

If the graph of $y = \sqrt{x+3}$ is reflected in the line $y = x$, then the range of this graph becomes the domain of the new graph, $\{x \mid x \geq 0, x \in \mathrm{R}\}$.
Choice **D** is correct.

Unit 1 Test Page 160 Question 6

If the graph of a polynomial function of degree 3 passes through (2, 4) and has x-intercepts of –2 and 3 only, then the factors are $(x + 2)$ and $(x - 3)$. The equation of the function is of the form $f(x) = a(x + 2)(x - 3)^2$, since $f(x)$ must be positive in the interval $-2 < x < 3$.
Determine the value of a so that the graph passes through the point (2, 4).
$4 = a(2 + 2)(2 - 3)^2$
$4 = a(4)(1)$
$4 = 4a$
$a = 1$
So, the equation of the function that satisfies these conditions is $f(x) = (x + 2)(x - 3)^2$ or $f(x) = x^3 - 4x^2 - 3x + 18$:.
Choice **C** is correct.

Unit 1 Test Page 160 Question 7

Cosider A: Evaluate $P(-1)$.
$P(x) = -x^3 - 4x^2 + x + 4$
$P(-1) = -(-1)^3 - 4(-1)^2 + (-1) + 4$
$P(-1) = 1 - 4 - 1 + 4$
$P(-1) = 0$
Since the remainder is zero, $x + 1$ is a factor of $P(x)$.
Choice **A** is correct.

Unit 1 Test Page 161 Question 8

Let $P(x) = x^4 + k$. Solve $P(-2) = 3$ to determine the value of k.
$P(x) = x^4 + k$
$3 = (-2)^4 + k$
$3 = 16 + k$
$k = -13$
When $x^4 + k$ is divided by $x + 2$, the remainder is 3. The value of k is –13.

Unit 1 Test Page 161 Question 9

For the function $g(x) = f(x + 2) - 3$, only the value of the parameter k, -3, affects the range.
If the range of the function $y = f(x)$ is $\{y \mid y \geq 11, y \in \mathrm{R}\}$, then the range of the new function $g(x) = f(x + 2) - 3$ is $\mathbf{\{y \mid y \geq 8, y \in R\}}$.

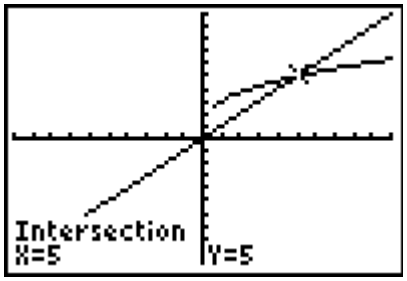
Unit 1 Test Page 161 Question 10

The mapping $(x, y) \rightarrow (x - 2, y + 3)$ represents a translation of 2 units to the left and 3 units up. So, $h = -3$ and $k = 3$.
The graph of the function $f(x) = |x|$ is transformed so that the point (x, y) becomes $(x - 2, y + 3)$. The equation of the transformed function is $g(x) = \mathbf{|x + 2| + 3}$.

Unit 1 Test Page 161 Question 11

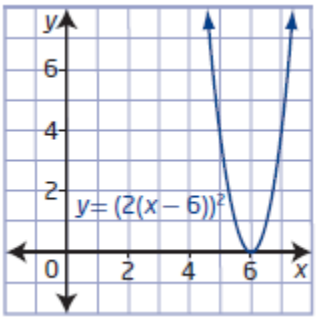
Solve $x = \sqrt{2x - 1} + 2$ graphically.

The root of the equation $x = \sqrt{2x - 1} + 2$ is $\mathbf{x = 5}$.

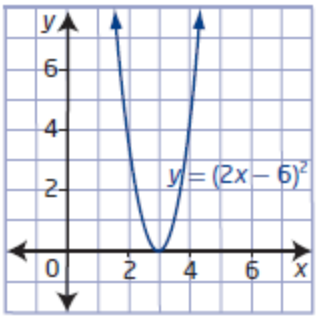


Unit 1 Test Page 161 Question 12

a) The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of $\frac{1}{2}$: $y = (2x)^2$.
Then, translated horizontally 6 units to the right:
$y = (2(x - 6))^2$.



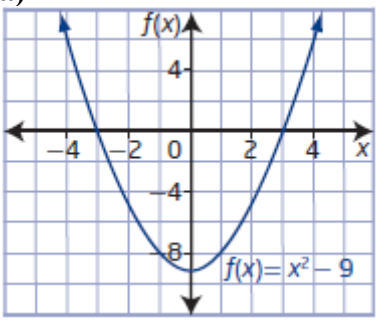
b) The graph of $y = x^2$ is translated horizontally 6 units to the right: $y = (x - 6)^2$.
Then, stretched horizontally about the y-axis by a factor of $\frac{1}{2}$: $y = (2x - 6)^2$.



c) From part b), rewrite $y = (2x - 6)^2$ as $y = (2(x - 3))^2$. The two graphs have the same shape but the vertex of the parabola in part a) is located at (6, 0) while the vertex of the parabola in part b) is located at (3, 0).

Unit 1 Test Page 161 Question 13

a)



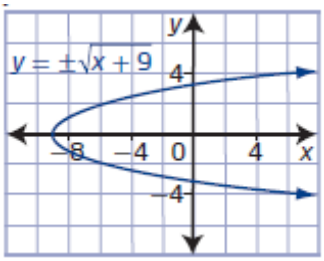
b) $f(x) = x^2 - 9$

$y = x^2 - 9$

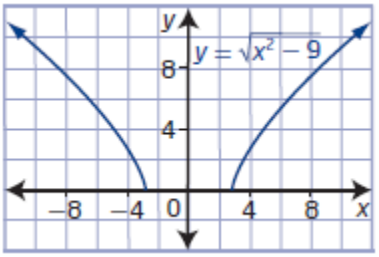
$x = y^2 - 9$

$x + 9 = y^2$

$y = \pm\sqrt{x+9}$



c) The equation of $y = \sqrt{f(x)}$ is $y = \sqrt{x^2 - 9}$.



d) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \geq -9, y \in \mathrm{R}\}$

inverse of $f(x)$: domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$ and range $\{y \mid y \in \mathrm{R}\}$

$\sqrt{f(x)}$: domain $\{x \mid x \leq -3$ and $x \geq 3, x \in \mathrm{R}\}$ and range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

Unit 1 Test Page 161 Question 14

For quadrant II: reflection in the y-axis, $y = f(-x)$.

For quadrant III: reflection in the y-axis and then the x-axis, $y = -f(-x)$.

For quadrant IV: reflection in the x-axis, $y = -f(x)$.

Unit 1 Test Page 161 Question 15

a) Mary should have subtracted 4 from both sides before squaring each side. She also incorrectly squared the right side in step 2.

$$2x = \sqrt{x+1} + 4$$
$$2x - 4 = \sqrt{x+1}$$
$$4x^2 - 16x + 16 = x + 1$$
$$4x^2 - 17x + 15 = 0$$
$$(4x - 5)(x - 3) = 0$$
$4x - 5 = 0$ or $x - 3 = 0$
$4x = 5$ $x = 3$
$x = \frac{5}{4}$ or 1.25

Check. Substitute $x = 1.25$ and $x = 3$ into the original equation to identify any extraneous roots.

Left Side	Right Side	Left Side	Right Side
$2x$	$\sqrt{x+1}+4$	$2x$	$\sqrt{x+1}+4$
$= 2(1.25)$	$= \sqrt{1.25+1}+4$	$= 2(3)$	$= \sqrt{3+1}+4$
$= 2.5$	$= 5.5$	$= 6$	$= 6$
Left Side ≠ Right Side		Left Side = Right Side	

The solution is $x = 3$.

b) Yes, John's method of graphing two functions representing each side of the equation can lead to a correct answer. He must find the x-coordinate of the point of intersection.

Unit 1 Test Page 161 Question 16

Solve $P(-3) = 0$ to determine the value(s) of k.
$P(x) = x^4 + 3x^3 + cx^2 - 7x + 6$
$0 = (-3)^4 + 3(-3)^3 + c(-3)^2 - 7(-3) + 6$
$0 = 81 - 81 + 9c + 21 + 6$
$-27 = 9c$
$c = -3$

For $P(x) = x^4 + 3x^3 - 3x^2 - 7x + 6$, $P(-3) = 0$. Use synthetic division to find the other factors.

+3	1	3	–3	–7	6
–		3	0	–9	6
×	1	0	–3	2	0

Repeat the process with the remaining factor $x^3 - 3x + 2$. Test factors of the constant term, 2: $P(1) = 0$.

–1	1	0	–3	2
–		–1	–1	2
×	1	1	–2	0

The remaining factor $x^2 + x - 2$ can be factored.
So, $P(x) = (x + 3)(x - 1)^2(x + 2)$.

Unit 1 Test Page 161 Question 17

a) For $P(x) = x^3 - 7x - 6$, the possible integral zeros of the polynomial are the factors of the constant term, –6: ±1, ±2, ±3, and ±6.

b) Test the values from part a) to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	0	–7	–6
–		1	–1	–6
×	1	–1	–6	0

The remaining factor $x^2 - x - 6$ can be factored.
So, $P(x) = (x + 1)(x - 3)(x + 2)$.

c) The x-intercepts are –1, 3, and –2. The y-intercept is –6.

d) Since the leading coefficient is positive, the graph extends down into quadrant III and up into quadrant I. The function is greater than or equal to zero for values of x in the intervals $-2 \le x \le -1$ and $x \ge 3$.

Chapter 4 Trigonometry and the Unit Circle

Section 4.1 Angles and Angle Measure

Section 4.1 Page 175 Question 1

a) -4π is a clockwise rotation

b) $750°$ is a counterclockwise rotation

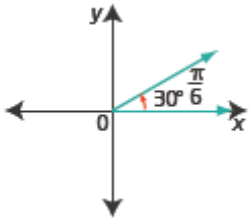
c) $-38.7°$ is a clockwise rotation

d) 1 radian is a counterclockwise rotation

Section 4.1 Page 175 Question 2

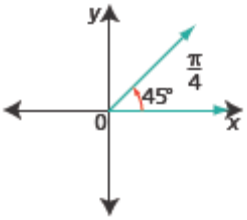
a) $30° = 30\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{6}$



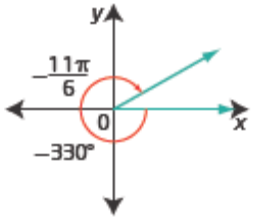
b) $45° = 45\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{4}$



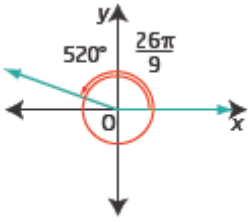
c) $-330° = -330\left(\frac{\pi}{180}\right)$

$= -\frac{11\pi}{6}$



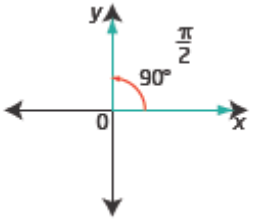
d) $520° = 520\left(\frac{\pi}{180}\right)$

$= \frac{26\pi}{9}$



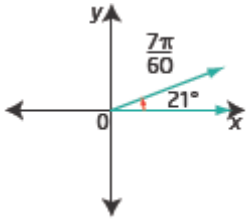
e) $90° = 90\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{2}$



f) $21° = 21\left(\frac{\pi}{180}\right)$

$= \frac{7\pi}{60}$



Section 4.1 Page 175 Question 3

a) $60° = 60\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{3}$

≈ 1.05

b) $150° = 150\left(\frac{\pi}{180}\right)$

$= \frac{5\pi}{6}$

≈ 2.62

c) $-270° = -270\left(\frac{\pi}{180}\right)$

$= -\frac{3\pi}{2}$

≈ -4.71

d) $72° = 72\left(\frac{\pi}{180}\right)$

$= \frac{2\pi}{5}$

≈ 1.26

e) $-14.8°$

$= -14.8\left(\frac{\pi}{180}\right)$

$= -\frac{148\pi}{1800}$

$= -\frac{37\pi}{450}$

≈ -0.26

f) $540° = 540\left(\frac{\pi}{180}\right)$

$= 3\pi$

≈ 9.42

Section 4.1 Page 175 Question 4

a) $\frac{\pi}{6} = \frac{180°}{6}$

$= 30°$

b) $\frac{2\pi}{3} = \frac{2(180°)}{3}$

$= 120°$

c) $-\frac{3\pi}{8} = -\frac{3(180°)}{8}$

$= -67.5°$

d) $-\frac{5\pi}{2} = -\frac{5(180°)}{2}$

$= -450°$

e) $1 = 1\left(\frac{180°}{\pi}\right)$

$= \frac{180°}{\pi}$

$\approx 57.3°$

f) $2.75 = 2.75\left(\frac{180°}{\pi}\right)$

$= \frac{495°}{\pi}$

$\approx 157.6°$

Section 4.1 Page 175 Question 5

a) $\frac{2\pi}{7} = \frac{2(180°)}{7}$

$= \frac{360°}{7}$

$\approx 51.429°$

b) $\frac{7\pi}{13} = \frac{7(180°)}{13}$

$= \frac{1260°}{13}$

$\approx 96.923°$

c) $\frac{2}{3} = \frac{2}{3}\left(\frac{180°}{\pi}\right)$

$= \frac{120°}{\pi}$

$\approx 38.197°$

d) $3.66 = 3.66\left(\frac{180°}{\pi}\right)$

$= \frac{658.8°}{\pi}$

$\approx 209.703°$

e) $-6.14 = -6.14\left(\frac{180°}{\pi}\right)$

$= -\frac{1105.2°}{\pi}$

$\approx -351.796°$

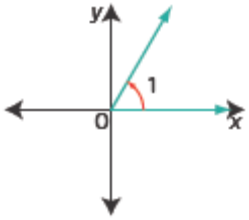
f) $-20 = -20\left(\frac{180°}{\pi}\right)$

$= -\frac{3600°}{\pi}$

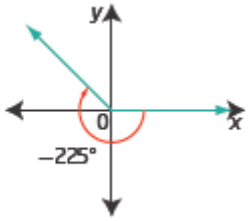
$\approx -1145.916°$

Section 4.1 Page 175 Question 6

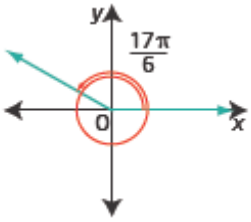
a) An angle that measures 1 radian is in quadrant I.



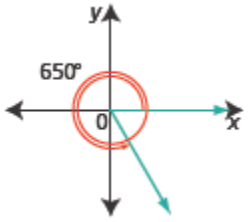
b) An angle that measures 225° is in quadrant II.



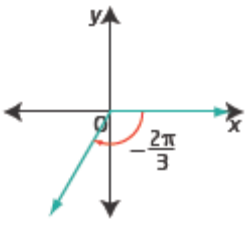
c) An angle that measures $\frac{17\pi}{6}$ is in quadrant II.



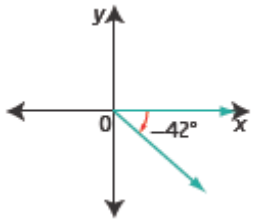
d) An angle that measures 650° is in quadrant IV.



e) An angle that measures $-\frac{2\pi}{3}$ is in quadrant III.



f) An angle that measures –42° is in quadrant IV.



Section 4.1 Page 176 Question 7

a) $72° + 360° = 432°$ $72° - 360° = -288°$

For an angle of 72°, one positive coterminal angle is 432° and one negative coterminal angle is –288°.

b) $\frac{3\pi}{4} + 2\pi = \frac{11\pi}{4}$ $\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$

For an angle of $\frac{3\pi}{4}$, one positive coterminal angle is $\frac{11\pi}{4}$ and one negative coterminal angle is $-\frac{5\pi}{4}$.

c) $-120° + 360° = 240°$ $-120° - 360° = -480°$

For an angle of –120°, one positive coterminal angle is 240° and one negative coterminal angle is –480°.

d) $\frac{11\pi}{2} - 2\pi = \frac{7\pi}{2}$ $\frac{11\pi}{2} - 6\pi = -\frac{\pi}{2}$

For an angle of $\frac{11\pi}{2}$, one positive coterminal angle is $\frac{7\pi}{2}$ and one negative coterminal angle is $-\frac{\pi}{2}$.

e) $-205° + 360° = 155°$ $-205° - 360° = -565°$

For an angle of –205°, one positive coterminal angle is 155° and one negative coterminal angle is –565°.

f) $7.8 - 2\pi \approx 1.5$ $7.8 - 4\pi \approx -4.8$

For an angle of –7.8, one positive coterminal angle is 1.5 and one negative coterminal angle is –4.8.

Section 4.1 Page 176 Question 8

a) The angles $\frac{5\pi}{6}$ and $\frac{17\pi}{6}$ are coterminal because

$$\frac{5\pi}{6} + 2\pi = \frac{5\pi}{6} + \frac{12\pi}{6}$$
$$= \frac{17\pi}{6}$$

b) The angles $\frac{5\pi}{2}$ and $\frac{17\pi}{6}$ are not coterminal because $\frac{5\pi}{2}$ is coterminal with $\frac{\pi}{2}$ which falls on the positive *y*-axis, while $\frac{17\pi}{6}$ is coterminal with $\frac{5\pi}{6}$, which is in quadrant II.

c) The angles 410° and –410° are not coterminal because 410° is coterminal with 50° and so is in quadrant I, while –410° is coterminal with 310° and is in quadrant IV.

d) The angles 227° and –493° are coterminal because –493° is coterminal with –493° + 2(360°) which is 227°.

Section 4.1 Page 176 Question 9

a) The coterminal angles for 135° are 135° ± (360°)*n*, where *n* is any natural number.

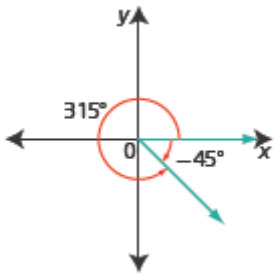
b) The coterminal angles for $-\frac{\pi}{2}$ are $-\frac{\pi}{2} \pm 2\pi n$, where *n* is any natural number.

c) The coterminal angles for –200° are –200° ± (360°)*n*, where *n* is any natural number.

d) The coterminal angles for 10 radians are 10 ± 2π*n*, where *n* is any natural number.

Section 4.1 Page 176 Question 10

Example: Choose –45°.
–45° = –45° + 360°
= 315°
In general, all angles coterminal with –45° are given by –45° ± (360°)*n*, where *n* is any natural number.



Section 4.1 Page 176 Question 11

a) $65° + 360° = 425°$
In the domain $0° \le \theta < 720°$, the angle 425° is coterminal with 65°.

b) $-40° + 360° = 320°$
In the domain $-180° \le \theta < 360°$, the angle 320° is coterminal with –40°.

c) $-40° + 360° = 320°$ $\quad -40° - 360° = -400°$
$-40° + 2(360°) = 680°$
In the domain $-720° \le \theta < 720°$, the angles –400°, 320°, and 680° are coterminal with –40°.

d) $\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$

In the domain $-2\pi \le \theta < 2\pi$, the angle $-\frac{5\pi}{4}$ is coterminal with $\frac{3\pi}{4}$.

e) $-\frac{11\pi}{6} - 2\pi = -\frac{23\pi}{6}$ $\quad -\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$ $\quad -\frac{11\pi}{6} + 4\pi = \frac{13\pi}{6}$

In the domain $-4\pi \le \theta < 4\pi$, the angles $-\frac{23\pi}{6}$, $\frac{\pi}{6}$ and $\frac{13\pi}{6}$ are coterminal with $-\frac{11\pi}{6}$.

f) $\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$ $\quad \frac{7\pi}{3} - 4\pi = -\frac{5\pi}{3}$

In the domain, $-2\pi \le \theta < 4\pi$, the angles $\frac{\pi}{3}$ and $-\frac{5\pi}{3}$ are coterminal with $\frac{7\pi}{3}$.

g) $2.4 - 2\pi \approx -3.9$
In the domain $-2\pi \le \theta < 2\pi$, the angle –3.9 is coterminal with 2.4.

h) $-7.2 + 2\pi \approx -0.9$ $\quad -7.2 + 4\pi \approx 5.4$ $\quad -7.2 - 2\pi \approx -13.5$ (outside specified domain)
In the domain $-4\pi \le \theta < 2\pi$, the angles –0.9 and 5.4 are coterminal with –7.2.

Section 4.1 Page 176 Question 12

a) Use a proportion with $r = 9.5$ and central angle 1.4 radians.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{arc length}}{2\pi(9.5)} = \frac{1.4}{2\pi}$$

$$\begin{aligned}\text{arc length} &= 1.4(9.5)\\ &= 13.3\end{aligned}$$

The arc length is 13.3 cm.

b) Use the formula $a = \theta r$, with $r = 1.37$ and $\theta = 3.5$.

$$\begin{aligned}a &= 3.5(1.37)\\ &= 4.795\end{aligned}$$

The arc length is 4.80 m, to the nearest hundredth of a metre.

c) Use a proportion with $r = 7$ and central angle 130°.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{arc length}}{2\pi(7)} = \frac{130}{360}$$

$$\begin{aligned}\text{arc length} &= \frac{13(14\pi)}{36}\\ &\approx 15.88\end{aligned}$$

The arc length is 15.88 cm, to the nearest hundredth of a centimetre.

d) Use a proportion with $r = 6.25$ and central angle 282°.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{arc length}}{2\pi(6.25)} = \frac{282}{360}$$

$$\begin{aligned}\text{arc length} &= \frac{282(12.5\pi)}{360}\\ &\approx 30.76\end{aligned}$$

The arc length is 30.76 in., to the nearest hundredth of an inch.

Section 4.1 Page 176 Question 13

a) Use the formula $a = \theta r$, with $a = 9$ and $r = 4$.

$9 = \theta(4)$

$\frac{9}{4} = \theta$

$2.25 = \theta$

The central angle is 2.25 radians.

b) Use the formula $a = \theta r$, with $\theta = 1.22$ and $r = 9$.

$a = 1.22(9)$

$= 10.98$

The arc length is 10.98 ft.

c) Use the formula $a = \theta r$, with $a = 15$ and $\theta = 3.93$.

$15 = 3.93r$

$\frac{15}{3.93} = r$

$3.82 \approx r$

The radius is 3.82 cm, to the nearest hundredth of a centimetre.

d) Use a proportion with $r = 7$ and central angle 140°.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{arc length}}{2\pi(7)} = \frac{140}{360}$$

$$\text{arc length} = \frac{14(14\pi)}{36}$$

$$\approx 17.10$$

The arc length is 17.10 m, to the nearest hundredth of a metre.

Section 4.1 Page 176 Question 14

a) Use the formula $a = \theta r$, with $r = 5$ and $\theta = \frac{5\pi}{3}$.

$$a = \left(\frac{5\pi}{3}\right)5$$

$$= \frac{25\pi}{3}$$

$$\approx 26.18$$

The arc length of the sector watered is $\frac{25\pi}{3}$ m or 26.18 m, to the nearest tenth of a metre.

b) Use a proportion with $r = 5$ and central angle $\frac{5\pi}{3}$.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{area of sector}}{\pi(5)^2} = \frac{\frac{5\pi}{3}}{2\pi}$$

$$\text{area of sector} = \frac{5(25\pi)}{6}$$

The area of the sector watered is $\frac{125\pi}{6}$, or approximately 65.45 m^2.

c) The sprinkler makes one revolution every 15 s, so in 2 min it will make 8 revolutions. In 2 min the sprinkler will rotate through $8(2\pi)$ radians, which is 16π radians, or $8(360°)$ which is 2880°.

Section 4.1 Page 177 Question 15

a) One revolution in 24 h is the same as:
$\frac{360°}{24}$ per hour, which is 15°/h; or π radians in 12 h, which is $\frac{\pi}{12}$ radians/h

b) 1000 rpm = $1000(2\pi)$ radians/min

$$= \frac{1000(2\pi)}{60} \text{ radians/s}$$

The angular velocity of the motor is $\frac{100\pi}{3}$ radians per second.

c) 10 revolutions in 4 s is $\frac{10(360°)}{4}$ or 900° per second.

In one minute, this is 60(900°) or 54 000°.
The angular velocity of the bicycle wheel is 54 000°/min.

Section 4.1 Page 177 Question 16

a) Use the formula $a = \theta r$, with $a = 170$ and $r = 72$.

$170 = \theta(72)$

$\frac{170}{72} = \theta$

$2.36 \approx \theta$

The central angle of the cable swing is 2.36 radians, to the nearest hundredth of a radian.

b) $2.36 \text{ radians} = 2.36\left(\frac{180°}{\pi}\right)$

$\approx 135.3°$

The measure of the central angle is 135.3°, to the nearest tenth of a degree.

Section 4.1 Page 177 Question 17

	Revolutions	Degrees	Radians
a)	1 rev	**360°**	**2π**
b)	**0.75 rev**	270°	**$\frac{3\pi}{2}$ or 4.7**
c)	**0.4 rev**	**150°**	$\frac{5\pi}{6}$
d)	**–0.3 rev**	**–97.4°**	–1.7
e)	**–0.1 rev**	–40°	**$-\frac{2\pi}{9}$ or –0.7**
f)	0.7 rev	**252°**	**$\frac{7\pi}{5}$ or 4.4**
g)	–3.25 rev	**–1170°**	**$-\frac{13\pi}{2}$ or –20.4**
h)	**$\frac{23}{18}$ or 1.3 rev**	460°	**$\frac{23\pi}{9}$ or 8.0**
i)	**$-\frac{3}{16}$ or –0.2 rev**	**–67.5°**	$-\frac{3\pi}{8}$

Section 4.1 Page 177 Question 18

Joran's answer includes the given angle, obtained when $n = 0$. Jasmine's answer is better as it excludes the actual given angle and just generates all positive and negative coterminal angles.

Section 4.1 Page 177 Question 19

a) $360° = 400$ grads

So, $1° = \frac{400}{360}$ grads

Then, $50° = 50\left(\frac{\overset{10}{\cancel{400}}}{\underset{9}{\cancel{360}}}\right)$

$= \frac{500}{9}$

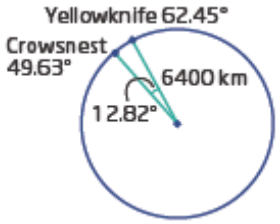
≈ 55.6 grads

b) Use the equivalence $360° = 400$ grads. To convert from degrees to gradians, multiply the number of degrees by $\frac{400}{360}$. To convert from gradians to degrees, multiply the number of gradians by $\frac{360}{400}$.

c) The gradian was developed in France along with the metric system. A right angle, or 90° is 100 gradians and so fractions of a right angle can be expressed in decimal form in gradians.

Section 4.1 Page 178 Question 20

a) The central angle is $62.45° - 49.63°$, or $12.82°$.



b) Use a proportion with $r = 6400$ and central angle $12.82°$.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{arc length}}{2\pi(6400)} = \frac{12.82}{360}$$

$$\text{arc length} = \frac{12.82(1280\pi)}{36}$$

$$\approx 1432.01$$

The distance between Yellowknife and Crowsnest Pass is 1432.01 km, to the nearest hundredth of a kilometre.

c) Example: Bowden and Aidrie are both approximately 114° W. Bowden is at 51.93° N and Airdrie is at 51.29° N.
So, the central angle is 51.93° – 51.29° or 0.64°.
Use a proportion with $r = 6400$ and central angle 0.64°.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{complete rotation}}$$

$$\frac{\text{arc length}}{2\pi(6400)} = \frac{0.64}{360}$$

$$\text{arc length} = \frac{0.64(1280\pi)}{36}$$

$$\approx 71.49$$

The distance between Bowden and Aidrie is 71.49 km, to the nearest hundredth of a kilometre.

Section 4.1 Page 178 Question 21

a) $133.284 \text{ km/h} = \frac{133.284(1000)}{60} \text{ m/min}$

$$= 2221.4 \text{ m/min}$$

Sam Whittingham's speed, in the 200-m flying start race, was 2221.4 m/min.

b) The bicycle wheel circumference is $\pi(0.6)$ m.
So, the number of wheel turns in 2221.4 m is $\frac{2221.4}{0.6\pi}$.
Then, the angular speed of the wheel is $\left(\frac{2221.4}{0.6\pi}\right)(2\pi)$ or approximately 7404.7 radians per minute.

Section 4.1 Page 178 Question 22

Speed of the water wheel is 15 rpm or $15(3\pi)$ m/min.
Convert the speed to kilometres per hour.

$$15(3\pi) \text{ m/min} = \frac{15(3\pi)(60)}{1000} \text{ km/h}$$

$$\approx 8.5 \text{ km/h}$$

The speed of the water wheel is approximately 8.5 km/h.

Section 4.1 Page 178 Question 23

In one revolution about the sun, Earth travels 93 000 000(2π) miles.
Convert 93 000 000(2π) miles in 365 days to a speed in miles per hour.

This is $\frac{93\ 000\ 000(2\pi)}{365(24)}$ miles per hour.

The speed of Earth is approximately 66 705.05 mph.

Section 4.1 Page 178 Question 24

a) $69.375° = 69° + \left(\frac{375}{1000}\right)60'$

$= 69° + 22.5'$

$= 69°\ 22'\ 30''$

b) i) $40.875° = 40° + \left(\frac{875}{1000}\right)60'$

$= 40° + 52.5'$

$= 40°\ 52'\ 30''$

ii) $100.126° = 100° + \left(\frac{126}{1000}\right)60'$

$= 100° + 7.56'$

$= 100°\ 7'\ + \left(\frac{56}{100}\right)60''$

$= 100°\ 7'\ 33.6''$

iii) $14.565° = 14° + \left(\frac{565}{1000}\right)60'$

$= 14° + 33.9'$

$= 14°\ 33'\ + \left(\frac{9}{10}\right)60''$

$= 14°\ 33'\ 54''$

iv) $80.385 = 80° + \left(\frac{385}{1000}\right)60'$

$= 80° + 23.1'$

$= 80°\ 23'\ + \left(\frac{1}{10}\right)60''$

$= 80°\ 23'\ 6''$

Section 4.1 Page 179 Question 25

a) $69°\ 22'\ 30'' = 69°\ 22' + \left(\frac{30}{60}\right)'$

$= 69°\ 22.5'$

$= 69° + \left(\frac{22.5}{60}\right)'$

$= 69.375°$

b) i) $45^\circ\ 30'\ 30'' = 45^\circ\ 30' + \left(\frac{30}{60}\right)'$

$= 45^\circ\ 30.5'$

$= 45^\circ + \left(\frac{30.5}{60}\right)'$

$\approx 45.508^\circ$

ii) $72^\circ\ 15'\ 45'' = 72^\circ\ 15' + \left(\frac{45}{60}\right)'$

$= 72^\circ\ 15.75'$

$= 72^\circ + \left(\frac{15.75}{60}\right)'$

$\approx 72.263^\circ$

iii) $105^\circ\ 40'\ 15'' = 105^\circ\ 40' + \left(\frac{15}{60}\right)'$

$= 105^\circ\ 40.25'$

$= 105^\circ + \left(\frac{40.25}{60}\right)'$

$\approx 105.671^\circ$

iv) $28^\circ\ 10' = 28^\circ\ 10'$

$= 28^\circ + \left(\frac{10}{60}\right)'$

$\approx 28.167^\circ$

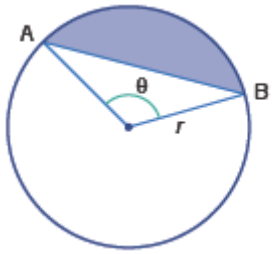
Section 4.1 Page 179 Question 26

First, write an expression for the area of the sector.
Use a proportion with central angle θ.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\theta}{2\pi}$$

$$\text{area of sector} = \frac{\theta(\pi r^2)}{2\pi}$$

$$= \frac{\theta r^2}{2}$$



Next, derive an expression for the area of the triangle.
The perpendicular height from the centre of the circle to AB will bisect the triangle into two congruent right triangles. The height will be $r\cos\frac{\theta}{2}$. The length AB will be $2r\sin\frac{\theta}{2}$.

$$\text{Area of triangle} = \frac{(2r\sin\frac{\theta}{2})(r\cos\frac{\theta}{2})}{2}$$

$$= r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= \frac{1}{2}r^2\sin\theta$$

Then, area of shaded segment = area of sector – area of triangle

$$= \frac{\theta r^2}{2} - \frac{1}{2} r^2 \sin\theta$$
$$= \frac{r^2}{2}(\theta - \sin\theta)$$

Section 4.1 Page 179 Question 27

a) At 4:00, the minute hand is at 12 and the hour hand is at 4.

$$\text{Angle} = \frac{4}{12}(360°)$$
$$= 120°$$

The angle between the hand of a clock at 4:00 is 120°.

b) At 4:10 the hour hand will have moved $\left(\frac{10}{60}\right)30°$, or 5° past the 4. The minute hand will have rotated $\left(\frac{10}{60}\right)360°$, or 60° from the 12. The angle between the hands at 4:10 is 120° + 5° – 60°, or 65°.

c) Example: The hands are at right angles to each other at 3:00 and at 9:00.

d) The hand are at right angles twice between 4:00 and 5:00.
Represent the time as 4 + x, where x is the number of minutes past the hour.
Then, the hour hand will have moved $\left(\frac{x}{60}\right)30°$. The minute hand will have rotated $\left(\frac{x}{60}\right)360°$. The angle between the hands is 90° when

$$120° + \left(\frac{x}{60}\right)30° - \left(\frac{x}{60}\right)360° = 90°$$

Solve the equation:

$$120 + \frac{x}{2} - 6x = 90$$
$$60 = 11x$$
$$x = \frac{60}{11}$$
$$x \approx 5.5$$

To the nearest minute, the hands will be at right angles to each other at 4:05.

There is another such time, when the minute hand is beyond the hour hand.
In this case, the minute hand is ahead of the hour hand, so the equation to solve is

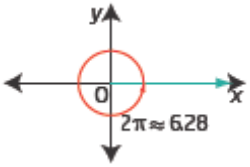
$$6x - 120 - \frac{x}{2} = 90$$
$$\frac{11x}{2} = 210$$
$$11x = 420$$
$$x \approx 38$$

To the nearest minute, the hands will be at right angles to each other at 4:38.

e) As shown in part d) above, one time occurs shortly after 4:05.

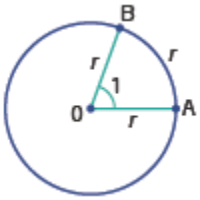
Section 4.1 Page 179 Question C1

One revolution is 2π radians, which is approximately 6.28 radians. So, 6 radians is a little less than 360°.



Section 4.1 Page 179 Question C2

One degree is $\frac{1}{360}$th of a complete revolution; so it is a very small angle. On the other hand, 1 radian is an amount of rotation that cuts off an arc with length equal to the radius. It is a little less than $\frac{1}{6}$th of a complete revolution.



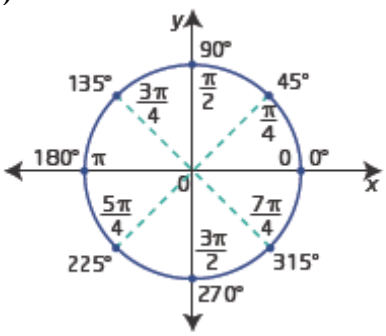
Section 4.1 Page 179 Question C3

a) 860° – 720° = 140°
The reference angle is 180° – 140°, or 40°.
An expression for all coterminal angles is $140° \pm 360°n$, $n \in \mathrm{N}$.

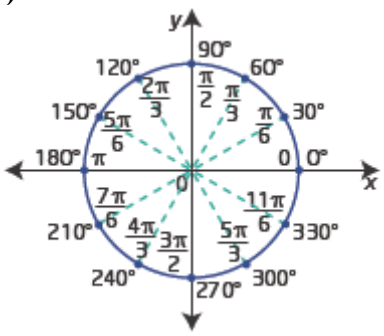
b) $(-7 + 4\pi)$ rad ≈ 5.566 rad
The reference angle is 2π – 5.566 rad, or 0.72 rad, to the nearest hundredth. An expression for all coterminal angles is $0.72 \pm 2\pi n$, $n \in \mathrm{N}$.

Section 4.1 Page 179 Question C4

a)

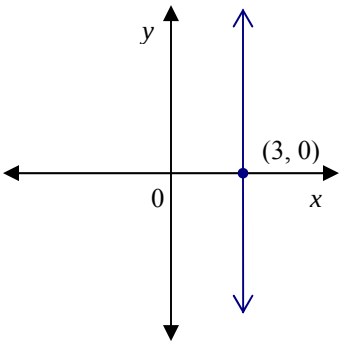


b)

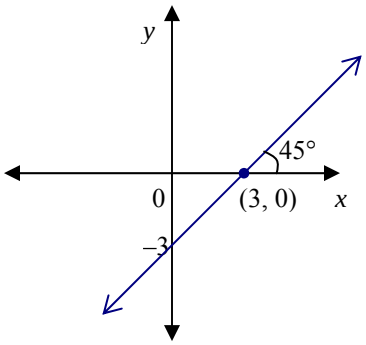


Section 4.1 Page 179 Question C5

a) $x = 3$



b) $y = x - 3$



Section 4.2 The Unit Circle

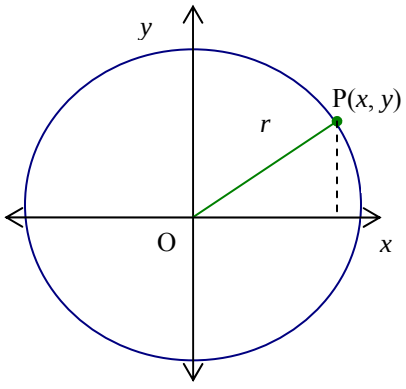
Section 4.2 Page 186 Question 1

a) In $x^2 + y^2 = r^2$, substitute $r = 4$.
$x^2 + y^2 = 16$

b) In $x^2 + y^2 = r^2$, substitute $r = 3$.
$x^2 + y^2 = 9$

c) In $x^2 + y^2 = r^2$, substitute $r = 12$.
$x^2 + y^2 = 144$

d) In $x^2 + y^2 = r^2$, substitute $r = 2.6$.
$x^2 + y^2 = 2.6^2$
$x^2 + y^2 = 6.76$



Section 4.2 Page 186 Question 2

A point is on the unit circle if $x^2 + y^2 = 1$.

a) For $\left(-\frac{3}{4}, \frac{1}{4}\right)$,

$$\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{9}{16} + \frac{1}{16}$$
$$= \frac{10}{16} \text{ or } \frac{5}{8}$$
$$\neq 1$$

Therefore, the point $\left(-\frac{3}{4}, \frac{1}{4}\right)$ is not on the unit circle.

b) For $\left(\frac{\sqrt{5}}{8}, \frac{7}{8}\right)$,

$$\left(\frac{\sqrt{5}}{8}\right)^2 + \left(\frac{7}{8}\right)^2 = \frac{5}{64} + \frac{49}{64}$$
$$= \frac{54}{64} \neq 1$$

Therefore, the point $\left(\frac{\sqrt{5}}{8}, \frac{7}{8}\right)$ is not on the unit circle.

c) For $\left(-\frac{5}{13}, \frac{12}{13}\right)$,

$$\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{25}{169} + \frac{144}{169}$$
$$= \frac{169}{169}$$
$$= 1$$

Therefore, the point $\left(-\frac{5}{13}, \frac{12}{13}\right)$ is on the unit circle.

d) For $\left(\frac{4}{5}, -\frac{3}{5}\right)$,

$$\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25}$$
$$= \frac{25}{25}$$
$$= 1$$

Therefore, the point $\left(\frac{4}{5}, -\frac{3}{5}\right)$ is on the unit circle.

e) For $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$,

$$\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

Therefore, the point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ is on the unit circle.

f) For $\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$,

$$\left(\frac{\sqrt{7}}{4}\right)^2 + \left(\frac{3}{4}\right)^2 = \frac{7}{16} + \frac{9}{16}$$
$$= \frac{16}{16}$$
$$= 1$$

Therefore, the point $\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$ is on the unit circle.

Section 4.2 Page 187 Question 3

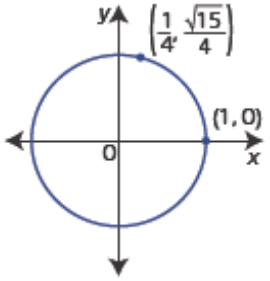
a) $\left(\frac{1}{4}\right)^2 + y^2 = 1$

$$y^2 = 1 - \frac{1}{16}$$

$$y^2 = \frac{15}{16}$$

$$y = \pm\frac{\sqrt{15}}{4}$$

In quadrant I, $y = \frac{\sqrt{15}}{4}$.

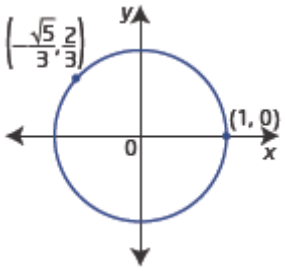


b) $x^2 + \left(\frac{2}{3}\right)^2 = 1$

$$x^2 = 1 - \frac{4}{9}$$

$$x = \pm\frac{\sqrt{5}}{3}$$

In quadrant II, $x = -\frac{\sqrt{5}}{3}$.



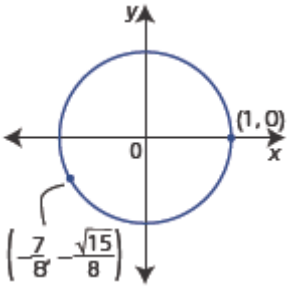
c) $\left(-\frac{7}{8}\right)^2 + y^2 = 1$

$$y^2 = 1 - \frac{49}{64}$$

$$y^2 = \frac{15}{64}$$

$$y = \pm\frac{\sqrt{15}}{8}$$

In quadrant III, $y = -\frac{\sqrt{15}}{8}$.

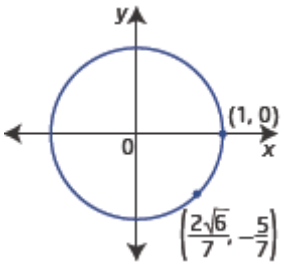


d) $x^2 + \left(-\frac{5}{7}\right)^2 = 1$

$$x^2 = 1 - \frac{25}{49}$$

$$x^2 = \frac{24}{49}$$

$$x = \pm\frac{\sqrt{24}}{7}$$



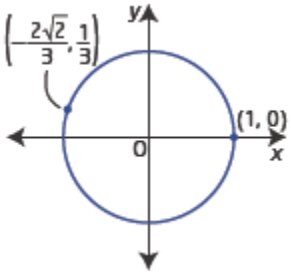
In quadrant IV, $x = \frac{\sqrt{24}}{7}$ or $\frac{2\sqrt{6}}{7}$.

e) $x^2 + \left(\frac{1}{3}\right)^2 = 1$

$$x^2 = 1 - \frac{1}{9}$$

$$x^2 = \frac{8}{9}$$

$$x = \pm\frac{\sqrt{8}}{3}$$



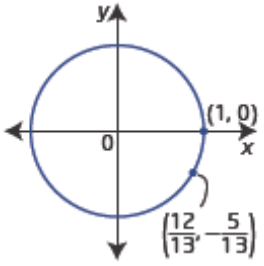
For $x < 0$, $x = -\frac{\sqrt{8}}{3}$ or $-\frac{2\sqrt{2}}{3}$.

f) $\left(\frac{12}{13}\right)^2 + y^2 = 1$

$$y^2 = 1 - \frac{144}{169}$$

$$y^2 = \frac{25}{169}$$

$$y = \pm\frac{5}{13}$$



The point is not in quadrant I. Since it has a positive x-value, the point must be in quadrant IV with $y = -\frac{5}{13}$.

Section 4.2 Page 187 Question 4

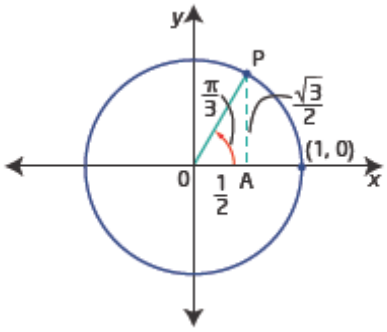
a) A rotation of π radians takes the terminal arm of the angle on to the x-axis to the left of the origin.
So, P(π) = (–1, 0).

b) A rotation of $-\frac{\pi}{2}$ radians takes the terminal arm of the angle onto the *y*-axis below the origin.

So, $P\left(-\frac{\pi}{2}\right) = (0, -1)$.

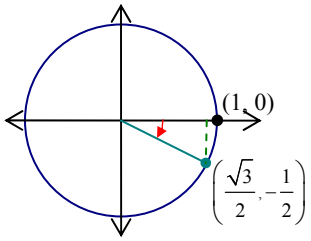
c) A rotation of $\frac{\pi}{3}$ radians takes the terminal arm of the angle into the first quadrant as shown.

So, $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.



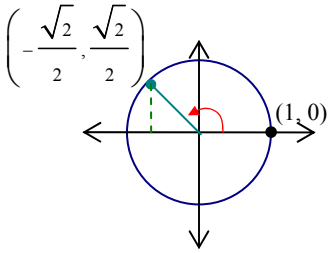
d) A rotation of $-\frac{\pi}{6}$ radians takes the terminal arm of the angle into quadrant IV as shown.

So, $P\left(-\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.



e) A rotation of $\frac{3\pi}{4}$ radians takes the terminal arm of the angle into the second quadrant as shown.

So, $P\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.



f) A rotation of $-\frac{7\pi}{4}$ radians takes the terminal arm of the angle into the first quadrant and is coterminal with $\frac{\pi}{4}$.

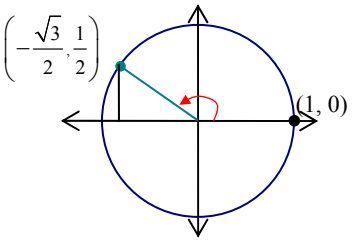
So, $P\left(-\frac{7\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

g) A rotation of 4π is two complete turns and is coterminal with 0 radians.
So, $P(4\pi) = (1, 0)$.

h) A rotation of $\frac{5\pi}{2}$ radians is one complete turn plus one-half turn and takes the terminal arm of the angle onto the *y*-axis above the origin.

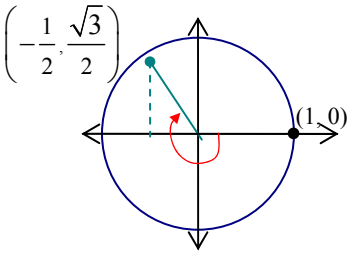
So, $P\left(\frac{5\pi}{2}\right) = (0, 1)$.

i) A rotation of $\frac{5\pi}{6}$ radians takes the terminal arm of the angle into quadrant II as shown.



So, $P\left(-\frac{\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

j) A rotation of $-\frac{4\pi}{3}$ radians takes the terminal arm of the angle into quadrant II, with reference angle $\frac{\pi}{3}$ as shown.



So, $P\left(-\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

Section 4.2 Page 187 Question 5

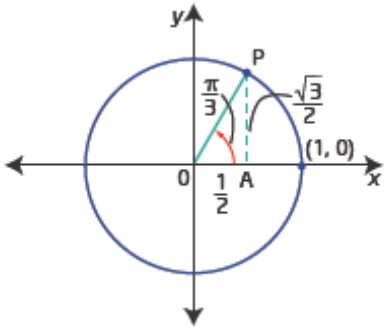
a) (0, –1) is on the *y*-axis, below the origin, so $\theta = \frac{3\pi}{2}$.

b) (1, 0) is on the *x*-axis, to the right of the origin, so $\theta = 0$.

c) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is in quadrant I, and since *x* and *y* are equal the measure of the central angle is $\frac{\pi}{4}$.

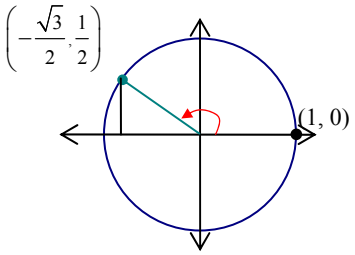
d) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is in quadrant II, and since *x* and *y* have equal length, the measure of the reference angle is $\frac{\pi}{4}$. In quadrant II, $\theta = \frac{3\pi}{4}$.

e) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is in quadrant I and $\theta = \frac{\pi}{3}$ as shown.



f) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ is in quadrant IV, and so $\theta = 2\pi - \frac{\pi}{3}$, or $\frac{5\pi}{3}$.

g) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ is in quadrant II as shown and $\theta = \frac{5\pi}{6}$.



h) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ is similar to part g), except the terminal arm is in quadrant III.

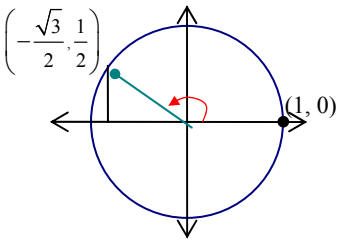
So, $\theta = \frac{7\pi}{6}$.

i) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ is in quadrant III, and since *x* and *y* are equal the measure of the central angle is $\pi + \frac{\pi}{4}$. So, $\theta = \frac{5\pi}{4}$.

j) (–1, 0) in on the *x*-axis, to the left of the origin. So, $\theta = \pi$.

Section 4.2 Page 187 Question 6

If $P(\theta) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, then θ is in quadrant II as shown. One positive measure for θ is $\frac{5\pi}{6}$ and a coterminal negative angle is $\frac{5\pi}{6} - 2\pi$, or $-\frac{7\pi}{6}$.



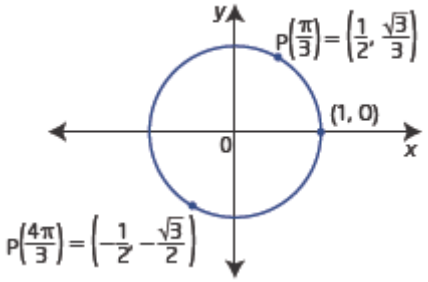
Section 4.2 Page 187 Question 7

a) Example: Choose $\theta = \frac{\pi}{3}$, then

$\theta + \pi = \frac{\pi}{3} + \pi$, or $\frac{4\pi}{3}$.

$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and

$P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



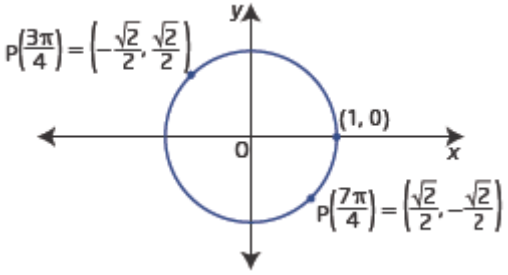
b) Example: Choose $\theta = \frac{3\pi}{4}$, then

$\theta + \pi = \frac{3\pi}{4} + \pi$, or $\frac{7\pi}{4}$.

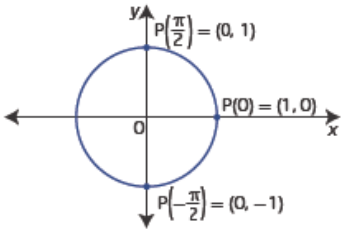
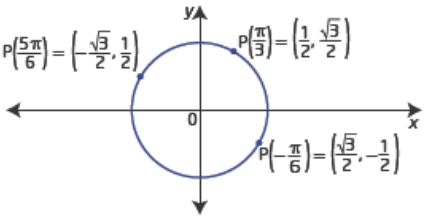
$P\left(\frac{3\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, or $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

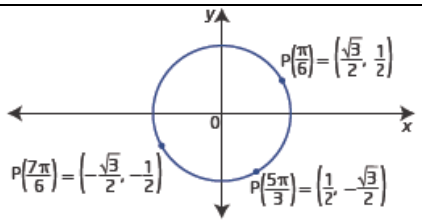
and

$P\left(\frac{7\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, or $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

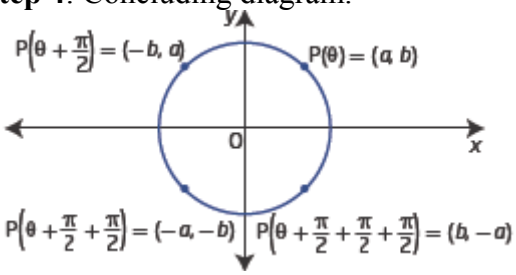


Section 4.2 Page 187 Question 8

Point	**Step 2:** $+\frac{1}{4}$ **turn**	**Step 3:** $-\frac{1}{4}$ **turn**	**Step 4: Description**	**Diagram**
$P(0) = (1, 0)$	$P\left(\frac{\pi}{2}\right) = (0, 1)$	$P\left(-\frac{\pi}{2}\right) = (0, -1)$	*x*- and *y*-values change places and take appropriate signs for the new quadrant	
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$P\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = P\left(-\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	*x*- and *y*-values change places and take appropriate signs for the new quadrant	

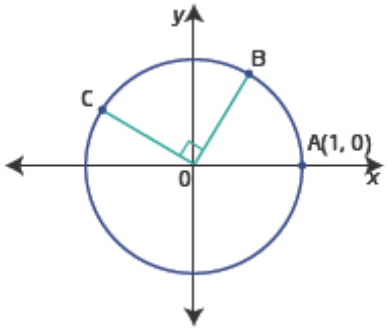
$P\left(\frac{5\pi}{3}\right)$ $=\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$P\left(\frac{5\pi}{3}+\frac{\pi}{2}\right)$ $=P\left(\frac{\pi}{6}\right)$ $=\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{5\pi}{3}-\frac{\pi}{2}\right)$ $=P\left(\frac{7\pi}{6}\right)$ $=\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	*x*- and *y*-values change places and take appropriate signs for the new quadrant	

Step 4: Concluding diagram.



Section 4.2 Page 188 Question 9

a) The diagram shows a unit circle: $x^2 + y^2 = 1$.



b) C is related to B by a 90° rotation, so as shown in the previous question, the coordinates switch and the signs are adjusted. In quadrant I, both coordinates are positive. So the coordinates of B are $\left(\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$.

c) $\overset{\frown}{AC} = \overset{\frown}{AB} + \frac{\pi}{2}$

$\overset{\frown}{AC} = \theta + \frac{\pi}{2}$

d) If P(θ) = B, then $P\left(\theta - \frac{\pi}{2}\right)$ is related by a rotation of $\frac{\pi}{2}$ clockwise, which puts it in quadrant IV.

e) The maximum value for the *x*-coordinates or the *y*-coordinates is 1.
The minimum value for the *x*-coordinates or the *y*-coordinates is –1.

Section 4.2 Page 188 Question 10

a) Mya is correct, because in quadrant I the *x*-coordinates start at 1 for an angle of 0° and decrease to a minimum of 0 for an angle of 90°.

b) Check Mya's answer by substituting $x = 0.807$ and $y = 0.348\ 751$ into the equation for the unit circle, $x^2 + y^2 = 1$. If the sum of the squares on the left side is not equal to 1, then Mya has made an error and her calculation needs checking. Observe that Mya forgot to the take the square root.

Left Side $= (0.807)^2 + (0.348\ 751)^2$
$= 0.772\ 876\ 26$
$\neq 1$

Recalculate: when $x = 0.807$

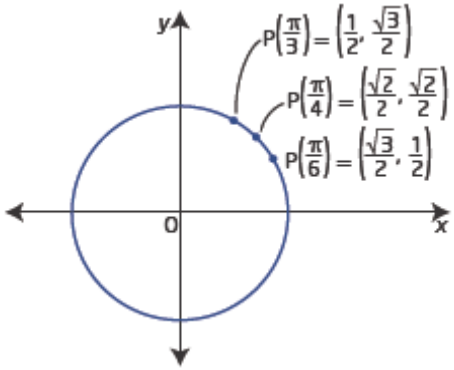
$(0.807)^2 + y^2 = 1$
$y^2 = 1 - (0.807)^2$
$y = \sqrt{1-(0.807)^2}$
$y \approx 0.590\ 551$

c) Substitute $y = 0.2571$ in $x^2 + y^2 = 1$

$x^2 + (0.2571)^2 = 1$
$x^2 = 1 - (0.2571)^2$
$x = \sqrt{1-(0.2571)^2}$
$x \approx 0.9664$

Section 4.2 Page 188 Question 11

a)



b) The denominators of the coordinates are all 2.

c) The numerators of the x-coordinates are decreasing as P(θ) increases, while the y-coordinates are increasing. This makes sense, in the quadrant I, because the terminal arm is getting closer to the y-axis as the angle increases. At $P\left(\frac{\pi}{4}\right)$ the x-coordinate and the y-coordinate are the same.

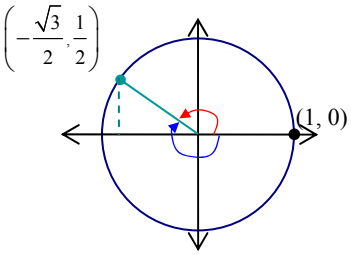
d) Square roots are derived from the special right triangles with acute angles $\frac{\pi}{4}, \frac{\pi}{4}$, and $\frac{\pi}{3}, \frac{\pi}{6}$.

e) Example. Remember the patterns of side ratios for the special right triangles and that for angles less than $\frac{\pi}{4}$, the *x*-coordinate is greater than the *y*-coordinate.

Section 4.2 Page 188 Question 12

a) The interval $-2\pi \le \theta < 4\pi$ represents three rotations around the unit circle: one complete clockwise rotation starting at 0 and two complete counterclockwise rotations. For every point on the unit circle there will be three coterminal angles in this interval.

b) If $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, then θ is in quadrant II.



In the interval $-2\pi \le \theta < 0$, $\theta = -\frac{4\pi}{3}$.

In the interval $0 \le \theta < 2\pi$, $\theta = \frac{2\pi}{3}$.

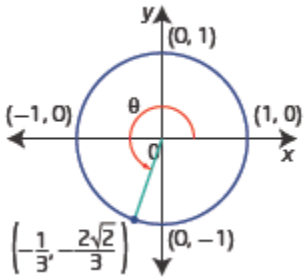
In the interval $2\pi \le \theta < 4\pi$,

$\theta = 2\pi + \frac{2\pi}{3}$, or $\frac{8\pi}{3}$.

c) The terminal arm of the three angles is in the same position on the unit circle and all three angles have the same reference angle. The angles are "coterminal".

Section 4.2 Page 188 Question 13

a) If $P(\theta) = \left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$, then θ is an angle with terminal arm in quadrant III. The location of P, at the intersection of the terminal arm and the unit circle, is given by $x = -\frac{1}{3}$ and $y = -\frac{2\sqrt{2}}{3}$.



b) θ terminates in quadrant III.

c) $P\left(\theta + \frac{\pi}{2}\right)$ will be in quadrant IV and for a rotation of $+\frac{\pi}{2}$ the coordinates of P from part a) switch and the signs are adjusted for quadrant IV.

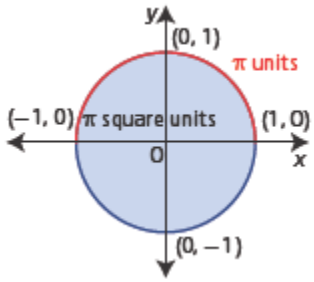
$P\left(\theta + \frac{\pi}{2}\right) = \left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$

d) $P\left(\theta-\frac{\pi}{2}\right)$ will be in quadrant II and for a rotation of $-\frac{\pi}{2}$ the coordinates of P from part a) switch and the signs are adjusted for quadrant II.

$$P\left(\theta-\frac{\pi}{2}\right)=\left(-\frac{2\sqrt{2}}{3},\frac{1}{3}\right)$$

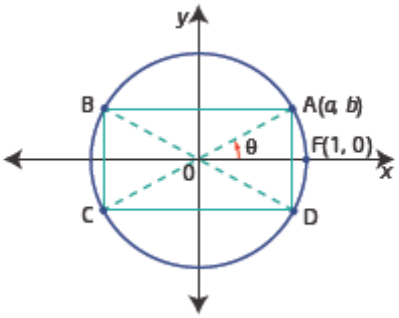
Section 4.2 Page 189 Question 14

π units is a length. On the unit circle it is the arc of an angle from (1, 0) to (–1, 0).
π square units is an area. It is the area of the unit circle because when $r = 1$ in $A = \pi r^2$ the area is $A = \pi(1)^2$, or π square units.



Section 4.2 Page 189 Question 15

a) Since ABCD is a rectangle, opposite sides are the same length. So, the coordinates of the other vertices are:
B(–*a*, *b*)
C(–*a*, –*b*)
D(*a*, –*b*)



b) i) Since π is half a complete rotation, θ + π will have terminal arm OC.
The angle will pass through C.

ii) Similarly, – π is half a complete rotation, in the clockwise direction, so θ – π will have terminal arm OC. The angle will pass through C.

iii) –θ is the ∠FOD. A rotation of π from OD will have terminal arm OB.
The angle –θ + π will pass through B.

iv) A rotation of –π from OD will have terminal arm OB.
The angle –θ – π will pass through B.

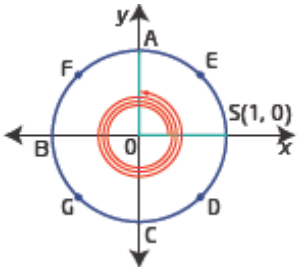
c) The answers would be the same because, since the angles are expressed in radians, arc FA is the same as θ.

Section 4.2 Page 189 Question 16

a) In a counterclockwise direction, arc SG is created by a rotation of $\frac{5\pi}{4}$.

The arc length of SG is $\frac{5\pi}{4}$.

b) $P\left(\frac{13\pi}{2}\right)$ represents a point on the unit circle obtained by rotating an angle in standard position through 3 complete rotations plus half a rotation. We know this because $\frac{13\pi}{2} = \frac{12\pi}{2} + \frac{\pi}{2}$, or $6\pi + \frac{\pi}{2}$ and 6π is 3 complete rotations of 2π each. The point on the unit circle corresponding to this amount of rotation is A.



c) P(5) is in quadrant IV. Since $P\left(\frac{3\pi}{2}\right)$ is approximately P(4.71) and P(2π) is approximately P(6.28), so P(5) is between C and D.

Section 4.2 Page 189 Question 17

a) $y = -3x$ ①
$x^2 + y^2 = 1$ ②
Substitute from ① into ②.

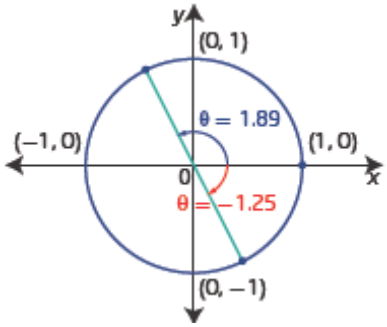
$$x^2 + (-3x)^2 = 1$$
$$10x^2 = 1$$
$$x = \pm\sqrt{\frac{1}{10}}$$

Substitute in ① to find the corresponding y-values

$$y = -3\left(\pm\frac{1}{\sqrt{10}}\right)$$
$$y = \frac{3}{\sqrt{10}} \text{ or } y = -\frac{3}{\sqrt{10}}$$

The points of intersection are $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$, or $(0.1\sqrt{10}, -0.3\sqrt{10})$, and $\left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$, or $(-0.1\sqrt{10}, 0.3\sqrt{10})$.

b) The first point of intersection above is in quadrant IV and the second point is a rotation of π away, in quadrant II. Since the first coordinate is cos θ, the measure of reference angle θ is approximately 1.25 radians.



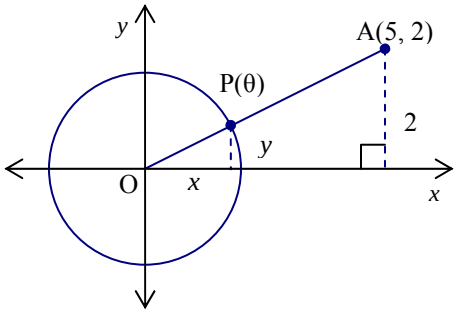
Section 4.2 Page 189 Question 18

a) First, use the Pythagorean theorem to determine the length of the hypotenuse OA.

$OA^2 = 5^2 + 2^2$

$OA = \sqrt{29}$

Next, compare sides of the similar right triangles.



$$\frac{x}{1} = \frac{5}{\sqrt{29}} \qquad \frac{y}{1} = \frac{2}{\sqrt{29}}$$

$$x = \frac{5}{\sqrt{29}} \qquad y = \frac{5}{\sqrt{29}}$$

The exact coordinates of P(θ) are $\left(\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$.

b) The radius of the larger circle passing through A is the length of OA found in part a). It is $\sqrt{29}$.

c) The equation for the larger circle passing through A is $x^2 + y^2 = 29$.

Section 4.2 Page 190 Question 19

Consider P(θ) = (*x*, *y*).

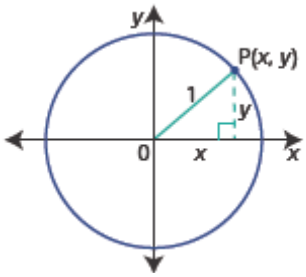
In the unit circle, the hypotenuse is 1.

Then, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$$= \frac{x}{1} \qquad = \frac{y}{1}$$

$$= x \qquad = y$$

So, P(θ) = (cos θ, sin θ).



Section 4.2 Page 190 Question 20

a) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ are the coordinates of a point on the unit circle in quadrant I, describing a rotation of $\frac{\pi}{4}$. The equivalent polar coordinates are $\left(1, \frac{\pi}{4}\right)$.

b) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{3}\right)$ are the coordinates of a point on a circle in quadrant III.

First, determine the radius of the circle:

$$\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{3}\right)^2 = r^2$$

$$\frac{3}{4} + \frac{1}{9} = r^2$$

$$\frac{31}{36} = r^2$$

$$r = \frac{\sqrt{31}}{6}$$

Next, determine the angle measure.

$$\tan\theta = \frac{y}{x}$$

$$= -\frac{1}{3} \div \left(-\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1}{3}\left(-\frac{2}{\sqrt{3}}\right)$$

$$= \frac{2}{2\sqrt{3}}$$

So, $\theta \approx 0.367$

This is the measure of the reference angle, so in quadrant III the angle is $\pi + 0.367$ or 3.509.

The equivalent polar coordinates are $\left(\frac{\sqrt{31}}{6}, 3.509\right)$.

c) (2, 2) are the coordinates of a point on a circle in quadrant I. Since $x = y$ this is a rotation of $\frac{\pi}{4}$.

Determine the radius:

$$2^2 + 2^2 = r^2$$

$$8 = r^2$$

$r = 2\sqrt{2}$

The equivalent polar coordinates are $\left(2\sqrt{2}, \frac{\pi}{4}\right)$.

d) (4, –3) are the coordinates of a point on a circle in quadrant IV.
First, determine the radius:

$4^2 + (-3)^2 = r^2$

$25 = r^2$

$r = 5$

Next determine the angle measure.

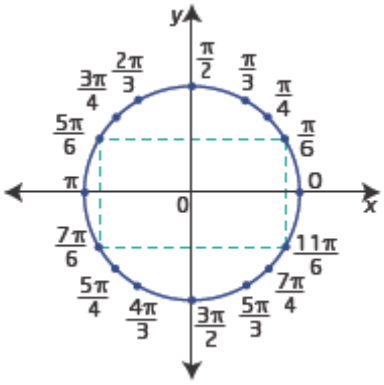
$\tan\theta = \frac{3}{4}$

$\theta \approx 0.644$

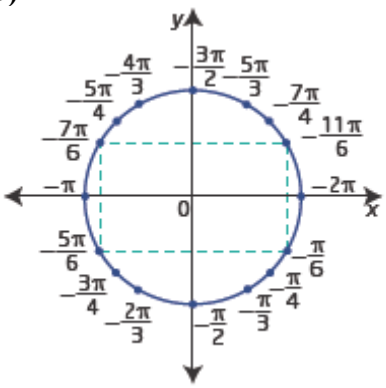
This is the measure of the reference angle, so in quadrant IV the angle is $2\pi - 0.644$ or 5.640.
The equivalent polar coordinates are (5, 5.640).

Section 4.2 Page 190 Question C1

a)



b)



c) Using the special right triangle for $\frac{\pi}{6}$, the vertex in quadrant I has coordinates $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Then adjust the coordinates for the vertices of the rectangle in other quadrants.

In quadrant II, $P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

In quadrant III, $P\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

In quadrant IV, $P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

d) Example: Divide the unit circle into eighths to mark off multiples of $\frac{\pi}{4}$. Use the ratio of sides of the special isosceles right triangle with hypotenuse 1 ($\frac{\sqrt{2}}{2} : \frac{\sqrt{2}}{2} : 1$) adjusting the signs of coordinates for each quadrant. Think of hours on a clock face to mark off multiples of $\frac{\pi}{6}$. Then, use the ratio of sides of the special right triangle with hypotenuse 1 ($\frac{\sqrt{3}}{2} : \frac{1}{2} : 1$) adjusting the signs of coordinates for each quadrant.

Section 4.2 Page 190 Question C2

a) Let n represent the measure of $\angle BOA$. Then, $\angle ABO = \angle BAO = 2n$.
Arc AB is the same as the measure of $\angle BAO$, in radians.
Use the angle sum of a triangle.

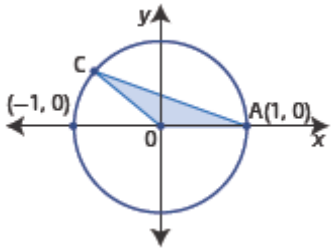
$$n + 2n + 2n = \pi$$
$$5n = \pi$$
$$n = \frac{\pi}{5}$$

The measure of arc AB is $\frac{\pi}{5}$.

b) If $P(C) = P\left(B + \frac{\pi}{2}\right)$, then C must be in quadrant II.

The measure of arc AC, and of $\angle COA$, is

$$\frac{\pi}{5} + \frac{\pi}{2} = \frac{7\pi}{10}$$



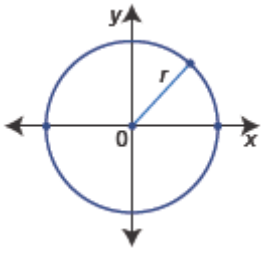
Then in ΔOAC,

$$\angle CAO + \angle ACO = \pi - \frac{7\pi}{10}$$
$$= \frac{3\pi}{10}$$

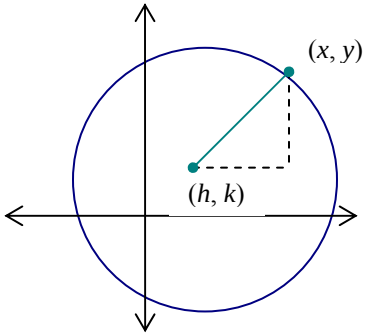
Since $\angle CAO = \angle ACO$, $\angle CAO = \frac{3\pi}{20}$

Section 4.2 Page 190 Question C3

a) $x^2 + y^2 = r^2$



b) Example: If the centre of the circle is moved to (h, k), then a new right triangle can be used to determine the equation. Its horizontal length will be $(x - h)$, and its vertical side will be $(y - k)$. Then, using the Pythagorean theorem $(x - h)^2 + (y - k)^2 = r^2$.



Section 4.2 Page 190 Question C4

a) Area of circle $= \pi(1^2)$
Area of the square $= 2^2$

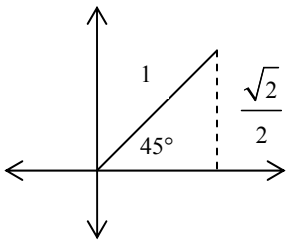
$$\text{Percent of paper cut off} = \left(\frac{4-\pi}{4}\right)100$$
$$\approx 21.5\%$$

b) Circumference : perimeter of square $= 2\pi : 8$
$$= \pi : 4$$

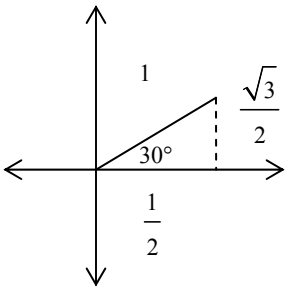
4.3 Trigonometric Ratios

Section 4.3 Page 201 Question 1

a) $\sin 45° = \frac{y}{r}$
$$= \frac{\sqrt{2}}{2}$$



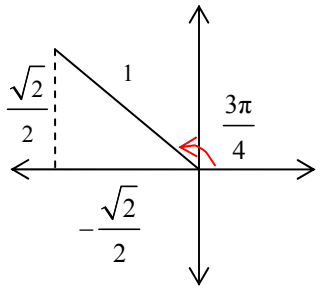
b) $\tan 30° = \frac{x}{y} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
$$= \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$



c) $\cos\frac{3\pi}{4} = \frac{x}{r}$

$= \frac{-\frac{\sqrt{2}}{2}}{1}$

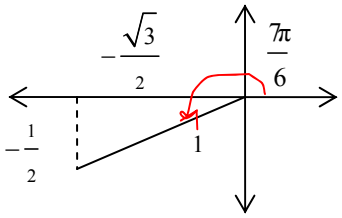
$= -\frac{\sqrt{2}}{2}$



d) $\cot\frac{7\pi}{6} = \frac{y}{x}$

$= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= \sqrt{3}$



e) Refer to the diagram in part d), because $\frac{7\pi}{6} = 210°$.

$\csc 210° = \frac{r}{y}$

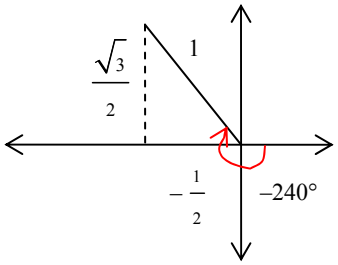
$= \frac{1}{-\frac{1}{2}}$

$= -2$

f) $\sec(-240°) = \frac{r}{x}$

$= \frac{1}{-\frac{1}{2}}$

$= -2$



g) A point on the terminal arm of $\frac{3\pi}{2}$ is $(0, -1)$.

Therefore, $\tan\frac{3\pi}{2} = \frac{y}{x}$

$= \frac{-1}{0}$

= undefined

h) A point on the terminal arm of π is $(-1, 0)$.

Therefore, $\sec \pi = \frac{r}{x}$

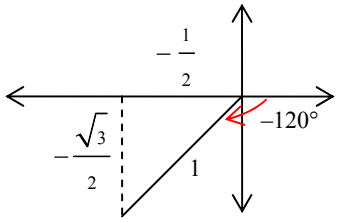
$= \frac{1}{-1}$

$= -1$

i) $\cot(-120°) = \frac{x}{y}$

$= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$



j) A rotation of 390° is coterminal with 30°.

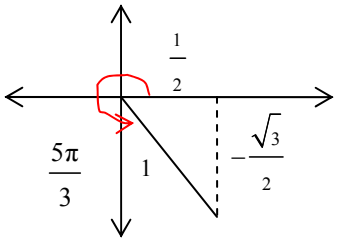
So, $\cos 390° = \cos 30°$

$= \frac{\sqrt{3}}{2}$

k) $\sin \frac{5\pi}{3} = \frac{y}{r}$

$= \frac{-\frac{\sqrt{3}}{2}}{1}$

$= -\frac{\sqrt{3}}{2}$



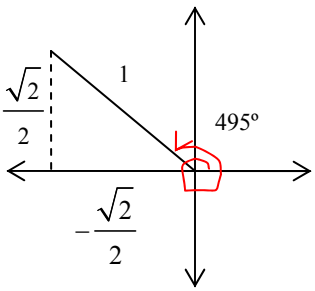
l) A rotation of 495° is coterminal with 495° – 360°, or 135°.

Therefore, $\csc 495° = \csc 135°$

$= \frac{r}{y}$

$= \frac{1}{\frac{\sqrt{2}}{2}}$

$= \frac{2}{\sqrt{2}}$ or $\sqrt{2}$



Section 4.3 Page 201 Question 2

Use a calculator. Verify that the sign is correct for the quadrant.

a) $\cos 47° \approx 0.68$

b) $\cot 160° = \dfrac{1}{\tan 160°}$
≈ -2.75

c) $\sec 15° = \dfrac{1}{\cos 15°}$
≈ 1.04

d) $\csc 4.71 = \dfrac{1}{\sin 4.71}$
≈ -1.00

e) $\sin 5 \approx -0.96$

f) $\tan 0.94 \approx 1.37$

g) $\sin \dfrac{5\pi}{7} \approx 0.78$

h) $\tan 6.9 \approx 0.71$

i) $\cos 302° \approx 0.53$

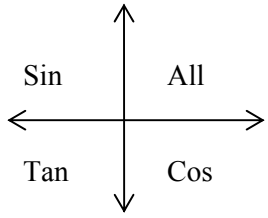
j) $\sin\left(-\dfrac{11\pi}{19}\right) \approx -0.97$

k) $\cot 6 = \dfrac{1}{\tan 6}$
≈ -3.44

l) $\sec(-270°) = \dfrac{1}{\cos(-270°)}$
$= \dfrac{1}{0}$
$=$ undefined

Section 4.3 Page 202 Question 3

The diagram shows a memory aid for the ratios that are positive, i.e. greater than 0, in each quadrant.



a) $\cos \theta > 0$ in quadrants I and IV

b) $\tan \theta < 0$ in quadrants II and IV

c) $\sin \theta < 0$ in quadrants III and IV

d) $\sin \theta > 0$ in quadrants I and II, $\cot \theta < 0$ in quadrants II and IV, so both conditions are true in quadrant II.

e) $\cos \theta < 0$ in quadrants II and III, $\csc \theta > 0$ in quadrants I and II, so both conditions are true in quadrant II.

f) sec θ > 0 in quadrants I and IV, tan θ > 0 in quadrants I and III, so both conditions are true in quadrant I.

Section 4.3 Page 202 Question 4

a) 250° = 180° + 70°, so 250° is in quadrant III. In quadrant III, sine is negative.
So, sin 250° = –sin 70°.

b) 290° = 360° – 70°, so 290° is in quadrant IV. In quadrant IV, tangent is negative.
So, tan 290° = –tan 70°.

c) 135° = 180° – 45°, so 135° is in quadrant II. In quadrant II, cosine and secant are negative. So, sec 135° = –sec 45°.

d) 4 radians is in quadrant III and its reference angle is 4 – π. In quadrant III, cosine is negative. So, cos 4 = –cos (4 – π).

e) 3 radians is in quadrant II and its reference angle is π – 3. In quadrant II, sine and cosecant are positive. So, csc 3 = csc (π – 3).

f) 4.95 radians is in quadrant III and its reference angle is 4.95 – π. In quadrant III, tangent and cotangent are positive. So, cot 4.95 = cot (4.95 – π).

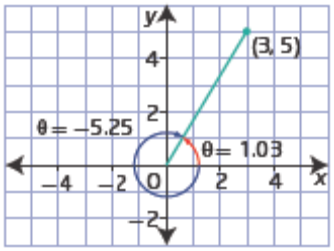
Section 4.3 Page 202 Question 5

a) (3, 5) is in quadrant I.

Use $\tan\theta = \frac{y}{x}$

$= \frac{5}{3}$

Then, the reference angle is θ ≈ 1.03.
A negative coterminal angle is 1.03 – 2π ≈ –5.25.

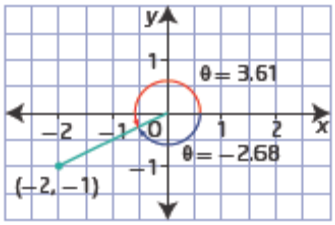


b) (–2, –1) is in quadrant III.

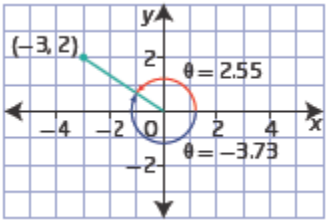
Use $\tan\theta = \frac{y}{x}$

$= \frac{-1}{-2}$ or 0.5

Then, the reference angle is θ ≈ 0.4636….
One positive angle, in quadrant III, is π + θ or approximately 3.61. A negative coterminal angle is 3.61 – 2π ≈ –2.68.



c) (–3, 2) is in quadrant II.

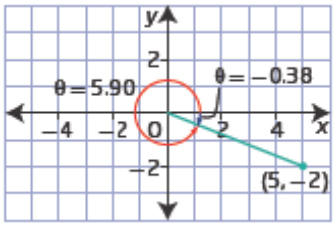


Use $\tan\theta = \frac{y}{x}$

$= \frac{2}{-3}$

Then, a reference angle is $\theta \approx 0.588\ldots$.
One positive angle, in quadrant II, is $\pi - \theta$ or approximately 2.55.
A negative coterminal angle is $-\pi - \theta$ or approximately –3.73.

d) (5, –2) is in quadrant IV.



Use $\tan\theta = \frac{y}{x}$

$= \frac{-2}{5}$

Then, a reference angle is $\theta \approx -0.38$. This angle is in quadrant IV.
A positive coterminal angle is $2\pi + \theta$ or approximately 5.90.

Section 4.3 Page 202 Question 6

a) $\cos\theta = \frac{x}{r}$. 300° is in quadrant IV, so the *x*-coordinate of a point on the terminal arm is positive. Therefore, cos 300° is positive.

b) $\sin\theta = \frac{y}{r}$**.** 4 radians is in quadrant III, so the *y*-coordinate of a point on the terminal arm is negative. Therefore, sin 4 is negative.

c) $\cot\theta = \frac{x}{y}$. 156° is in quadrant II, so the *x*-coordinate of a point on the terminal arm is negative and the *y*-coordinate is positive. Therefore, cot 156° is negative.

d) $\csc\theta = \frac{r}{y}$. –235° is in quadrant II, so the *y*-coordinate of a point on the terminal arm is positive. Therefore, cos 300° is positive.

e) $\tan\theta = \frac{y}{x}$. $\frac{13\pi}{6}$ is coterminal with $\frac{\pi}{6}$ and is in quadrant I, so the *x*-coordinate of a point on the terminal arm is positive and the *y*-coordinate is positive. Therefore, $\tan\frac{13\pi}{6}$ is positive.

f) $\sec\theta = \frac{r}{x}$. $\frac{17\pi}{3}$ is coterminal with $\frac{5\pi}{3}$ and is in quadrant IV, so the *x*-coordinate of a point on the terminal arm is positive. Therefore, $\sec\frac{17\pi}{3}$ is positive.

Section 4.3 Page 202 Question 7

a) $\sin^{-1} 0.2 \approx 0.2014$
This means that an angle of 0.2014 radians has a sine ratio of 0.2.

b) $\tan^{-1} 7 \approx 1.4289$
This means that an angle of 1.4289 radians has a tangent ratio of 7.

c) $\sec 450° = \frac{1}{\cos 450°}$ which is undefined because 450° is coterminal with 90° and $\cos 90° = 0$.

d) $\cot(-180°) = \frac{1}{\tan(-180°)}$ which is undefined because (–180°) is coterminal with 180° and $\tan 180° = 0$.

Section 4.3 Page 202 Question 8

a) Since $P(\theta) = \left(\frac{3}{5}, y\right)$ lies on the unit circle,

$x^2 + y^2 = 1$

$$\left(\frac{3}{5}\right)^2 + y^2 = 1$$

$$y^2 = 1 - \frac{9}{25}$$

$$y^2 = \frac{16}{25}$$

$$y = \pm\frac{4}{5}$$

For P(θ) to be in quadrant IV, y must be negative. So, $y = -\frac{4}{5}$.

b) $\tan\theta = \frac{y}{x}$

$= \frac{-\frac{4}{5}}{\frac{3}{5}}$

$= -\frac{4}{3}$

c) $\csc\theta = \frac{r}{y}$

$= \frac{1}{-\frac{4}{5}}$

$= -\frac{5}{4}$

$= -1.25$

Section 4.3 Page 202 Question 9

a) $\cos 60° + \sin 30° = \frac{1}{2} + \frac{1}{2}$

$= 1$

b) $(\sec 45°)^2 = \frac{1}{(\cos 45)^2}$

$= 1 \div \left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 \div \frac{1}{2}$

$= 2$

c) $\left(\cos\frac{5\pi}{3}\right)\left(\sec\frac{5\pi}{3}\right)$

$= \left(\cos\frac{5\pi}{3}\right)\left(\frac{1}{\left(\cos\frac{5\pi}{3}\right)}\right)$

$= 1$

d) $(\tan 60°)^2 - (\sec 60°)^2$

$= \left(\sqrt{3}\right)^2 - \left(1 \div \left(\frac{1}{2}\right)\right)^2$

$= 3 - 4$

$= -1$

e) $\left(\cos\frac{7\pi}{4}\right)^2 + \left(\sin\frac{7\pi}{4}\right)^2$

$= \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

f) $\left(\cot\frac{5\pi}{6}\right)^2 = 1 \div \left(\tan\frac{5\pi}{6}\right)^2$

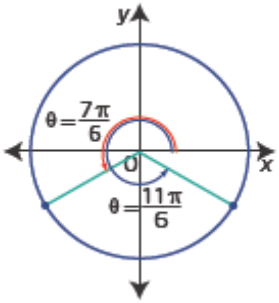
$= 1 \div \left(-\frac{1}{\sqrt{3}}\right)^2$

$= 1 \div \frac{1}{3}$

$= 3$

Section 4.3 Page 202 Question 10

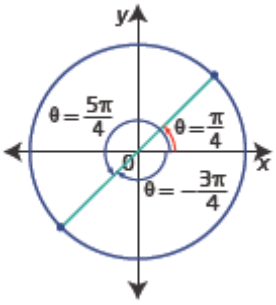
a) $\sin\theta = -\frac{1}{2}$, $0 \le \theta < 2\pi$, means that θ is in quadrant III or IV. The reference angle is $\frac{\pi}{6}$.



In quadrant III, $\theta = \pi + \frac{\pi}{6}$, or $\frac{7\pi}{6}$.

In quadrant IV, $\theta = 2\pi - \frac{\pi}{6}$, or $\frac{11\pi}{6}$.

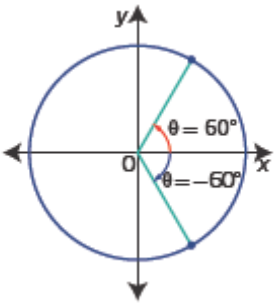
b) $\cot\theta = 1$, $-\pi \le \theta < 2\pi$, means that θ is in quadrant I or III. The reference angle is $\frac{\pi}{4}$.



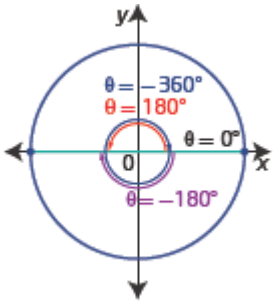
In quadrant I, $\theta = \frac{\pi}{4}$.

In quadrant III, for a positive rotation, $\theta = \pi + \frac{\pi}{4}$, or $\frac{5\pi}{4}$. There is also a negative angle in the given domain that falls in quadrant III; $\theta = -\pi + \frac{\pi}{4}$, or $-\frac{3\pi}{4}$.

c) $\sec\theta = 2$, $-180° \le \theta < 90°$; $\cos\theta = \frac{1}{2}$ means that θ is in quadrant I or IV. The reference angle is 60°.
In quadrant I, $\theta = 60°$.
The negative rotation that is in quadrant IV is –60°.

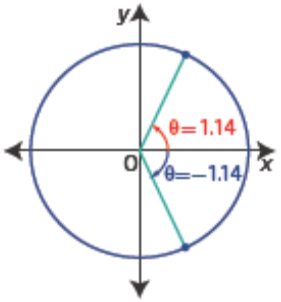


d) If $\cos^2\theta = 1$, then $\cos\theta = \pm 1$.
In the domain $-360° \le \theta < 360°$,
$\theta = 0°, 180°, -180°, -360°$.

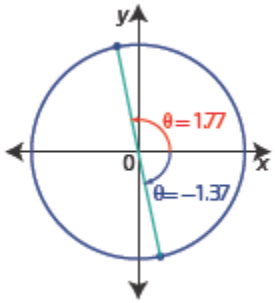


Section 4.3 Page 202 Question 11

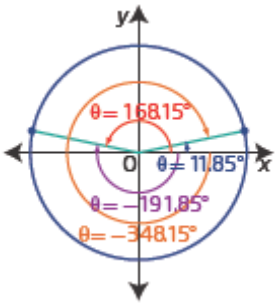
a) $\cos\theta = 0.42$, gives a reference angle of $\theta \approx 1.14$.
Cosine is positive in quadrants I and IV.
So, in the domain $-\pi \le \theta \le \pi$,
$\theta \approx 1.14$ in quadrant I and
$\theta \approx -1.14$ in quadrant IV.



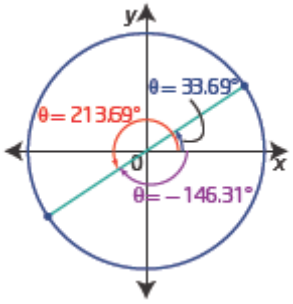
b) $\tan\theta = -4.87$, gives a value of $\theta \approx -1.37$.
Tangent is negative in quadrants II and IV. So, in the domain $-\frac{\pi}{2} \le \theta \le \pi$,
In quadrant II, $\theta \approx \pi - 1.37$, so $\theta \approx 1.77$.
In quadrant IV, $\theta \approx -1.37$.



c) $\csc\theta = 4.87$, means $\sin\theta = \frac{1}{4.87}$, or $\sin\theta \approx 0.2053$.
This gives a value of $\theta \approx 11.85°$.
Sine is positive in quadrants I and II. Consider the domain $-360° \le \theta < 180°$.
For positive rotations: in quadrant I, $\theta \approx 11.85°$ and in quadrant II, $\theta \approx 180° - 11.85°$, or 168.15°.
For negative rotations:
in quadrant I, $\theta \approx 11.85° - 360°$, or −348.15°, and
in quadrant II, $-180° - 11.85°$, or −191.85°.

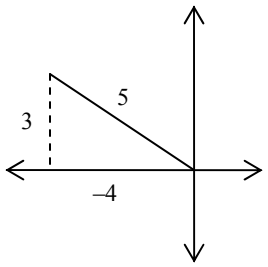


d) $\cot\theta = 1.5$, means $\tan\theta = \frac{1}{1.5}$, or $\tan\theta \approx 0.6666$.
This gives a value of $\theta \approx 33.69°$.
Tangent, and cotangent, is positive in quadrants I and III. Consider the domain $-180° \le \theta < 360°$.
For positive rotations: in quadrant I, $\theta \approx 33.69°$ and in quadrant III, $\theta \approx 33.69° + 180°$ or 213.69°.
For negative rotations:
in quadrant III, $\theta \approx -180° + 33.69°$ or −146.31°.



Section 4.3 Page 202 Question 12

a) Given $\sin\theta = \frac{3}{5}$, and $\frac{\pi}{2} < \theta < \pi$, then θ must be in quadrant II, as shown. The x-coordinate must be 3, since this is a 3-4-5 right triangle.



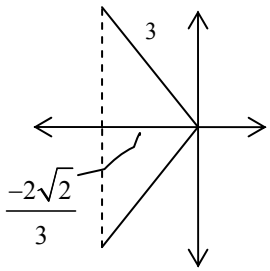
The other five trigonometric ratios are:

$$\cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$
$$= -\frac{4}{5} \qquad = -\frac{3}{4}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$
$$= \frac{5}{3} \qquad = -\frac{5}{4} \qquad = -\frac{4}{3}$$

b) Given $\cos\theta = \frac{-2\sqrt{2}}{3}$, and $-\pi \le \theta \le \frac{3\pi}{2}$, then θ must be in quadrant II or III, with $x = -2\sqrt{2}$ and $r = 3$, as shown.

Determine the y-coordinate.



$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = r^2$$
$$\left(-2\sqrt{2}\right)^2 + y^2 = 3^2$$
$$8 + y^2 = 9$$
$$y = \pm 1$$

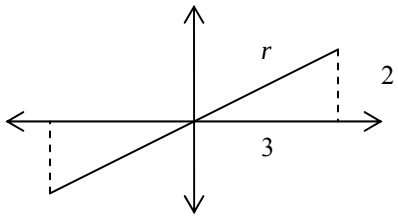
The other five trigonometric ratios are:

$$\sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x}$$
$$= \pm\frac{1}{3} \qquad = \frac{\pm 1}{-2\sqrt{2}}$$
$$= \pm\frac{\sqrt{2}}{4}$$

$$\csc\theta = \pm 3 \qquad \sec\theta = \frac{3}{-2\sqrt{2}} \qquad \cot\theta = \pm 2\sqrt{2}$$
$$= -\frac{3\sqrt{2}}{4}$$

c) Given $\tan\theta = \frac{2}{3}$ and $-360° < \theta < 180°$, θ is in quadrant I or III, as shown.

Determine the measure of r.



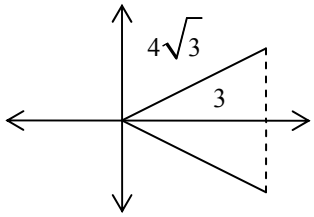
$$x^2 + y^2 = r^2$$
$$3^2 + 2^2 = r^2$$
$$13 = r^2$$
$$r = \sqrt{13}$$

$$\sin\theta = \frac{y}{r} = \pm\frac{2}{\sqrt{13}} \qquad \cos\theta = \frac{x}{r} = \pm\frac{3}{\sqrt{13}}$$

$$\csc\theta = \pm\frac{\sqrt{13}}{2} \qquad \sec\theta = \pm\frac{\sqrt{13}}{3} \qquad \cot\theta = \frac{3}{2}$$

d) Given $\sec\theta = \frac{4\sqrt{3}}{3}$, $\cos\theta = \frac{3}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$, and $-180° \le \theta \le 180°$, then θ is in quadrant I or IV, with $x = 3$ and $r = 4\sqrt{3}$.

Determine the y-coordinate.



$$x^2 + y^2 = r^2$$
$$3^2 + y^2 = \left(4\sqrt{3}\right)^2$$
$$9 + y^2 = 48$$
$$y = \pm\sqrt{39}$$

$$\sin\theta = \frac{y}{r} = \pm\frac{\sqrt{39}}{4\sqrt{3}} \text{ or } \pm\frac{\sqrt{13}}{4} \qquad \tan\theta = \frac{y}{x} = \pm\frac{\sqrt{39}}{3}$$

$$\csc\theta = \pm\frac{4\sqrt{3}}{\sqrt{39}} \text{ or } \pm\frac{4\sqrt{13}}{13} \qquad \cot\theta = \pm\frac{3}{\sqrt{39}} \text{ or } \pm\frac{\sqrt{39}}{13}$$

Section 4.3 Page 203 Question 13

Sketch the angle with point B(–2, –3) on its terminal arm. It is in quadrant III.

Use the Pythagorean theorem to calculate the measure of r in the right triangle with $x = -2$ and $y = -3$. The measure of r is $\sqrt{13}$. The exact value of cos θ can be determined as

$$\cos\theta = \frac{x}{r}$$
$$= \frac{-2}{\sqrt{13}} \text{ or } -\frac{2\sqrt{13}}{13}$$

Section 4.3 Page 203 Question 14

a) $\frac{4900}{360} = 13\frac{220}{360}$, so the angle of 4900° is 13 complete revolutions plus 220°.

b) 220° puts the terminal arm in quadrant III.

c) Since 220° is 40° past 180°, the reference angle is 40°.

d) sin 4900° ≈ –0.643, cos 4900° ≈ –0.766, tan 4900° ≈ 0.839,
csc 4900° ≈ –1.556, sec 4900° ≈ –1.305, cot 4900° ≈ 1.192

Section 4.3 Page 203 Question 15

a) sin ($\cos^{-1}$ 0.6) = $\frac{4}{5}$ or 0.8.

To evaluate $\cos^{-1}$ 0.6 find an angle whose cosine is 0.6. In other words, cos θ = $\frac{x}{r} = \frac{6}{10}$ or $\frac{3}{5}$. This means θ can be determined using a 3-4-5 right triangle. Then, the sine of this angle is sin θ = $\frac{y}{r} = \frac{4}{5}$.

b) cos ($\sin^{-1}$ 0.6) is very similar to the result in part a), the only difference being the orientation of the 3-4-5 right triangle. So, the positive value of cos ($\sin^{-1}$ 0.6) is 0.8.

Section 4.3 Page 203 Question 16

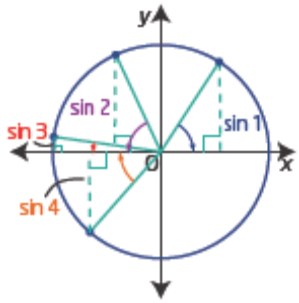
a) Jason is not correct. He used degree mode, when he should have chosen radian mode.

b) First choose radian mode. Determine cos $\left(\frac{40\pi}{7}\right)$ and then use the reciprocal key to determine sec $\left(\frac{40\pi}{7}\right)$. The correct answer is approximately 1.603 875 472.

Section 4.3 Page 203 Question 17

a) sin 1 ≈ 0.841, sin 2 ≈ 0.909, sin 3 ≈ 0.141, sin 4 ≈ –0.757
So the values in increasing order are sin 4, sin 3, sin 1, sin 2.

b) 4 radians is in quadrant III, so sin 4 is negative. 3 radians is very close to π, so close to 0, but positive. The value of π/2 is approximately 1.57, so sin 2 is closer to the *y*-axis, where sine has value 1, than is sin 1.



c) Cosine uses the *x*-coordinates which increase from left to right on the diagram. So the order is cos 3, cos 4, cos 2, cos 1.
Check: cos 3 ≈ –0.999, cos 4 ≈ –0.654, cos 2 ≈ –0.416, cos 1 ≈ 0.540

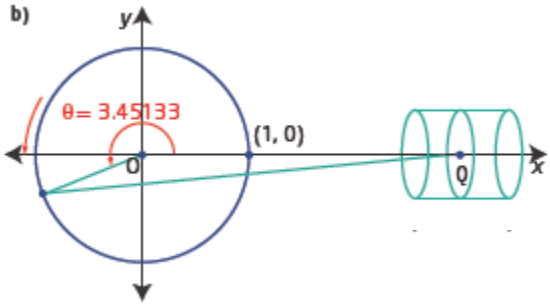
Section 4.3 Page 203 Question 18

a) As P moves around the wheel it can move from closest to the piston at (1, 0) to furthest away at (–1, 0). So the maximum distance that Q can move is 2 units.

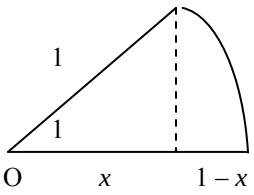
b) At a speed of rotation is 1 radian/s, after 1 min the wheel will have travelled through 60 radians.

$$\frac{60}{2\pi} \approx 9.55$$

The wheel will have made 9 complete turns plus 0.55 of the next turn. This will put P in quadrant III at an angle of rotation of (0.55)2π or approximately 3.451 33 radians.



c) After 1 s, P will have moved 1 radian and pulled Q to the left a distance of 1 – cos 1, or 0.46 units, to the nearest hundredth.



Section 4.3 Page 203 Question 19

a) A(–3, 4), domain $0 < \theta \leq 4\pi$

Using the coordinates of A, $\tan\theta = \frac{4}{-3}$.

The reference angle is $\theta \approx 0.93$.
A(–3, 4) is in quadrant II, so in the domain $0 < \theta \leq 4\pi$,
$\theta \approx \pi - 0.93$, or 2.21, and $\theta \approx 2\pi + 2.21$, or 8.50.

b) B(5, –1), $-360° \leq \theta < 360°$

Using the coordinates of B, $\tan\theta = \frac{-1}{5} = -0.2$.

The reference angle is $\theta \approx 11.31°$.
B(5, –1) is in quadrant IV, so in the domain $-360° \leq \theta < 360°$:
for negative rotations: $\theta \approx -11.31°$
for positive rotations: $\theta \approx 360° - 11.31°$, or 348.69°.

c) C(–2, –3), domain $-\frac{3\pi}{2} < \theta < \frac{7\pi}{2}$

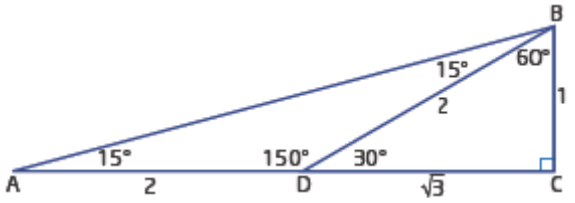
Using the coordinates of C, $\tan\theta = \frac{-3}{-2} = 1.5$.

The reference angle is $\theta \approx 0.98$.

C(–2, –3) is in quadrant III, so in the domain $-\frac{3\pi}{2} < \theta < \frac{7\pi}{2}$:

for negative rotations: $\theta \approx -\pi + 0.98$, or –2.16
for positive rotations: $\theta \approx \pi + 0.98$, or 4.12 and $3\pi + 0.98$, or 10.41

Section 4.3 Page 203 Question 20

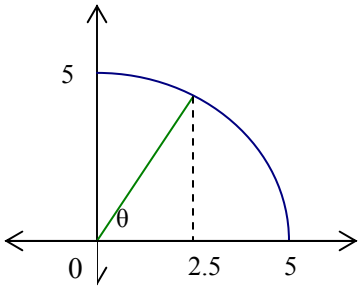


ΔBCD is a 30°-60°-90° triangle, so it sides have the proportions shown.
ΔABD is isosceles, so AD = BD = 2 units.
Then in ΔABC,

$$\tan 15° = \frac{BC}{AC}$$
$$= \frac{1}{\sqrt{3}+2}$$

Section 4.3 Page 204 Question 21

Note: The text answers show a rationale using degrees. This solution provides an alternate, using radians.



The distance between (0, 5) and (5, 0) is $\frac{\pi}{2}$.

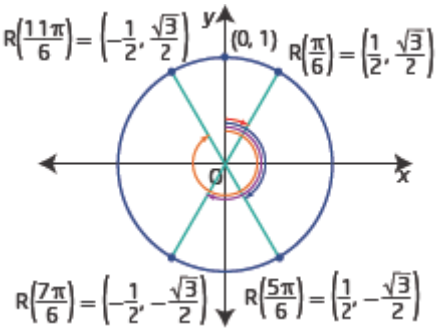
For the angle of rotation, θ, with $x = 2.5$,

$\cos\theta = \frac{2.5}{5} = \frac{1}{2}$. Therefore, $\theta = \frac{\pi}{3}$.

So, moving from (0, 5) to the point on the curve where $x = 2.5$, the angle of rotation is $-\frac{\pi}{6}$. This angle has an arc length of $\frac{\pi}{6}$ which is one-third of $\frac{\pi}{2}$.

Section 4.3 Page 204 Question 22

a)



b) The new angle of rotation, $R\left(\frac{\pi}{6}\right)$, has the same terminal arm as the angle in standard position $P\left(\frac{\pi}{3}\right)$. So, $R\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The new angle of rotation, $R\left(\frac{5\pi}{6}\right)$, has the same terminal arm as the angle in standard position $P\left(\frac{5\pi}{3}\right)$. So, $R\left(\frac{5\pi}{6}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

c) The angles in the new system are related to angles in standard position by determining which quadrant the terminal arm is in and then determining the positive rotation to reach that terminal arm.

$$R\left(\frac{\pi}{6}\right)=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right),\ R\left(\frac{5\pi}{6}\right)=\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right),\ R\left(\frac{7\pi}{6}\right)=\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right),\ R\left(\frac{11\pi}{6}\right)=\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$$

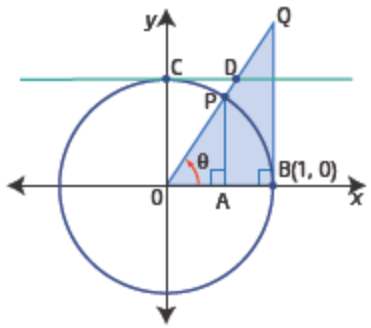
d) Bearings are measured clockwise from 0°. The new system is the same as bearings, except that bearings are measured in degrees.

Section 4.3 Page 204 Question 23

a) In ΔOBQ, $\cos\theta = \frac{OB}{OQ} = \frac{1}{OQ}$.

Therefore, $\sec\theta = \frac{1}{\cos\theta} = OQ$.

b)



In ΔOCD, $\angle ODC = \theta$ (alternate angles).

Then, $\sin\theta = \frac{OC}{OD} = \frac{1}{OD}$.

So, $\csc\theta = \frac{1}{\sin\theta} = OD$.

Similarly, $\cot\theta = = CD$.

Section 4.3 Page 204 Question C1

a) Paula is correct, sine ratios are increasing in quadrant I.

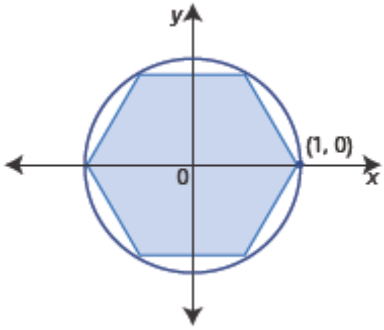
Examples: $\sin 0 = 0$, $\sin\frac{\pi}{6} = 0.5$, $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx 0.707$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \approx 0.866$, $\sin\frac{\pi}{2} = 1$.

b) In quadrant II, sine is decreasing. In the unit circle, sine is given by the *y*-coordinate. As the end of the terminal arm moves past the *y*-axis, the *y*-coordinate decreases, from 1 to 0.

c) The sine ratio increases in quadrant IV. The y-coordinate has a minimum value of -1 at $\frac{3\pi}{2}$ and then its value increases to 0 at 0.

Section 4.3 Page 204 Question C2

In this regular hexagon, the diagonals will intersect at the origin and each of the six angles at the center will be 60°.

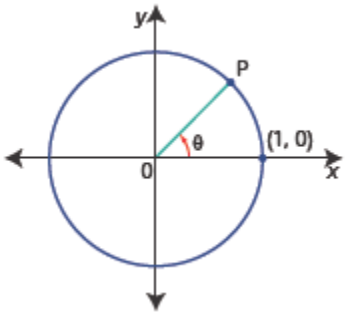


The vertex in quadrant I is at 60° and has coordinates $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The vertex in quadrant II is at 120° and has coordinates $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The next vertex is at 180° and has coordinates (–1, 0). The vertex in quadrant III is at 240° and has coordinates $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. The vertex in quadrant IV is at 300° and has coordinates $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

Section 4.3 Page 205 Question C3

a) If the coordinates of P are (x, y) then

$$\text{slope of OP} = \frac{y}{x}$$
$$= \tan\theta$$



b) Yes, the formula is true in all four quadrants. In quadrant II and IV the slope will be negative, as expected.

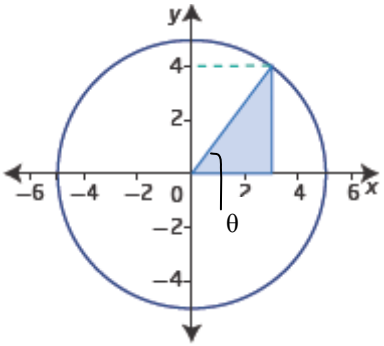
c) An equation for the line OP, where O is the origin, is $y = (\tan\theta)x$.

d) For any line, the equation is $y = mx + b$, where m is the slope and b is the y-intercept. The slope can be defined in terms of a unit circle as $\tan\theta$, if the circle is not centred at the origin then a vertical translation of b units is needed. The equation is $y = (\tan\theta)x + b$.

Section 4.3 Page 205 Question C4

a) $\sin\left(\sin^{-1}\left(\frac{4}{5}\right)\right) = \sin\theta$

$= \frac{4}{5}$



b) $\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) = \cos\theta$

$= \frac{3}{5}$

c) $\csc\left(\cos^{-1}\left(-\frac{3}{5}\right)\right) = \csc\theta$ Sine and cosecant are positive in quadrant II.

$= \frac{1}{\sin\theta}$

$= \frac{5}{4}$

d) $\sin\left(\tan^{-1}\left(-\frac{4}{3}\right)\right) = \sin(360° - \theta)$ Sine is negative in quadrant IV.

$= -\frac{4}{5}$

4.4 Introduction to Trigonometric Equations

Section 4.4 Page 211 Question 1

a) The given sine ratio is positive, so in the domain $0 \le \theta < 2\pi$, there will be two solutions, one is quadrant I and one is quadrant II.

b) The given cosine ratio is positive. The domain $-2\pi \le \theta < 2\pi$ is two complete rotations. There will be four solutions, two in quadrant I and two in quadrant IV.

c) The given tangent ratio is negative. This is true in quadrants II and IV. So, in the domain $-360° \le \theta \le 180°$ there will be three solutions, two in quadrant II and one is quadrant IV.

d) The given secant ratio, and thus the cosine ratio, is positive. This is true in quadrants I and IV. In the domain $-180° \le \theta < 180°$ there will be two solutions, one in quadrant I and one in quadrant IV.

Section 4.4 Page 211 Question 2

a) For $\theta = \frac{\pi}{3}$, the general solution is $\theta = \frac{\pi}{3} + 2\pi n,$ where $n \in \mathrm{I}$.

b) For $\theta = \frac{5\pi}{3}$, the general solution is $\theta = \frac{5\pi}{3} + 2\pi n,$ where $n \in \mathrm{I}$.

Section 4.4 Page 211 Question 3

a) $2\cos\theta - \sqrt{3} = 0$

$$2\cos\theta = \sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}$$

Cosine is positive in quadrants I and IV. So, in the interval $0 \le \theta < 2\pi,$ $\theta = \frac{\pi}{6}$ and $\theta = \frac{11\pi}{6}$.

b) csc θ is undefined when sin θ = 0. So, in the domain $0° \le \theta < 360°$, $\theta = 0°$ and $\theta = 180°$.

c) $5 - \tan^2\theta = 4$

$$1 = \tan^2\theta$$

$$\tan\theta = \pm 1$$

So, the reference angle for θ is 45°.
In the domain $-180° \le \theta \le 360°$, $\theta = -135°, -45°, 45°, 135°, 225°,$ and 315°.

d) $\sec\theta + \sqrt{2} = 0$

$$\sec\theta = -\sqrt{2}$$

$$\cos\theta = -\frac{1}{\sqrt{2}}$$

The reference angle is $\frac{\pi}{4}$.

Cosine is negative in quadrants II and III.

So, in the domain $-\pi \le \theta \le \frac{3\pi}{2}$, $\theta = -\frac{3\pi}{4}$, $\frac{3\pi}{4}$, and $\frac{5\pi}{4}$.

Section 4.4 Page 211 Question 4

a) $\tan \theta = 4.36$

$\theta = \tan^{-1} 4.36$

$\theta \approx 1.35$

Tangent is positive in quadrants I and III.

So, in the domain $0 \le \theta < 2\pi$, $\theta \approx 1.35$ and $\theta \approx \pi + 1.33$, or 4.49.

b) $\cos \theta = -0.19$

$\theta = \cos^{-1} (-0.19)$

$\theta \approx 1.76$

Cosine is negative in quadrants II and III.

So, in the domain $0 \le \theta < 2\pi$, $\theta \approx 1.76$ and $\theta \approx 2\pi - 1.76$, or 4.52.

c) $\sin \theta = 0.91$

$\theta = \sin^{-1} 0.91$

$\theta \approx 1.14$

Sine is positive in quadrants I and II.

So, in the domain $0 \le \theta < 2\pi$, $\theta \approx 1.14$ and $\theta \approx \pi - 1.14$, or 2.00

d) $\cot \theta = 12.3$

$\theta = \tan^{-1}\left(\frac{1}{12.3}\right)$

$\theta \approx 0.08$

Cotangent and tangent are positive in quadrants I and III.

So, in the domain $0 \le \theta < 2\pi$, $\theta \approx 0.08$ and $\theta \approx \pi + 0.08$, or 3.22.

e) $\sec \theta = 2.77$

$\theta = \cos^{-1}\left(\frac{1}{2.77}\right)$

$\theta \approx 1.20$

Secant and cosine are positive in quadrants I and IV.

So, in the domain $0 \le \theta < 2\pi$, $\theta \approx 1.20$ and $\theta \approx 2\pi - 1.20$, or 5.08.

f) $\csc \theta = -1.57$

$$\theta = \sin^{-1}\left(-\frac{1}{1.57}\right)$$

$$\theta \approx -0.69$$

Cosecant and sine are negative in quadrants III and VI.
So, in the domain $0 \le \theta < 2\pi$, $\theta \approx \pi + 0.69$ or 3.83 and $\theta \approx 2\pi - 0.69$ or 5.59

Section 4.4 Page 211 Question 5

a) $3 \cos \theta - 1 = 4 \cos \theta$

$$-1 = \cos \theta$$

In the domain $0 \le \theta < 2\pi$, $\theta = \pi$.

b) $\sqrt{3} \tan \theta + 1 = 0$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

Tangent is negative in quadrants II and IV.
In the domain $-\pi \le \theta \le 2\pi$, $\theta = -\frac{\pi}{6}$, $\theta = \frac{5\pi}{6}$, and $\theta = \frac{11\pi}{6}$.

c) $\sqrt{2} \sin x - 1 = 0$

$$\sin x = \frac{1}{\sqrt{2}}$$

Sine is positive in quadrants I and II.
In the domain $-360° < \theta \le 360°$, $\theta = -315°$, $-225°$, $45°$, and $135°$.

d) $3 \sin x - 5 = 5 \sin x - 4$

$$-1 = 2 \sin x$$

$$\sin x = -\frac{1}{2}$$

Sine is negative in quadrants III and IV.
In the domain $-360° \le x < 180°$, $x = -150°$ and $-30°$.

e) $3 \cot x + 1 = 2 + 4 \cot x$

$$-1 = \cot x$$

$$\tan x = -1$$

Tangent is negative in quadrants II and IV.
In the domain $-180° < x < 360°$, $x = -45°$, $135°$, and $315°$.

f) $\sqrt{3}\sec\theta + 2 = 0$

$$\sec\theta = -\frac{2}{\sqrt{3}}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

Cosine is negative in quadrants II and III.

In the domain $-\pi \le \theta \le 3\pi$, $\theta = -\frac{5\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{17\pi}{6}$.

Section 4.4 Page 212 Question 6

	Domain	Interval
a)	$-2\pi \le \theta \le 2\pi$	$\theta \in [-2\pi, 2\pi]$
b)	$-\frac{\pi}{3} \le \theta \le \frac{7\pi}{3}$	$\theta \in \left[-\frac{\pi}{3}, \frac{7\pi}{3}\right]$
c)	$0° \le \theta \le 270°$	$\theta \in [0°, 270°]$
d)	$0 \le \theta < \pi$	$\theta \in [0, \pi)$
e)	$0° < \theta < 450°$	$\theta \in (0°, 450°)$
f)	$-2\pi < \theta \le 4\pi$	$\theta \in (-2\pi, 4\pi]$

Section 4.4 Page 212 Question 7

a) $2\cos^2\theta - 3\cos\theta + 1 = 0$

$(2\cos\theta - 1)(\cos\theta - 1) = 0$

$2\cos\theta - 1 = 0$ or $\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = 1$

In the domain $0 \le \theta < 2\pi$,

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ or $\theta = 0$

b) $\tan^2\theta - \tan\theta - 2 = 0$

$(\tan\theta - 2)(\tan\theta + 1) = 0$

$\tan\theta - 2 = 0$ or $\tan\theta + 1 = 0$

$\tan\theta = 2$ or $\tan\theta = -1$

In the domain $0° \le \theta < 360°$,

$\theta \approx 63.435°, 243.435°$ or $\theta = 135°, 315°$

c) $\sin^2 \theta - \sin \theta = 0$

$\sin \theta (\sin \theta - 1) = 0$

$\sin \theta = 0$ or $\sin \theta - 1 = 0$

$\sin \theta = 1$

In the domain $\theta \in [0, 2\pi)$,

$\theta = 0, \pi$ or $\theta = \frac{\pi}{2}$

d) $\sec^2 \theta - 2 \sec \theta - 3 = 0$

$(\sec \theta - 3)(\sec \theta + 1) = 0$

$\sec \theta - 3 = 0$ or $\sec \theta + 1 = 0$

$\sec \theta = 3$ or $\sec \theta = -1$

In the domain $\theta \in [-180°, 180°)$,

$\theta \approx -70.529°, 70.529°$ or $\theta = -180°$

Section 4.4 Page 212 Question 8

Check for $\theta = 180°$:

Left Side	Right Side
$= 5 \cos^2 \theta$	$= -4 \cos \theta$
$= 5 \cos^2 180°$	$= -4 \cos 180°$
$= 5(-1)^2$	$= -4(-1)$
$= 5$	$= 4$

Left Side $\neq$ Right Side

So, $\theta = 180°$ is not a solution.

Check for $\theta = 270°$:

Left Side	Right Side
$= 5 \cos^2 \theta$	$= -4 \cos \theta$
$= 5 \cos^2 270°$	$= -4 \cos 270°$
$= 5(0)^2$	$= -4(0)$
$= 0$	$= 0$

Left Side = Right Side

So, $\theta = 270°$ is a solution.

Section 4.4 Page 212 Question 9

a) In step 1, they should not divide both sides by $\sin \theta$ because this may eliminate one possible solution and if $\sin \theta = 0$ this division is not permissible.

b) $2 \sin^2 \theta = \sin \theta$

$2 \sin^2 \theta - \sin \theta = 0$

$$\sin\theta(2\sin\theta - 1) = 0$$
$$\sin\theta = 0 \text{ or } 2\sin\theta - 1 = 0$$
$$\sin\theta = \frac{1}{2}$$

In the domain $0 < \theta \leq \pi$,

$$\theta = \pi \text{ or } \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Section 4.4 Page 212 Question 10

$\sin\theta = 0$ when $\theta = 0$, $\theta = \pi$, or $\theta = 2\pi$. However the interval $(\pi, 2\pi)$ means $\pi < \theta < 2\pi$. There are no values of θ for which $\sin\theta = 0$ in this interval.

Section 4.4 Page 212 Question 11

The equation $\sin\theta = 2$ has no solution, because $\sin\theta$ has a maximum value of 1. This is true for all values of θ, so the interval is irrelevant.

Section 4.4 Page 212 Question 12

The trigonometric equation $\cos\theta = \frac{1}{2}$ does have an infinite number of solutions. Cosine is positive in quadrants I and IV, so in one positive rotation $\theta = 60°$ and $\theta = 300°$. However, all coterminal angles have the same value. In general, $\theta = 60° + 360°n$ or $\theta = 300° + 360°n$, where $n \in \text{I}$.

Section 4.4 Page 212 Question 13

a) Check by substituting $\theta = \pi$ into the original equation.

Left Side $= 3\sin^2\theta - 2\sin\theta$
$= 3\sin^2\pi - 2\sin\pi$
$= 3(0)^2 - 2(0)$
$= 0$
$=$ Right Side

The solution $\theta = \pi$ is correct.

b) $3\sin^2\theta - 2\sin\theta = 0$
$\sin\theta(3\sin\theta - 2) = 0$
$\sin\theta = 0 \text{ or } 3\sin\theta - 2 = 0$

$$\sin\theta = \frac{2}{3}$$

In the interval $\theta \in [0, \pi]$,
$\theta = 0, \pi$ or $\theta \approx 0.7297, 2.4119$

Section 4.4 Page 212 Question 14

Use $n_1 \sin\theta_1 = n_2 \sin\theta_2$
Solve for θ_2 when $\theta_1 = 35°$, $n_1 = 1.000\ 29$, and $n_2 = 1.33$.

$$1.000\ 29 \sin 35° = 1.33 \sin\theta_2$$

$$\frac{1.000\ 29 \sin 35°}{1.33} = \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{1.000\ 29 \sin 35°}{1.33}\right)$$

$$\theta_2 \approx 25.56°$$

Section 4.4 Page 213 Question 15

a) For sales of 8300, substitute $y = 8.3$ into $y = 5.9 + 2.4\sin\left(\frac{\pi}{6}(t-3)\right)$.

$$8.3 = 5.9 + 2.4\sin\left(\frac{\pi}{6}(t-3)\right)$$

$$\frac{8.3 - 5.9}{2.4} = \sin\left(\frac{\pi}{6}(t-3)\right)$$

$$\sin^{-1}\left(\frac{2.4}{2.4}\right) = \frac{\pi}{6}(t-3)$$

$$\frac{\pi}{2} = \frac{\pi}{6}(t-3)$$

$$3 = t - 3$$

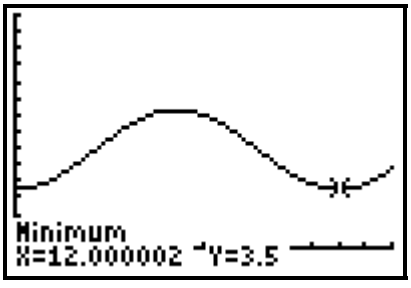
$$t = 6$$

Sales of 8300 air conditioners are expected in the sixth month, June.

b) Graph the function

$y = 5.9 + 2.4\sin\left(\frac{\pi}{6}(t-3)\right)$.

Minimum sales occur in the 12th month, December.



c) The formula seems reasonable. In western Canada you would expect sales of air conditioners to peak in summer and be least in winter.

Section 4.4 Page 213 Question 16

$9 \sin^2 \theta + 12 \sin \theta + 4 = 0$

$(3 \sin \theta + 2)(3 \sin \theta + 2) = 0$

$3 \sin \theta + 2 = 0$

$$\sin \theta = -\frac{2}{3}$$

$$\theta = \sin^{-1}\left(-\frac{2}{3}\right)$$

$$\approx -41.810\,314\,9$$

The reference angle is 41.8°, to the nearest tenth of a degree.
Sine is negative in quadrants III and IV. <This line was Nora's error>
The solution in quadrant III is 180° + 41.8° = 221.8°.
The solution in quadrant IV is 360° – 41.8° = 318.2°.

Section 4.4 Page 213 Question 17

Examples:
An equation such as $\cos x = 3$ has no solution because the maximum value of cosine is 1.
An equation such as $\sin x = -1$, $0 < x < \pi$, has no solution in the required interval, because $\sin^{-1}(-1) = \frac{3\pi}{2}$.

Section 4.4 Page 213 Question 18

Given $\cot \theta = \frac{3}{4}$, and θ is in quadrant III so $x = -3$ and $y = -4$. Using the Pythagorean theorem, $r = 5$.

Then, $\sec \theta = \frac{r}{x} = \frac{5}{-3}$.

Section 4.4 Page 213 Question 19

a) For the ball at sea level, substitute $h = 0$.

$$h = 1.4\sin\left(\frac{\pi t}{3}\right)$$

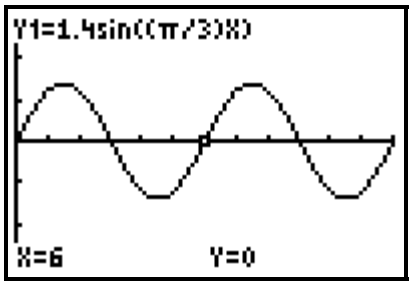
$$0 = 1.4\sin\left(\frac{\pi t}{3}\right)$$

$$0 = \sin\left(\frac{\pi t}{3}\right)$$

Therefore, $\frac{\pi t}{3} = 0, \frac{\pi t}{3} = \pi, \frac{\pi t}{3} = 2\pi, \ldots$

So, in the first 10 s, the beach ball is at sea level at 0 s, 3 s, 6 s, and 9 s.

b) The ball will reach its maximum height, for the first time, half way between 0s and 3s which is at 1.5 s. This will repeat every 6 s, so an expression for the time that the maximum occurs is $1.5 + 6n$, $n \in W$.



A graph of the function $y = 1.4\sin\left(\frac{\pi t}{3}\right)$ confirms this reasoning.

c) Since sine has minimum value –1, the minimum value of $y = 1.4\sin\left(\frac{\pi t}{3}\right)$ is –1.4. So the most the ball goes below sea level is 1.4 m.

Section 4.4 Page 213 Question 20

a) $I = 4.3 \sin 120\pi t$

Substitute $I = 0$, then $0 = 4.3 \sin 120\pi t$

$$0 = \sin 120\pi t$$

$$\sin\theta = 0 \text{ at } \theta = 0, \pi, 2\pi, \ldots$$

$$0 = 120\pi t \rightarrow t = 0$$

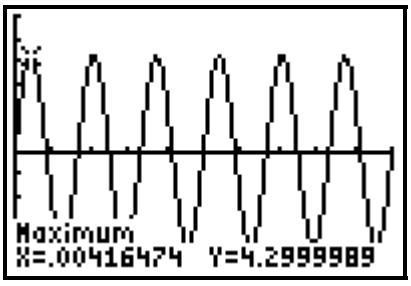
$$\pi = 120\pi t \rightarrow t = \frac{1}{120}$$

$$2\pi = 120\pi t \rightarrow t = \frac{1}{60}$$

Since the current must alternate from 0 to positive back to 0 and then negative back to 0, it will take $\frac{1}{60}$ s for one complete cycle or 60 cycles in one second.

b) Each complete cycle takes $\frac{1}{60}$ s, so to reach the first maximum will take one-quarter of that time period or $\frac{1}{240}$ s. As a decimal this is approximately 0.004 167 s.

The cycle repeats every $\frac{1}{60}$ s, so in general the current reaches its maximum value at $\left(\frac{1}{240}+\frac{1}{60}n\right)$ seconds, where $n \in$ W. A graph of $y = 4.3 \sin 120\pi t$ confirms these results.



c) The current is at its first minimum value at $\left(\frac{3}{4}\right)\left(\frac{1}{60}\right)$ or $\frac{1}{80}$ s. As a decimal, this is 0.0125. So in general, the current reaches it minimum value at $(0.0125 + \frac{1}{60}n)$ seconds, where $n \in$ W.

d) The maximum value of sine is 1, so the maximum value of this function is 4.3. The maximum current is 4.3 amps.

Section 4.4 Page 214 Question 21

$$\cos\left(x-\frac{\pi}{2}\right)=\frac{\sqrt{3}}{2}$$

The reference angle for $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$. Cosine is positive in quadrants I and IV.

So, in the domain $-\pi < x < \pi$,

$$x-\frac{\pi}{2}=\frac{\pi}{6} \text{ or } x-\frac{\pi}{2}=-\frac{\pi}{6}$$

$$x=\frac{2\pi}{3} \text{ or } x=\frac{\pi}{3}$$

Section 4.4 Page 214 Question 22

a) The left side of the equation, $\sin^2 \theta + \sin \theta - 1$, has the form $x^2 + x - 1$ and cannot be factored.

b) In the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, substitute $a = 1$, $b = 1$, $c = -1$.

$$\sin \theta = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

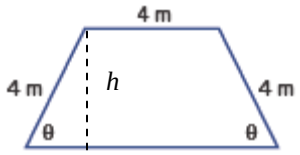
The only solution that makes sense is $\dfrac{-1+\sqrt{5}}{2}$; the other solution is less than -1 which is not a possible value for sine.

c) From b), $\theta = \sin^{-1}\left(\dfrac{-1+\sqrt{5}}{2}\right)$ or $\theta \approx 0.67$.

This is the solution in quadrant I. In the domain $0 < \theta \leq 2\pi$, there is another solution, in quadrant II. $\theta \approx \pi - 0.668 = 2.48$.

Section 4.4 Page 214 Question 23

a) The height of the trapezoid is $4 \sin \theta$ and the base of the trapezoid is $4 + 2 \cos \theta$.
Then, use the formula for the area of a trapezoid:



$$A = \left(\frac{\text{sum of parallel sides}}{2}\right)h$$

$$= \left(\frac{4+4+2(4\cos\theta)}{2}\right)4\sin\theta$$

$$= (8+8\cos\theta)2\sin\theta$$

$$= 16\sin\theta(1+\cos\theta)$$

b) Substitute $A = 12\sqrt{3}$ in the formula from part a).

$$12\sqrt{3} = 16\sin\theta(1+\cos\theta)$$

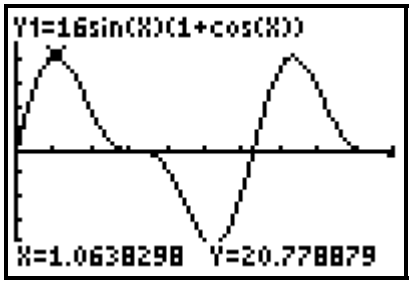
$$\frac{12\sqrt{3}}{16} = \sin\theta(1+\cos\theta)$$

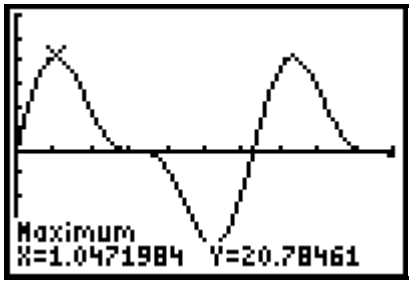
$$\frac{3\sqrt{3}}{4} = \sin\theta(1+\cos\theta)$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{2}\right) = \sin\theta(1+\cos\theta)$$

Then, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$ which is true when $\theta = \frac{\pi}{3}$.

c) Example: Graph the function $y = 16 \sin x(1 + \cos x)$ and identify the first maximum. The graph shown here is in radian mode.





Section 4.4 Page 214 Question C1

The methods and steps used to simplify linear and quadratic trigonometric equations are the same as those used for linear and quadratic equations. The major difference is the last steps when the inverse of trigonometric ratios have to be used and consideration of signs and domain is needed.

Section 4.4 Page 214 Question C2

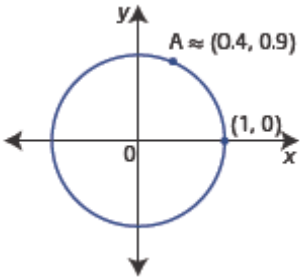
a) The point is on the unit circle if $x^2 + y^2 = 1$.
For A:
$0.384\,615\,384\,6^2 + 0.923\,076\,923\,1^2 = 1$
Therefore A is on the unit circle.

b) For a point on the unit circle,
$\cos\theta = x$
$= 0.384\,615\,384\,6$
≈ 0.385

$$\csc\theta = \frac{1}{y}$$
$$= \frac{1}{0.923\,076\,923\,1}$$
$$\approx 1.083$$

$$\tan\theta = \frac{y}{x}$$
$$= \frac{0.923\,076\,923\,1}{0.384\,615\,384\,6}$$
$$\approx 2.400$$

c) $\theta = \tan^{-1}(2.4)$
$\approx 67.4°$
This answer seems reasonable.
The *x*-coordinate is about one-third the *y*-coordinate, so the angle is about two-thirds of the rotation from 0° to 90°.



Section 4.4 Page 214 Question C3

a) Example: A non-permissible value is a value for which an expression is not defined. For a rational expression this is any value that would lead to division by 0. In the rational equation $\frac{5}{x-1} = 9,\ x \neq 1$.

b) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$, so tan θ is not defined for values that make cos θ have value 0.

c) In the interval $0 \leq \theta < 4\pi$, cos θ = 0 when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$. These are the non-permissible values.

d) In general, tan θ is not defined for $\theta = \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

Section 4.4 Page 214 Question C4

a) $2\sin^2\theta = 1 - \sin\theta$
$2\sin^2\theta + \sin\theta - 1 = 0$
$(2\sin\theta - 1)(\sin\theta + 1) = 0$
$2\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$
$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$
In the domain $0° \leq \theta < 360°$, θ = 30°, 150° or θ = 270°.

b) The solutions are exact because both values for sin θ are special values.

c) To check, substitute the solution into both sides of the original equation. Both sides should have the same value.
Example θ = 270°:

Left Side	Right Side
$= 2\sin^2 \theta$	$= 1 - \sin \theta$
$= 2\sin^2 270°$	$= 1 - \sin 270°$
$= 2(-1)^2$	$= 1 - (-1)$
$= 2(1)$	$= 1 + 1$
$= 2$	$= 2$

Left Side = Right Side

Chapter 4 Review

Chapter 4 Review Page 215 Question 1

a) The terminal arm of 100° is in quadrant II.

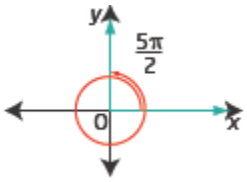
b) 500° – 360° = 140°; so the terminal arm of 500° is in quadrant II.

c) 10 – 2π = 3.71, 3.71 – π = 0.57; so the terminal arm of 10 radians is in quadrant III.

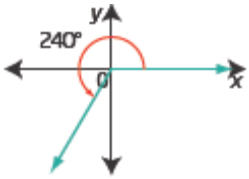
d) $\frac{29\pi}{6} = 4\pi + \frac{5\pi}{6}$; so the terminal arm of the angle is in quadrant II.

Chapter 4 Review Page 215 Question 2

a) $\frac{5\pi}{2} = \frac{5(180°)}{2}$
$= 450°$

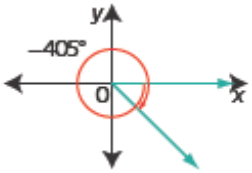


b) $240° = 240\left(\frac{\pi}{180}\right)$
$= \frac{4\pi}{3}$



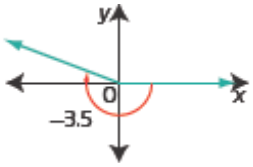
c) $-405^\circ = -405\left(\frac{\pi}{180}\right)$

$= -\frac{9\pi}{4}$



d) $-3.5 = -3.5\left(\frac{180^\circ}{\pi}\right)$

$= -\frac{630^\circ}{\pi}$



Chapter 4 Review Page 215 Question 3

a) $20^\circ = 20\left(\frac{\pi}{180}\right)$

≈ 0.35

b) $-185^\circ = -185\left(\frac{\pi}{180}\right)$

≈ -3.23

c) $-1.75 = -1.75\left(\frac{180^\circ}{\pi}\right)$

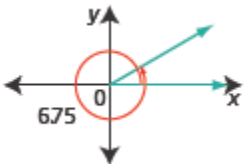
$\approx -100.27^\circ$

d) $\frac{5\pi}{12} = \frac{5(180^\circ)}{12}$

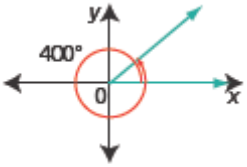
$= 75^\circ$

Chapter 4 Review Page 215 Question 4

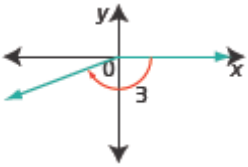
a) $6.75 - 2\pi \approx 0.4668$.
The given angle is coterminal with 0.467 and terminates in quadrant I.



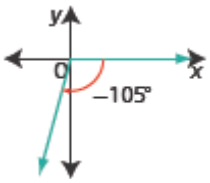
b) $400^\circ - 360^\circ = 40^\circ$.
The given angle is coterminal with 40° and terminates in quadrant I.



c) –3 is almost $-\pi$.
$2\pi - 3 \approx 3.28$
The given angle is coterminal with 3.28 and terminates in quadrant III.



d) –105° is a negative rotation, past –90°.
360° – 105° = 255°
The given angle is coterminal with 255°
and terminates in quadrant III.



Chapter 4 Review Page 215 Question 5

a) All angles coterminal with 250° are given by the expression 250° ± (360°)*n*, where *n* is any natural number.

b) All angles coterminal with $\frac{5\pi}{2}$ are given by the expression $\frac{5\pi}{2} \pm 2\pi n$, where *n* is any natural number.

c) All angles coterminal with –300° are given by the expression –300° ± (360°)*n*, where *n* is any natural number.

d) All angles coterminal with 6 radians are given by the expression $6 \pm 2\pi n$, where *n* is any natural number.

Chapter 4 Review Page 215 Question 6

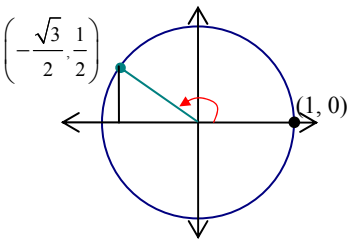
a) 80 000 rpm = 80 000(2π), or 160 000π radians per minute.

b) 80 000 rpm = 80 000(360) degrees per minute

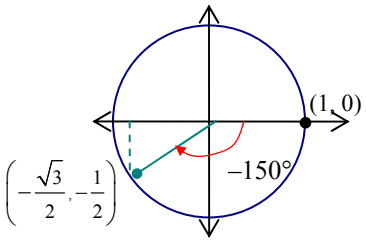
$$= \frac{80\ 000(360°)}{60}$$
$$= 480\ 000°/\text{s}$$

Chapter 4 Review Page 215 Question 7

a) $P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

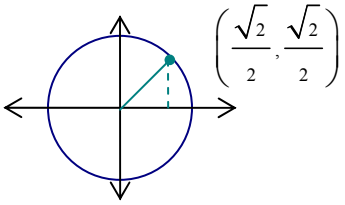


b) $P(-150°) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

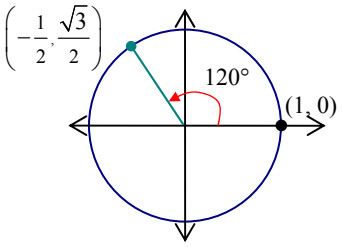


c) $-\frac{11\pi}{2}$ is coterminal with $\frac{\pi}{2}$, so $P\left(-\frac{11\pi}{2}\right) = (0, 1)$.

d) $P(45°) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

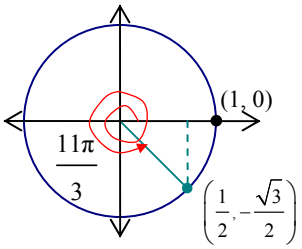


e) $P(120°) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



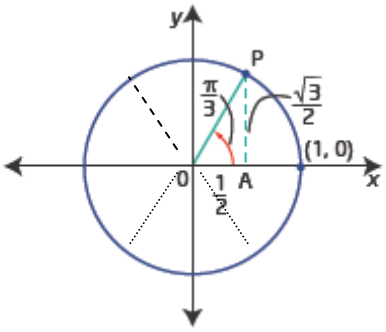
f) $\frac{11\pi}{3}$ is coterminal with $\frac{5\pi}{3}$.

So, $P\left(\frac{11\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.



Chapter 4 Review Page 216 Question 8

a) The diagram shows $P\left(\frac{\pi}{3}\right)$, with coordinates $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Then, $P\left(\frac{2\pi}{3}\right)$ will be the same distance from each axis as $P\left(\frac{\pi}{3}\right)$ is, but in quadrant II. So, its coordinates are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.



$P\left(\frac{4\pi}{3}\right)$ will be the same distance from each axis, but in quadrant III. So, its coordinates are $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Similarly, $P\left(\frac{5\pi}{3}\right)$ will be the same distance from each axis, but in quadrant IV. So, its coordinates are $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

b) If $P(\theta) = \left(-\frac{2\sqrt{2}}{3}, \frac{1}{3}\right)$, then θ is in quadrant II.

$P\left(\theta + \frac{\pi}{2}\right)$ will be in quadrant III and will have coordinates $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$.

c) $P\left(\frac{5\pi}{6}\right)$ is in quadrant II. Then, $P\left(\frac{5\pi}{6} + \pi\right)$, which is a rotation of π more, will be in quadrant IV.

$$P(\theta) = P\left(\frac{5\pi}{6} + \pi\right)$$
$$= P\left(\frac{11\pi}{6}\right)$$
$$= \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

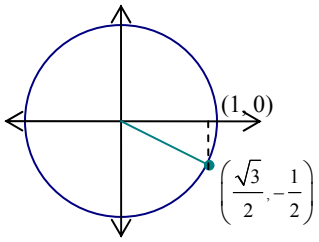
Chapter 4 Review Page 216 Question 9

In the interval $-2\pi \le \theta < 2\pi$:

a) (0, 1) is on the *y*-axis, above the origin, so $\theta = -\frac{3\pi}{2}$ and $\theta = \frac{\pi}{2}$.

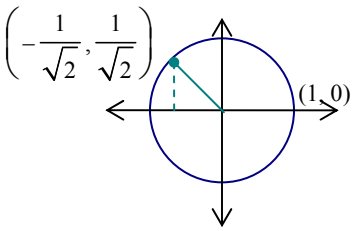
b) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ is in quadrant IV.

$\theta = -\frac{\pi}{6}$ and $\theta = \frac{11\pi}{6}$.



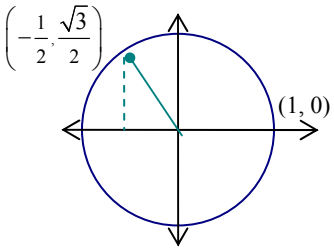
c) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is in quadrant II.

$\theta = -\frac{5\pi}{4}$ and $\theta = \frac{3\pi}{4}$.



d) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is in quadrant II.

$\theta = -\frac{4\pi}{3}$ and $\theta = \frac{2\pi}{3}$.

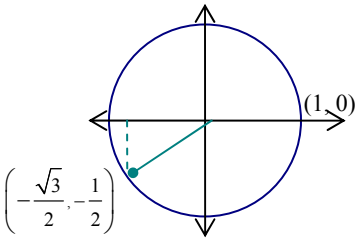


Chapter 4 Review Page 216 Question 10

In the domain $-180° < \theta \le 360°$:

a) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ is in quadrant III.

$\theta = -150°$ and $\theta = 210°$.

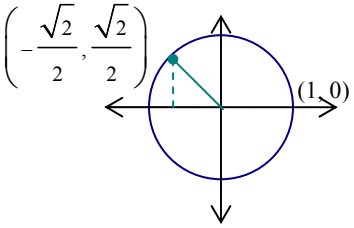


b) (–1, 0) is on the *x*-axis to the left of the origin.

$\theta = 180°$.

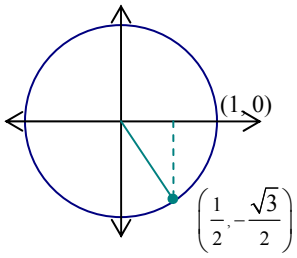
c) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is in quadrant II.

$\theta = 135°$.



d) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ is in quadrant IV.

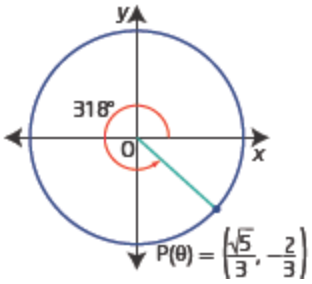
$\theta = –60°$ and $\theta = 300°$.



Chapter 4 Review Page 216 Question 11

a) Given $P(\theta) = \left(\frac{\sqrt{5}}{3}, -\frac{2}{3}\right)$, then θ is in quadrant IV.

$$\tan\theta = \frac{y}{x}$$
$$= -\frac{2}{3} \div \frac{\sqrt{5}}{3}$$
$$= -\frac{2}{\sqrt{5}}$$



Then, $\theta \approx –42°$ or 318°, or (in radians) $\theta \approx 5.55$.

b) θ terminates in quadrant IV.

c) $P(\theta + \pi)$ will be in quadrant II with coordinates $\left(-\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$.

d) $P\left(\theta + \frac{\pi}{2}\right)$ will be in quadrant I with coordinates $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$.

e) $P\left(\theta - \frac{\pi}{2}\right)$ will be in quadrant III with coordinates $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$.

Chapter 4 Review Page 216 Question 12

If $\cos\theta = \frac{1}{3}$, $0° \le \theta \le 270°$, then θ is in quadrant I.

$\cos\theta = \frac{x}{r}$, so $x = 1$ and $r = 3$.

Determine y:

$x^2 + y^2 = r^2$

$1^2 + y^2 = 3^2$

$y^2 = 8$

$y = \sqrt{8}$ or $2\sqrt{2}$

Then, $\sin\theta = \frac{y}{r} = \frac{2\sqrt{2}}{3}$ $\tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

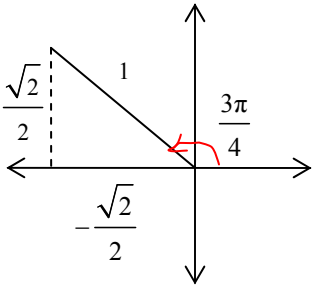
$\csc\theta = \frac{r}{y} = \frac{3}{2\sqrt{2}}$ or $\frac{3\sqrt{2}}{4}$ $\sec\theta = \frac{r}{x} = \frac{3}{1} = 3$ $\cot\theta = \frac{x}{y} = \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

Chapter 4 Review Page 216 Question 13

a) The terminal arm of $-\frac{3\pi}{2}$ is on the y-axis, above the origin. On the unit circle, its coordinates are (0, 1).

$$\sin\left(-\frac{3\pi}{2}\right) = \frac{y}{r}$$
$$= \frac{1}{1}$$
$$= 1$$

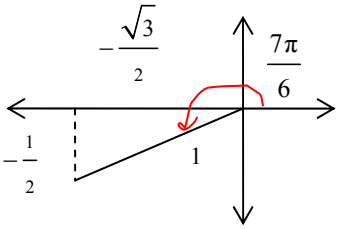
b) The terminal arm of $\frac{3\pi}{4}$ is in quadrant II. Its reference angle is $\frac{\pi}{4}$



$$\cos\frac{3\pi}{4} = \frac{x}{r}$$
$$= \frac{-\frac{\sqrt{2}}{2}}{1}$$
$$= -\frac{\sqrt{2}}{2}$$

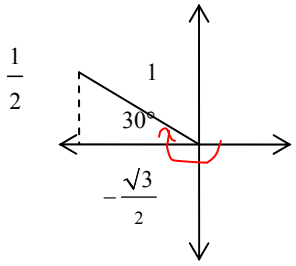
c) The terminal arm of $\frac{7\pi}{6}$ is in quadrant III. Its reference angle is $\frac{\pi}{6}$.

$$\cot\frac{7\pi}{6} = \frac{x}{y}$$
$$= \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$
$$= \sqrt{3}$$



d) The terminal arm of –210° is in quadrant II. Its reference angle is 30°.

$$\sec(-210°) = \frac{r}{x}$$
$$= \frac{1}{-\frac{\sqrt{3}}{2}}$$
$$= -\frac{2}{\sqrt{3}} \text{ or } -\frac{2\sqrt{3}}{3}$$

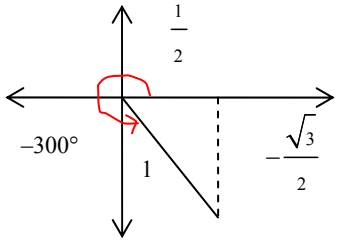


e) The terminal arm of 720° is coterminal with 0°. On the unit circle, its coordinates are (1, 0).

$$\tan 720° = = \frac{y}{x}$$
$$= \frac{0}{1}$$
$$= 0$$

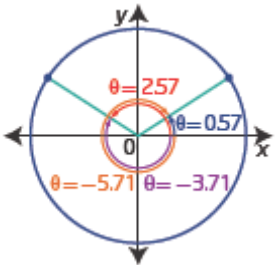
f) The terminal arm of 300° is in quadrant IV. Its reference angle is 60°.

$$\csc 300° = \frac{r}{y}$$
$$= \frac{1}{-\frac{\sqrt{3}}{2}}$$
$$= -\frac{2}{\sqrt{3}} \text{ or } -\frac{2\sqrt{3}}{3}$$

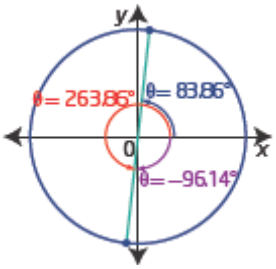


Chapter 4 Review Page 216 Question 14

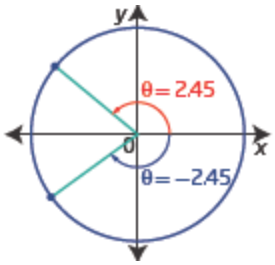
a) $\sin\theta = 0.54$, $-2\pi < \theta \leq 2\pi$. The sine value is positive, so θ is in quadrant I or II.
$\theta \approx 0.57, 2.57, -5.71, -3.71$



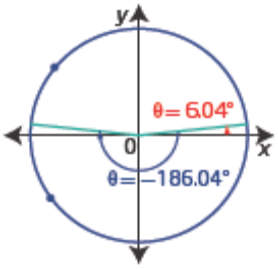
b) $\tan\theta = 9.3$, $-180° \leq \theta < 360°$. Since the tangent value is positive, θ is in quadrant I or III.
$\theta \approx 83.86°, 263.86°, -96.14°$



c) $\cos\theta = -0.77$, $-\pi \leq \theta < \pi$. The cosine value is negative, so θ is in quadrant II or III.
$\theta \approx 2.45, -2.45$



d) $\csc\theta = 9.5$, $-270° < \theta \leq 90°$. The cosecant value, and therefore the sine value, is positive, so θ is in quadrant I or II.
$\theta \approx 6.04°, -186.04°$



Chapter 4 Review Page 217 Question 15

a) $\sin 285° \approx -0.966$

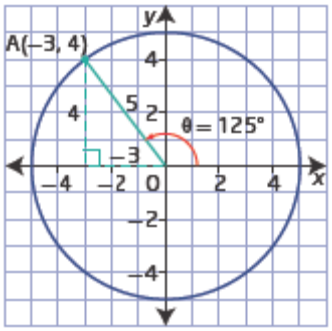
b) $\cot 130° \approx -0.839$

c) $\cos 4.5 \approx -0.211$

d) $\sec 7.38 \approx 2.191$

Chapter 4 Review Page 217 Question 16

a) Example: A(–3, 4) is in quadrant II. The angle of rotation measures approximately 125°.



b) $\cos\theta = \frac{x}{r}$

$= \frac{-3}{5}$ or -0.6

c) $\csc\theta + \tan\theta = \frac{r}{y} + \frac{y}{x}$

$= \frac{5}{4} + \frac{4}{-3}$

$= \frac{15 + (-16)}{12}$

$= -\frac{1}{12}$

d) $\theta = \cos^{-1}(-0.6)$

$= 126.9°$, or 2.2

Chapter 4 Review Page 217 Question 17

a) $\cos^2\theta + \cos\theta$

$= \cos\theta(\cos\theta + 1)$

b) $\sin^2\theta - 3\sin\theta - 4$

$= (\sin\theta - 4)(\sin\theta + 1)$

c) $\cot^2\theta - 9$

$= (\cot\theta - 3)(\cot\theta + 3)$

d) $2\tan^2\theta - 9\tan\theta + 10$

$= (2\tan\theta - 5)(\tan\theta - 2)$

Chapter 4 Review Page 217 Question 18

a) $\sin^{-1} 2$ is impossible because the sine value of an angle is never 2. Sine has a maximum value of 1.

b) tan 90° is not defined because tan = $\frac{y}{x}$ and 90° is on the *y*-axis, so *x* is 0. Division by 0 is not permissible.

Chapter 4 Review Page 217 Question 19

a) $4 \cos \theta - 3 = 0$

$\cos \theta = \frac{3}{4}$ or 0.75

The cosine ratio is positive in quadrants I and IV, so there are two solutions in the domain $0° < \theta \leq 360°$.

b) $\sin \theta + 0.9 = 0$

$\sin \theta = -0.9$

The sine ratio is negative in quadrants III and IV, so there are two solutions (both negative) in the domain $-\pi \leq \theta \leq \pi$.

c) $0.5 \tan \theta - 1.5 = 0$

$\tan \theta = 3$

The tangent ratio is positive in quadrants I and III, so there is one solution (in quadrant III) in the domain $-180° \leq \theta \leq 0°$.

d) csc θ is undefined. This occurs when $y = 0$. So, in the interval $\theta \in [-2\pi, 4\pi]$ there will be two angles which have their terminal arm on the *y*-axis in each of the three complete rotation, giving a total of six solutions.

Chapter 4 Review Page 217 Question 20

a) $\csc \theta = \sqrt{2}$, so $\sin \theta = \frac{1}{\sqrt{2}}$. The sine ratio is positive in quadrants I and II. The angle has reference angle 45°. Then, in the interval $\theta \in [0°, 360°]$, $\theta = 45°$ and 135°.

b) $2 \cos \theta + 1 = 0$, so $\cos \theta = -\frac{1}{2}$. The cosine ratio is negative in quadrants II and III. The reference angle is $\frac{\pi}{3}$. Then, in the domain $0 \leq \theta < 2\pi$, $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

c) $3 \tan \theta - \sqrt{3} = 0$, so $\tan \theta = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$. The tangent ratio is positive in quadrants I and III. The angle has reference angle 30°. Then, in the domain $-180° \le \theta < 360°$.
$\theta = 30°, 210°$, and $-150°$.

d) $\cot \theta + 1 = 0$, so $\cot \theta = -1$. This means $\tan \theta = -1$ too. The tangent ratio is negative in quadrants II and IV. The angle has reference angle $\frac{\pi}{4}$. Then, in the domain $-\pi \le \theta < \pi$,
$\theta = \frac{3\pi}{4}$ and $-\frac{\pi}{4}$.

Chapter 4 Review Page 217 Question 21

a) $\sin^2 \theta + \sin \theta - 2 = 0$
$(\sin \theta + 2)(\sin \theta - 1) = 0$
$\sin \theta = -2$ or $\sin \theta = 1$
The first equation yields no solution. In the domain $0 \le \theta < 2\pi$, $\sin \theta = 1$ for $\theta = \frac{\pi}{2}$.

b) $\tan^2 \theta + 3 \tan \theta = 0$
$\tan \theta (\tan \theta + 3) = 0$
$\tan \theta = 0$ or $\tan \theta = -3$
In the domain $0° < \theta \le 360°$,
$\theta = 180°, 360°$ or $\theta \approx 108.435°, 288.435°$

c) $6 \cos^2 \theta + \cos \theta = 1$
$6 \cos^2 \theta + \cos \theta - 1 = 0$
$(3 \cos \theta - 1)(2 \cos \theta + 1) = 0$
$\cos \theta = \frac{1}{3}$ or $\cos \theta = -\frac{1}{2}$
In the interval $\theta \in (0°, 360°)$,
$\theta \approx 70.529°, 289.471°$ or $\theta = 120°, 240°$

d) $\sec^2 \theta - 4 = 0$
$(\sec \theta - 2)(\sec \theta + 2) = 0$
$\sec \theta = \pm 2$
$\cos \theta = \pm \frac{1}{2}$
In the interval $\theta \in [-\pi, \pi]$, $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$.

Chapter 4 Review Page 217 Question 22

For $\sin\theta = \frac{\sqrt{3}}{2}$, the angle must be in quadrant I or II.

Examples:

a) In the domain $0 \le \theta < 2\pi$, $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$.

b) In the domain $-2\pi \le \theta < \frac{\pi}{2}$, $\theta = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}$.

c) In the domain $-720° \le \theta < 0°$, $\theta = -660°, -600°, -300°, -240°$.

d) In the domain $-270° \le \theta < 450°$, $\theta = -240°, 60°, 120°, 420°$.

Chapter 4 Review Page 217 Question 23

a) $\sin x = -\frac{1}{2}$, in radians. The sine ratio is negative, so x is in quadrants III or IV.

The reference angle is $\frac{\pi}{6}$.

In quadrant III, $x = \frac{7\pi}{6} + 2\pi n,\ n \in \mathrm{I}$

In quadrant IV, $x = \frac{11\pi}{6} + 2\pi n,\ n \in \mathrm{I}$

b) $\sin x = \sin^2 x$, in degrees.

$\sin x - \sin^2 x = 0$

$\sin x(1 - \sin x) = 0$

$\sin x = 0$ or $1 - \sin x = 0$

$1 = \sin x$

$x = 0°, 180°$ or $x = 90°$

In general, $x = (180°)n,\ n \in \mathrm{I}$ and $x = 90° + (360°)n,\ n \in \mathrm{I}$.

c) $\sec x + 2 = 0$, in degrees.

$\sec x = -2$, or $\cos x = -\frac{1}{2}$

The cosine ratio is negative, so x is in quadrants II or III.

The reference angle is 60°.

In quadrant II, $x = 120° + (360°)n,\ n \in \text{I}$
In quadrant III, $x = 240° + (360°)n,\ n \in \text{I}$

d) $(\tan x - 1)(\tan x - \sqrt{3}) = 0$, in radians.
$\tan x - 1 = 0$ or $\tan x - \sqrt{3} = 0$
$\tan x = 1$ or $\tan x = \sqrt{3}$
Then, $x = \frac{\pi}{4}, \frac{5\pi}{4}$ or $x = \frac{\pi}{3}, \frac{4\pi}{3}$
In general, $x = \frac{\pi}{4} + \pi n,\ n \in \text{I}$ and $x = \frac{\pi}{3} + \pi n,\ n \in \text{I}$.

Chapter 4 Practice Test

Chapter 4 Practice Test Page 218 Question 1

Given $\cos \theta = \frac{\sqrt{3}}{2}$, the reference angle for θ is $\frac{\pi}{6}$ and θ is in quadrant I or IV.
Of the possible answers, **D** is the best answer.

Chapter 4 Practice Test Page 218 Question 2

Given $\sin \theta = -\frac{\sqrt{3}}{2}$, $0° \le \theta < 360°$. The sine ratio is negative in quadrants III and IV and the reference angle is 60°. So, $\theta = 240°$ and 300°. **C** is the correct answer.

Chapter 4 Practice Test Page 218 Question 3

Given $\cot \theta = 1.4$, the cotangent ratio is positive in quadrants I and III.
In quadrant I, $\theta \approx 0.620$, and in quadrant III, $\theta \approx 3.762$.
Answer **A** is correct.

Chapter 4 Practice Test Page 218 Question 4

Given $P\left(-\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$. For a 90° counterclockwise rotation the *x*-coordinate and *y*-coordinate switch and signs are adjusted for the new quadrants. So $Q\left(-\frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$.

Answer **B** is correct.

Chapter 4 Practice Test Page 218 Question 5

$\sin\theta(\sin\theta + 1) = 0$
In the domain $-180° < \theta < 360°$, $\sin\theta = 0$ at 0° and 180°. $\sin\theta = -1$ at –90°and 270°.
There are 4 solutions in the given domain. Answer **B** is correct.

Chapter 4 Practice Test Page 218 Question 6

a) Determine the circumference, to find the distance travelled in one rotation of the tire.
$C = \pi d$
$= \pi(75)$

$$\text{Number of turns per second} = \frac{110(1000)(\cancel{100})}{\pi(75)(\cancel{60}_{6})(\cancel{60}_{6})}$$
$$\approx 12.968$$

So, a point on the tire moves through approximately 12.968(360°) or 4668.5° each second. In radians, this is approximately 81.5 radians each second.

b) If the diameter of the tire is 66 cm, then

$$\text{Number of turns per second} = \frac{110(1000)(\cancel{100})}{\pi(66)(\cancel{60}_{6})(\cancel{60}_{6})}$$
$$\approx 14.737$$

A point on the tire moves through approximately 14.737(2π) or 92. 6 radians each second.
The smaller tire has to make more turns to travel a given distance, so it will experience more tire wear.

Chapter 4 Practice Test Page 218 Question 7

a) The equation for any circle with centre the origin and radius 1 unit is
$x^2 + y^2 = 1$.

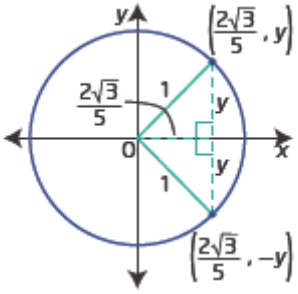
b) Substitute into the equation from part a) and solve for the missing coordinate.

i) $\left(\frac{2\sqrt{3}}{5}\right)^2 + y^2 = 1$

$$\frac{12}{25} + y^2 = 1$$

$$y^2 = \frac{13}{25}$$

$$y = \pm\frac{\sqrt{13}}{5}$$

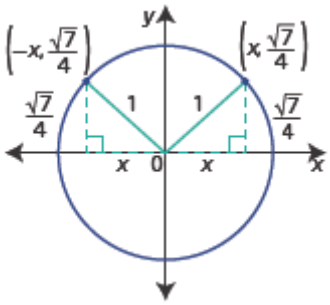


ii) $x^2 + \left(\frac{\sqrt{7}}{4}\right)^2 = 1$

$$x^2 + \frac{7}{16} = 1$$

$$x^2 = \frac{9}{16}$$

$$x = \pm\frac{3}{4}$$



For $x < 0$, the missing value of x is $-\frac{3}{4}$.

c) Example: If you know the y-coordinate of a point on the unit circle then you can use the equation $x^2 + y^2 = 1$ to determine the corresponding x-coordinate. Then, the cosine is the same as the x-coordinate because in $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, the adjacent is the x-coordinate and the hypotenuse is 1 in the unit circle.

Chapter 4 Practice Test Page 219 Question 8

a) The cosine is negative when the angle is in quadrant II or III. If you have determined that the solution in quadrant III is A, then $A - \pi$ is the measure of the reference angle. The solution in quadrant II is $\pi - (A - \pi)$ or $2\pi - A$.

b) The general solution is given by each solution plus $2\pi n$, where $n \in I$.

Chapter 4 Practice Test Page 219 Question 9

$$2\cos\theta + \sqrt{2} = 0$$
$$2\cos\theta = -\sqrt{2}$$
$$\cos\theta = -\frac{\sqrt{2}}{2}$$

The reference angle for θ is $\frac{\pi}{4}$. The cosine is negative when the angle is in quadrant II or III.

In quadrant II: $\theta = \frac{3\pi}{4} + 2\pi n$, where $n \in$ I.

In quadrant III: $\theta = \frac{5\pi}{4} + 2\pi n$, where $n \in$ I.

Chapter 4 Practice Test Page 219 Question 10

An angle measuring 3° is a very small angle with its terminal arm just above the *x*-axis in quadrant I, whereas 3 radians is a much larger angle, with its terminal arm close to the *y*-axis, in quadrant II.

Chapter 4 Practice Test Page 219 Question 11

a) –500° = –(360° + 140°), so the angle terminates in quadrant III.

b) The reference angle for –500° is 180° – 140°, which is 40°.

c) Since the angle is in quadrant III,
sin (–500°) ≈ –0.6, cos (–500°) ≈ –0.8, tan (–500°) ≈ 0.8
csc (–500°) ≈ –1.6, sec (–500°) ≈ –1.3, cot (–500°) ≈ 1.2

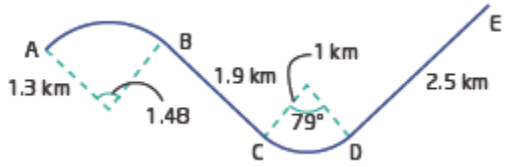
Chapter 4 Practice Test Page 219 Question 12

a) For $\frac{13\pi}{4}$, one positive coterminal angle is $\frac{5\pi}{4}$, one negative coterminal angle is $-\frac{3\pi}{4}$. All coterminal angles are given by $\frac{5\pi}{4} \pm 2\pi n$, where $n \in$ N.

b) For –575°, one positive coterminal angle is 145°, one negative coterminal angle is –215°. All coterminal angles are given by 145° ± (360°) *n*, where $n \in$ N.

Chapter 4 Practice Test Page 219 Question 13

For the arc lengths, AB and CD, use the formula $a = \theta r$.



For the arc length AB:

$AB = 1.48(1.3)$

For the arc length CD:

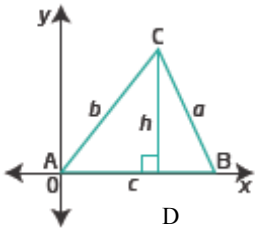
$CD = 2\pi(1)\frac{79}{360}$

Total length A to E $= 1.48(1.3) + 1.9 + 2\pi(1)\frac{79}{360} + 2.5$

≈ 7.7

The length of this stretch of road is 7.7 km, to the nearest tenth.

Chapter 4 Practice Test Page 219 Question 14



D

$\text{Area } \Delta ABC = \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}ch \qquad (1)$

In ΔACD, $\sin A = \frac{h}{b}$

$h = b\sin A$

Substitute for h in equation (1):

$\text{Area } \Delta ABC = \frac{1}{2}cb\sin A$

$= \frac{1}{2}bc\sin A$

Alternatively, in ΔBCD, $\sin B = \frac{h}{a}$

$h = a\sin B$

Substitute for h in equation (1):

$\text{Area } \Delta ABC = \frac{1}{2}ca\sin B$

$= \frac{1}{2}ac\sin B$

Chapter 4 Practice Test Page 219 Question 15

a) $3\tan^2\theta - \tan\theta - 4 = 0$

$(3\tan\theta - 4)(\tan\theta + 1) = 0$

$\tan\theta = \frac{4}{3}$ or $\tan\theta = -1$

Then, in the domain $-\pi < \theta < 2\pi$,

$\theta \approx -2.21, 0.93, 4.07$ or $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

b) $\sin^2\theta + \sin\theta - 1 = 0$

Use the quadratic formula with $a = 1$, $b = 1$ and $c = -1$.

$$\sin\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Then, in the domain $0 \le \theta < 2\pi$,

$\theta \approx 0.67, 2.48$

c) $\tan^2\theta = 4\tan\theta$

$\tan^2\theta - 4\tan\theta = 0$

$\tan\theta(\tan\theta - 4) = 0$

$\tan\theta = 0$ or $\tan\theta = 4$

Then, in the interval $\theta \in [0, 2\pi]$,

$\theta = 0, \pi, 2\pi$ or $\theta \approx 1.33, 4.47$

Chapter 4 Practice Test Page 219 Question 16

Use a ratio to determine the arc length.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{210}{360}$$

$$\frac{\text{arc length}}{2\pi(8)} = \frac{210}{360}$$

$$\text{arc length} = 16\pi\left(\frac{210}{360}\right)$$

$$= \frac{28\pi}{3}$$

$$\approx 29.32$$

Jack travelled 29.32 m, to the nearest hundredth of a metre.

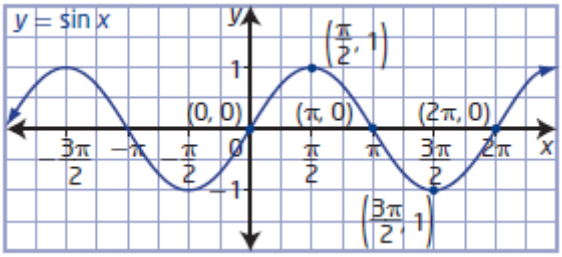
Chapter 5 Trigonometric Functions Graphs

Section 5.1 Graphing Sine and Cosine Functions

Section 5.1 Page 233 Question 1

a) One cycle of the sine function $y = \sin x$, from 0 to 2π, includes three x-intercepts, a maximum, and a minimum. These five key points divide the period into quarters: (0, 0), $\left(\frac{\pi}{2}, 1\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -1\right)$, and $(2\pi, 0)$.

b)



c) The x-intercepts of the graph of $y = \sin x$ for $-2\pi \le x \le 2\pi$ are -2π, $-\pi$, 0, π, and 2π.

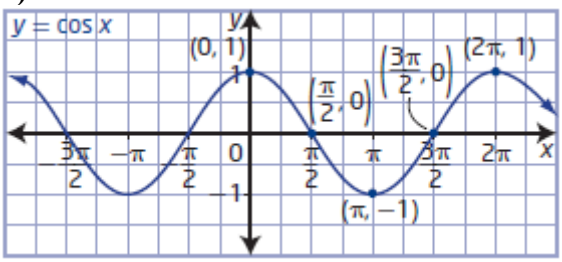
d) The y-intercept of the graph of $y = \sin x$ is 0.

e) For the graph of $y = \sin x$, the maximum value is 1 and the minimum value is -1.

Section 5.1 Page 233 Question 2

a) One cycle of the function $y = \cos x$, from 0 to 2π, includes two x-intercepts, two maximums, and a minimum. These five key points divide the period into quarters: (0, 1), $\left(\frac{\pi}{2}, 0\right)$, $(\pi, -1)$, $\left(\frac{3\pi}{2}, 0\right)$, and $(2\pi, 1)$.

b)



c) The x-intercepts of the graph of $y = \cos x$ for $-2\pi \le x \le 2\pi$ are $-\frac{3\pi}{2}$, $-\frac{\pi}{2}$, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$.

d) The y-intercept of the graph of $y = \cos x$ is 1.

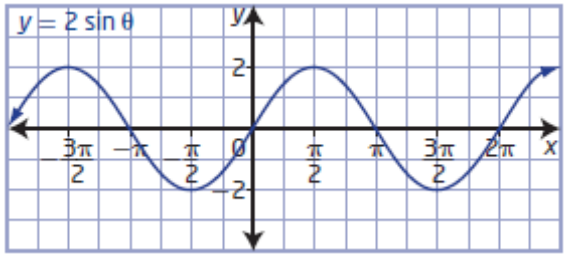
e) For the graph of $y = \cos x$, the maximum value is 1 and the minimum value is –1.

Section 5.1 Page 233 Question 3

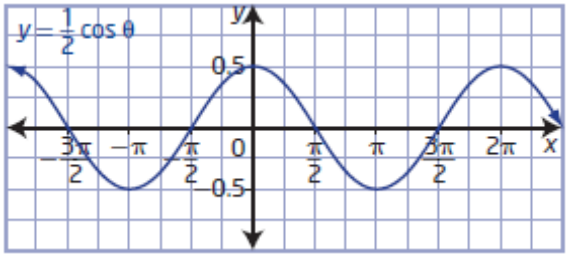
Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	-1	-1
amplitude	1	1
period	2π	2π
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid -1 \le y \le 1, y \in R\}$	$\{y \mid -1 \le y \le 1, y \in R\}$
y-intercept	0	1
x-intercepts	$\pi n, n \in I$	$\frac{\pi}{2} + \pi n, n \in I$

Section 5.1 Page 233 Question 4

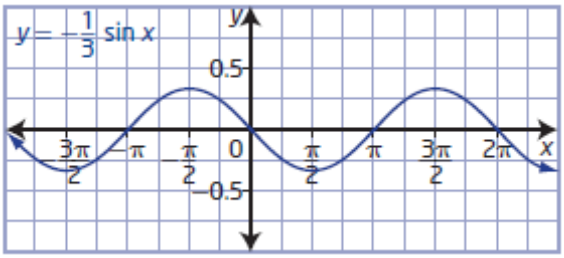
a) For the function $y = 2 \sin \theta$, $a = 2$. The amplitude is $|2|$, or 2.



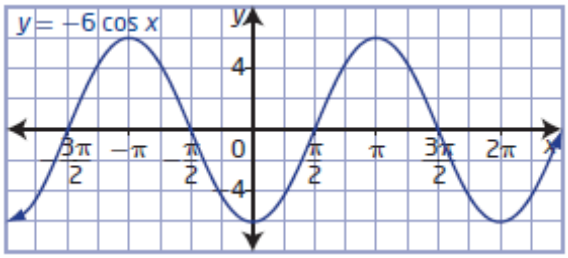
b) For the function $y = \frac{1}{2} \cos x$, $a = \frac{1}{2}$. The amplitude is $\left|\frac{1}{2}\right|$, or $\frac{1}{2}$.



c) For the function $y = -\frac{1}{3} \sin x$, $a = -\frac{1}{3}$. The amplitude is $\left|-\frac{1}{3}\right|$, or $\frac{1}{3}$.



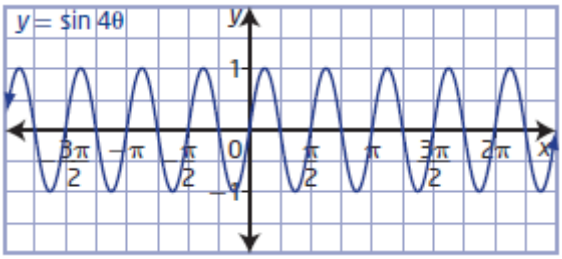
d) For the function $y = -6 \cos \theta$, $a = -6$. The amplitude is $|-6|$, or 6.



Section 5.1 Page 233 Question 5

a) For the function $y = \sin 4\theta$, $b = 4$.

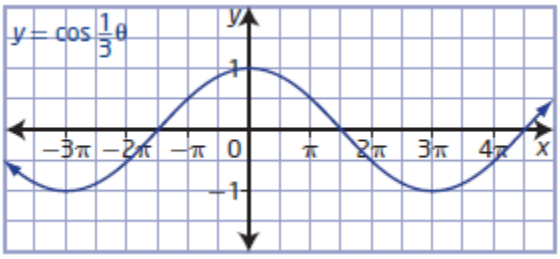
$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{|4|} \qquad = \frac{2\pi}{|4|}$$
$$= 90° \qquad = \frac{\pi}{2}$$



The period is 90° or $\frac{\pi}{2}$.

b) For the function $y = \cos \frac{1}{3}\theta$, $b = \frac{1}{3}$.

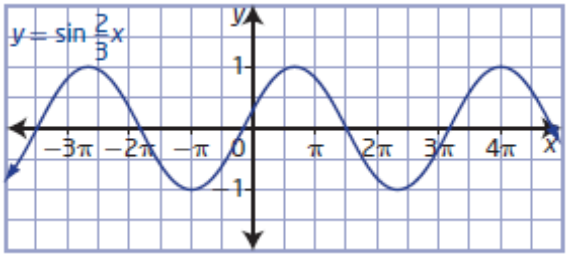
$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{\left|\frac{1}{3}\right|} \qquad = \frac{2\pi}{\left|\frac{1}{3}\right|}$$
$$= 1080° \qquad = 6\pi$$



The period is 1080° or 6π.

c) For the function $y = \sin \frac{2}{3}x$, $b = \frac{2}{3}$.

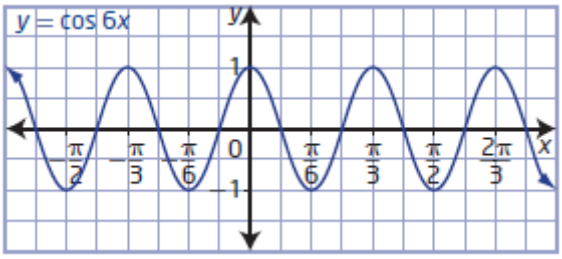
$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{\left|\frac{2}{3}\right|} \qquad = \frac{2\pi}{\left|\frac{2}{3}\right|}$$
$$= 540° \qquad = 3\pi$$



The period is 540° or 3π.

d) For the function $y = \cos 6x$, $b = 6$.

$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{|6|} \qquad = \frac{2\pi}{|6|}$$
$$= 60° \qquad = \frac{\pi}{3}$$



The period is 60° or $\frac{\pi}{3}$.

Section 5.1 Page 233 Question 6

a) For the function $y = 3 \cos x$, $a = 3$ and $b = 1$. The graph of this cosine function will have an amplitude of 3 and a period of 2π: choice **A**.

b) For the function $y = \cos 3x$, $a = 1$ and $b = 3$. The graph of this cosine function will have an amplitude of 1 and a period of $\frac{2\pi}{3}$: choice **D**.

c) For the function $y = -\sin x$, $a = -1$ and $b = 1$. The graph of this sine function will have an amplitude of 1, be reflected in the x-axis, and have a period of 2π: choice **C**.

d) For the function $y = -\cos x$, $a = -1$ and $b = 1$. The graph of this cosine function will have an amplitude of 1, be reflected in the x-axis, and have a period of 2π: choice **B**.

Section 5.1 Page 234 Question 7

a) For the function $y = 3 \sin x$, $a = 3$. The amplitude is $|3|$, or 3. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of 3.

b) For the function $y = -5 \sin x$, $a = -5$. The amplitude is $|-5|$, or 5. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of 5 and a reflection in the x-axis.

c) For the function $y = 0.15 \sin x$, $a = 0.15$. The amplitude is $|0.15|$, or 0.15. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of 0.15.

d) For the function $y = -\frac{2}{3} \sin x$, $a = -\frac{2}{3}$. The amplitude is $\left|-\frac{2}{3}\right|$, or $\frac{2}{3}$. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of $\frac{2}{3}$ and a reflection in the x-axis.

Section 5.1 Page 234 Question 8

a) For the function $y = \cos 2x$, $b = 2$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|} \\ &= \frac{360°}{|2|} \\ &= 180°\end{aligned}$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of $\frac{1}{2}$.

b) For the function $y = \cos(-3x)$, $b = -3$.

$$\text{Period} = \frac{360°}{|b|}$$
$$= \frac{360°}{|-3|}$$
$$= 120°$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of $\frac{1}{3}$ and a reflection in the y-axis.

c) For the function $y = \cos \frac{1}{4}x$, $b = \frac{1}{4}$.

$$\text{Period} = \frac{360°}{|b|}$$
$$= \frac{360°}{\left|\frac{1}{4}\right|}$$
$$= 1440°$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of 4.

d) For the function $y = \cos \frac{2}{3}x$, $b = \frac{2}{3}$.

$$\text{Period} = \frac{360°}{|b|}$$
$$= \frac{360°}{\left|\frac{2}{3}\right|}$$
$$= 540°$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of $\frac{3}{2}$.

Section 5.1 Page 234 Question 9

a) For the function $y = 2 \sin x$, $a = 2$ and $b = 1$. The amplitude is $|2|$, or 2.

$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{|1|} \qquad = \frac{2\pi}{|1|}$$
$$= 360° \qquad = 2\pi$$

b) For the function $y = -4 \sin 2x$, $a = -4$ and $b = 2$. The amplitude is $|-4|$, or 4.

$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{|2|} \qquad = \frac{2\pi}{|2|}$$
$$= 180° \qquad = \pi$$

c) For the function $y = \frac{5}{3} \sin\left(-\frac{2}{3}x\right)$, $a = \frac{5}{3}$ and $b = -\frac{2}{3}$. The amplitude is $\left|\frac{5}{3}\right|$, or $\frac{5}{3}$.

$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{\left|-\frac{2}{3}\right|} \qquad = \frac{2\pi}{\left|-\frac{2}{3}\right|}$$
$$= 540° \qquad = 3\pi$$

d) For the function $y = 3 \sin \frac{1}{2}x$, $a = 3$ and $b = \frac{1}{2}$. The amplitude is $|3|$, or 3.

$$\text{Period} = \frac{360°}{|b|} \qquad \text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{360°}{\left|\frac{1}{2}\right|} \qquad = \frac{2\pi}{\left|\frac{1}{2}\right|}$$
$$= 720° \qquad = 4\pi$$

Section 5.1 Page 234 Question 10

a) Use $\text{Amplitude} = \dfrac{\text{maximum value} - \text{minimum value}}{2}$.

The amplitude of graph A is $\frac{2-(-2)}{2}$, or 1, and the period is 4π.

The amplitude of graph B is $\frac{0.5-(-0.5)}{2}$, or 0.5, and the period is π.

b) Graph A has the pattern of a sine curve. Since the amplitude is 1, $a = 1$. Using the period of 4π and choosing b to be positive

$$\text{Period} = \frac{2\pi}{|b|}$$

$$4\pi = \frac{2\pi}{|b|}$$

$$b = \frac{1}{2}$$

So, the equation of the function in the form $y = a \sin bx$ is $y = \sin \frac{1}{2}x$.

Graph B has the pattern of a cosine curve. Since the amplitude is 0.5, $a = 0.5$. Using the period of π and choosing b to be positive

$$\text{Period} = \frac{2\pi}{|b|}$$

$$\pi = \frac{2\pi}{|b|}$$

$$b = 2$$

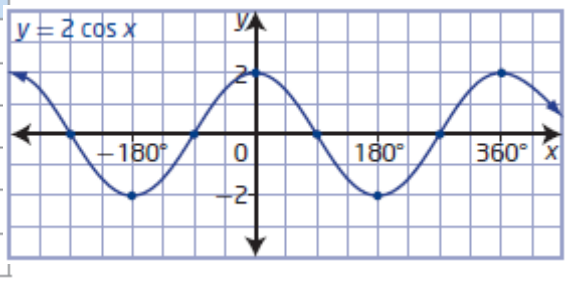
So, the equation of the function in the form $y = a \cos bx$ is $y = 0.5 \cos 2x$.

c) Since graph passes through (0, 0), the sine function is the better choice. Since graph B passes through (0, 1), the cosine function is the better choice.

Section 5.1 Page 234 Question 11

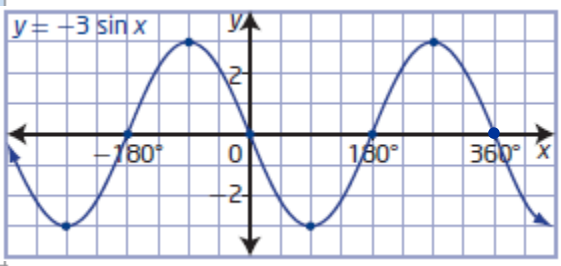
a) For $y = 2 \cos x$ in the interval $[-360°, 360°]$:

Property	Points on the Graph of $y = 2 \cos x$
maximum	(–360°, 2), (0°, 2), (360°, 2)
minimum	(–180°, –2), (180°, –2)
x-intercepts	(–270°, 0), (–90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, 2)
period	360°
range	$\{y \mid -2 \le y \le 2, y \in R\}$



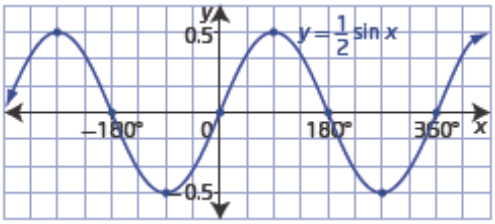
b) For $y = -3 \sin x$ in the interval $[-360°, 360°]$:

Property	Points on the Graph of $y = -3 \sin x$
maximum	(–90°, 3), (270°, 3)
minimum	(–270°, –3), (90°, –3)
x-intercepts	(–360°, 0), (–180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -3 \le y \le 3, y \in R\}$



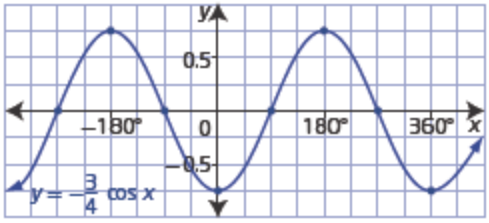
c) For $y = \frac{1}{2}\sin x$ in the interval [–360°, 360°]:

Property	Points on the Graph of $y = \frac{1}{2}\sin x$
maximum	(–270°, 0.5), (90°, 0.5)
minimum	(–90°, –0.5), (270°, –0.5)
x-intercepts	(–360°, 0), (–180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -0.5 \le y \le 0.5, y \in R\}$



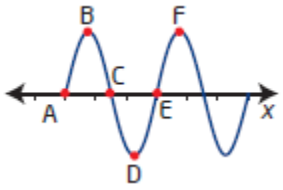
d) For $y = -\frac{3}{4}\cos x$ in the interval [–360°, 360°]:

Property	Points on the Graph of $y = -\frac{3}{4}\cos x$
maximum	(–180°, 0.75), (180°, 0.75)
minimum	(–360°, –0.75), (0°, –0.75), (360°, –0.75)
x-intercepts	(–270°, 0), (–90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, –0.75)
period	360°
range	$\{y \mid -0.75 \le y \le 0.75, y \in R\}$



Section 5.1 Page 234 Question 12

a) Given $y = 3\sin 2x$ and point A has coordinates (0, 0), find the coordinates of points B, C, D, and E.



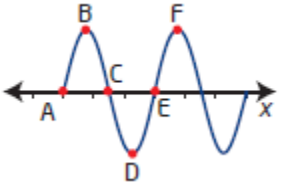
Since $a = 3$, the amplitude is $|3|$, or 3. Since $b = 2$, the period is $\frac{2\pi}{|2|}$, or π.

Point B, a maximum, occurs at one-quarter of the period: $\left(\frac{\pi}{4}, 3\right)$.

Points C and E, *x*-intercepts, occur at half the period and at the end of the period: $\left(\frac{\pi}{2}, 0\right)$ and $(\pi, 0)$.

Point D, a minimum, occurs at three-quarters of the period: $\left(\frac{3\pi}{4}, -3\right)$.

b) Given $y = 2\cos x$ and point B has coordinates (0, 2), find the coordinates of points C, D, E, and F.



Since $a = 2$, the amplitude is $|2|$, or 2. Since $b = 1$, the period is 2π.

Points C and E, x-intercepts, occur at one-quarter and three-quarters of the period: $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{3\pi}{2}, 0\right)$.
Point D, a minimum, occurs at half of the period: $(\pi, -2)$.
Point F, a maximum, occurs at the end of the period: $(2\pi, 2)$.

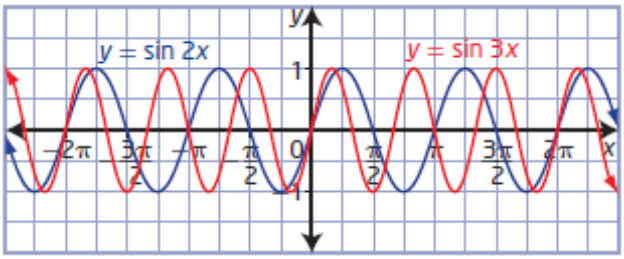
c) Given $y = \sin \frac{1}{2}x$ and point A has coordinates $(-4\pi, 0)$, find the coordinates of points B, C, D, and E.

Since $a = 1$, the amplitude is $|1|$, or 1. Since $b = \frac{1}{2}$, the period is $\frac{2\pi}{\left|\frac{1}{2}\right|}$, or 4π.

Point B, a maximum, occurs at one-quarter of the period: $(-3\pi, 1)$.
Points C and E, x-intercepts, occur at half the period and at the end of the period: $(-2\pi, 0)$ and $(0, 0)$.
Point D, a minimum, occurs at three-quarters of the period: $(-\pi, -1)$.

Section 5.1 Page 234 Question 13

The graphs of $f(x) = \sin 2x$ and $f(x) = \sin 3x$ the same amplitude of 1 but different periods, π and $\frac{2\pi}{3}$, respectively. So, the maximum, minimum, y-intercepts, domain, and range are the same for both graphs. However, the x-intercepts are different.



Section 5.1 Page 234–235 Question 14

a) Use Amplitude $= \frac{\text{maximum value} - \text{minimum value}}{2}$.

The amplitude of the sine wave is $\frac{5-(-5)}{2}$, or 5, and the period is $\frac{4\pi}{3}$.

b) The amplitude of the sine wave is $\frac{4-(-4)}{2}$, or 4, and the period is $\frac{2\pi}{3}$.

Section 5.1 Page 235 Question 15

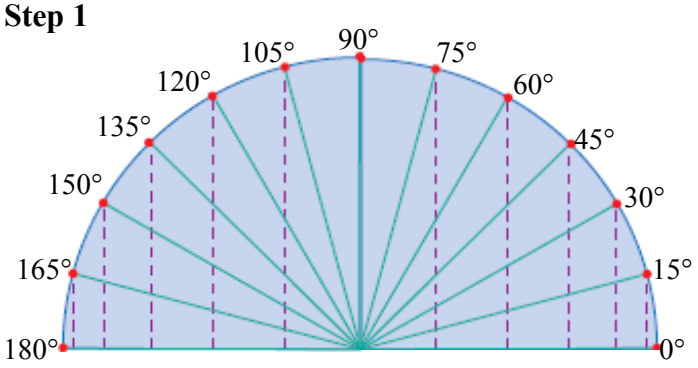
a) The amplitude of the graph is $\frac{120-80}{2}$, or 20 mmHg, and the period is 0.8 s.

b) The pulse rate for this person is $\frac{60}{0.8}$, or 75 beats per minute.

Section 5.1 Page 235 Question 16

Examples:

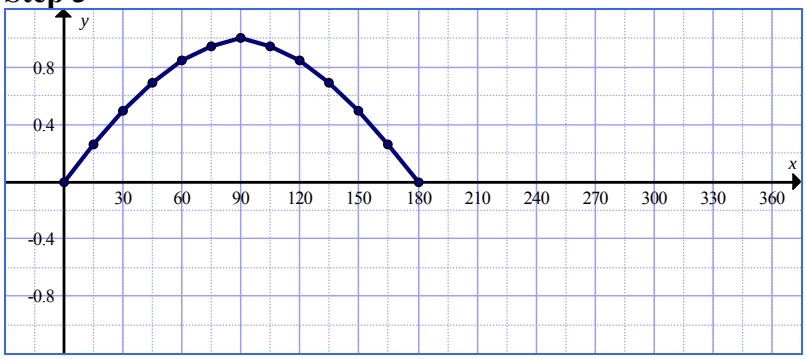
Step 1



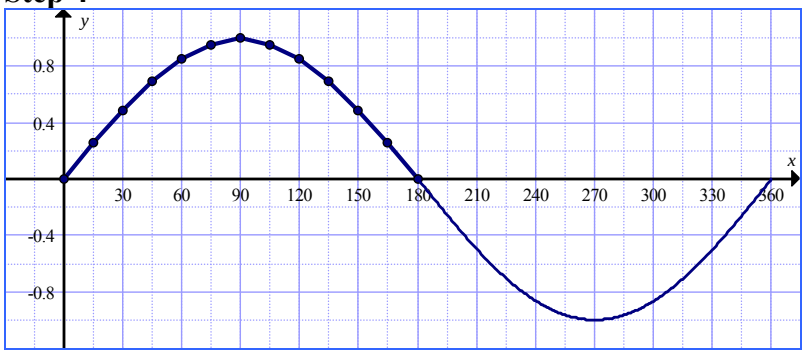
Step 2

Angle, x	Opposite (cm)	Hypotenuse (cm)	$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$
0°	0.0	3.9	$\sin 0° = 0$
15°	1.0	3.9	$\sin 15° \approx 0.26$
30°	1.9	3.9	$\sin 30° \approx 0.49$
45°	2.7	3.9	$\sin 45° \approx 0.69$
60°	3.3	3.9	$\sin 60° \approx 0.85$
75°	3.7	3.9	$\sin 75° \approx 0.95$
90°	3.9	3.9	$\sin 90° = 1$
105°	3.7	3.9	$\sin 105° \approx 0.95$
120°	3.3	3.9	$\sin 120° \approx 0.85$
135°	2.7	3.9	$\sin 135° \approx 0.69$
150°	1.9	3.9	$\sin 150° \approx 0.49$
165°	1.0	3.9	$\sin 165° \approx 0.26$
180°	0.0	3.9	$\sin 180° = 0$

Step 3

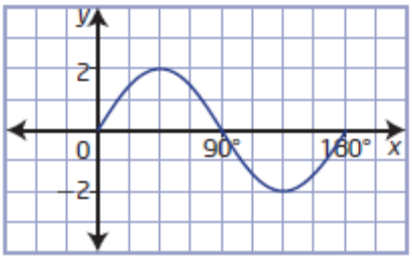


Step 4

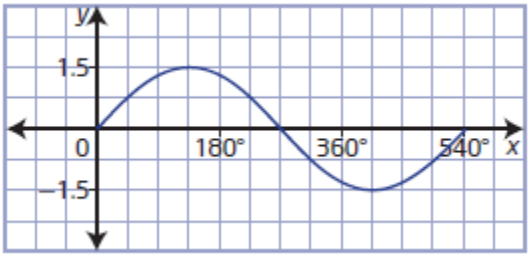


Section 5.1 Page 236 Question 17

a) For a sinusoidal curve with amplitude 2, period 180°, and passing through the point (0, 0), the five key points that divide the period into quarters are (0, 0), (45°, 2), (90°, 0), (270°, –2), and (180°, 0).

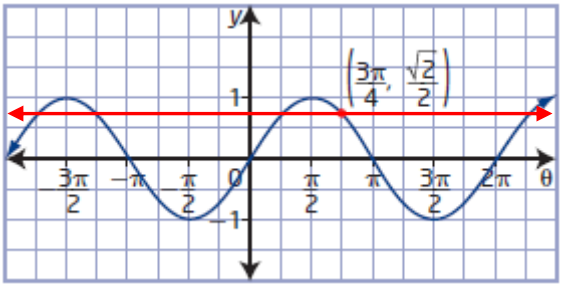


b) For a sinusoidal curve with amplitude 1.5, period 540°, and passing through the point (0, 0), the five key points that divide the period into quarters are (0, 0), (135°, 1.5), (270°, 0), (405°, –1.5), and (540°, 0).



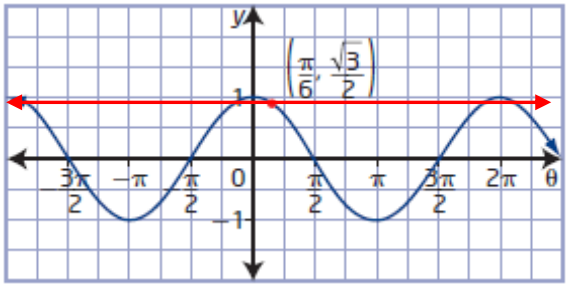
Section 5.1 Page 236 Question 18

a) Add the horizontal line $y = \frac{\sqrt{2}}{2}$ to the grid. The points of intersection with the graph of $y = \sin\theta$ will have the same y-coordinate as the point given.



$\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, and $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$

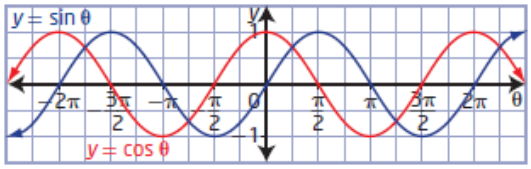
b) Add the horizontal line $y = \frac{\sqrt{3}}{2}$ to the grid. The points of intersection with the graph of $y = \cos\theta$ will have the same y-coordinate as the point given.



$\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-6, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, and $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$

Section 5.1 Page 236 Question 19

a) The graphs have the same maximum values, minimum values, period, domain, and range.



b) The graphs have different x- and y-intercepts.

c) Example: The graph of $y = \cos\theta$ will be the same as the graph of $y = \sin\theta$, if it is translated horizontally $\frac{\pi}{2}$ units to the right.

Section 5.1 Page 236 Question 20

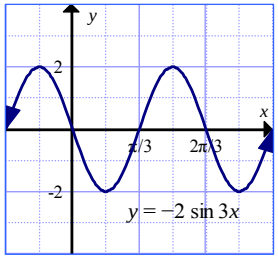
The function $y = f(x)$ has a period of 6. For $y = f\left(\frac{1}{2}x\right)$, $b = \frac{1}{2}$. So, the period for $y = f\left(\frac{1}{2}x\right)$ is $\frac{6}{\left|\frac{1}{2}\right|}$, or 12.

Section 5.1 Page 236 Question 21

One method for determining the period is to graph the function and observe the length of a cycle from the graph. Another method is to use the formula.

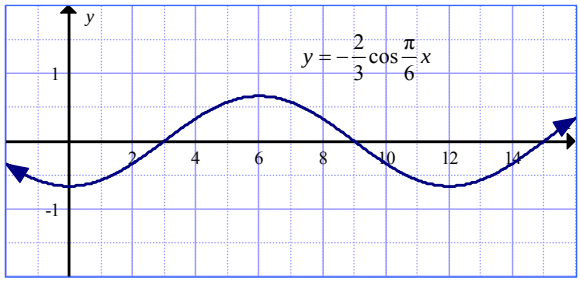
a) For the function $y = -2\sin 3x$, $b = 3$.

$$\text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{2\pi}{|3|}$$
$$= \frac{2\pi}{3}$$



b) For the function $y = -\frac{2}{3}\cos\frac{\pi}{6}x$, $b = \frac{\pi}{6}$.

$$\text{Period} = \frac{2\pi}{|b|}$$
$$= \frac{2\pi}{\left|\frac{\pi}{6}\right|}$$
$$= 12$$



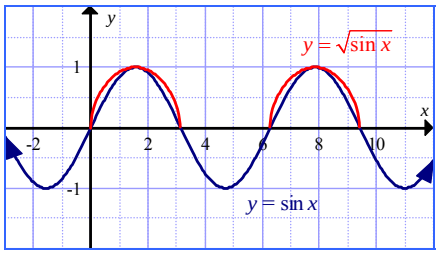
Section 5.1 Page 236 Question 22

If $\sin\theta = 0.3$ and the period of $y = \sin\theta$ is 2π, then $\sin(\theta + 2\pi) = \sin(\theta + 4\pi) = 0.3$.

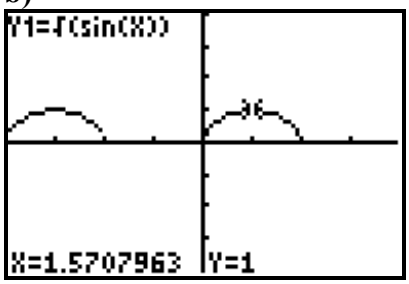
$\sin\theta + \sin(\theta + 2\pi) + \sin(\theta + 4\pi)$
$= 0.3 + 0.3 + 0.3$
$= 0.9$

Section 5.1 Page 236 Question 23

a) The graph of $y = \sqrt{\sin x}$ will consists the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

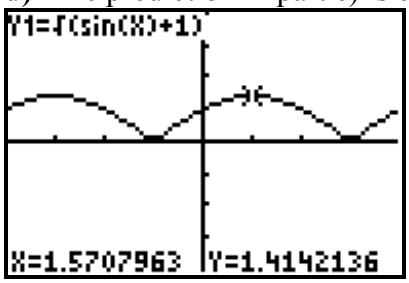


b)



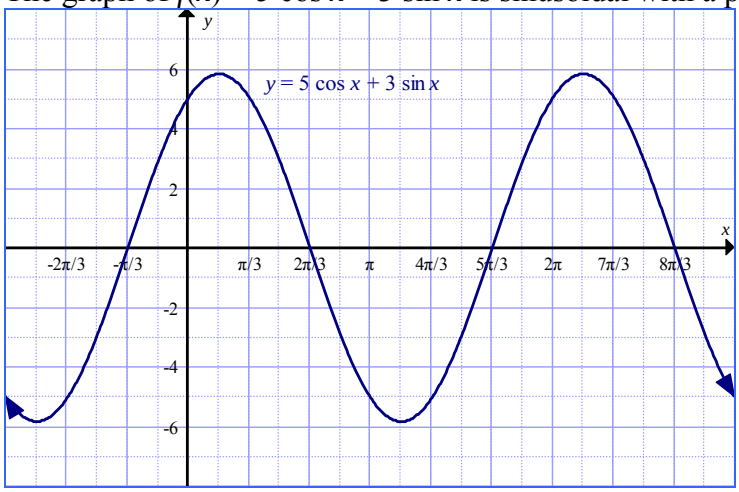
c) Since the range of $y = \sin x$ is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$, the range of $y = \sin x + 1$ will be $\{y \mid 0 \le y \le 2, y \in \mathrm{R}\}$. In other words, the entire graph of $y = \sin x + 1$ will lie on or above the x-axis. So, the graph of $y = \sqrt{\sin x + 1}$ will exist for all values of x, as compared to the graph of $y = \sqrt{\sin x}$.

d) The prediction in part c) is correct.



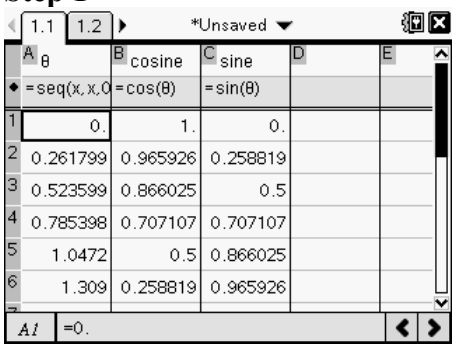
Section 5.1 Page 236 Question 24

The graph of $f(x) = 5 \cos x + 3 \sin x$ is sinusoidal with a period of 2π.

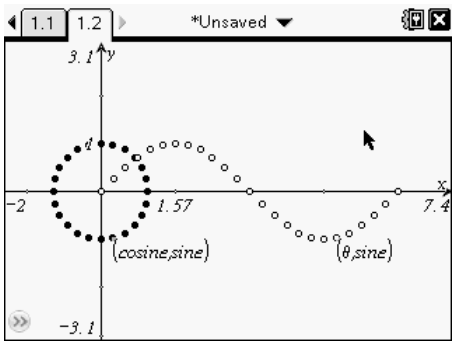


Section 5.1 Page 236–37 Question C1

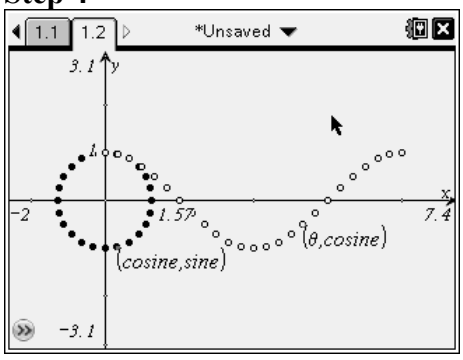
Step 1



Steps 2–3



Step 4



Step 5 a) The x-coordinate of each point on the unit circle represents cos θ. The y-coordinate of each point on the unit circle represents the sin θ.

b) The y-coordinates of the points on the sine graph are the same as the y-coordinates of the points on the unit circle. The y-coordinates of the points on the cosine graph are the same as the x-coordinates of the points on the unit circle.

Section 5.1 Page 237 Question C2

The value of $(\cos \theta)^2 + (\sin \theta)^2$ is 1.
The points on the unit circle have the coordinates (cos θ, sin θ). So, the sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

Section 5.1 Page 237 Question C3

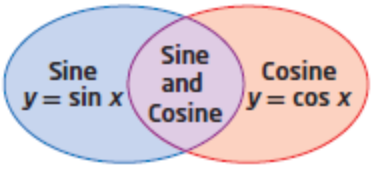
Given: The graph of $y = f(x)$ is sinusoidal with a period of 40° passing through the point (4, 0).
a) The value of $f(0)$ cannot be determined because the amplitude is not given.

b) The value of $f(4)$ is 0, as given.

c) The value of $f(84)$ is 0. Since the period is 40°, the graph will return to the same value every 40°. So, the y-coordinate at $x = 4$, $x = 44$, and $x = 84$ will be 0.

Section 5.1 Page 237 Question C4

a) The domain of both $y = \sin x$ and $y = \cos x$ is $\{x \mid x \in \mathrm{R}\}$.



b) The range of both $y = \sin x$ and $y = \cos x$ is $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$.

c) The period of both $y = \sin x$ and $y = \cos x$ is 2π.

d) The amplitude of both $y = \sin x$ and $y = \cos x$ is 1.

e) The graph of $y = \sin x$ has x-intercepts of $n(180°)$, $n \in \text{I}$.

f) The graph of $y = \cos x$ has x-intercepts of $90° + n(180°)$, $n \in \text{I}$.

g) The graph of $y = \cos x$ has y-intercept 1.

h) The graph of $y = \sin x$ has y-intercept 0.

i) The graph of $y = \cos x$ passes through the point (0, 1).

j) The graph of $y = \sin x$ passes through the point (0, 0).

k) The graph of $y = \cos x$ has a maximum value at (360°, 1).

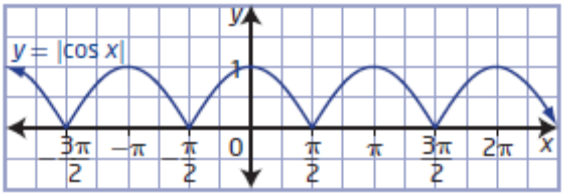
l) The graph of $y = \sin x$ has a maximum value at (90°, 1).

m) This is the graph of $y = \sin x$.

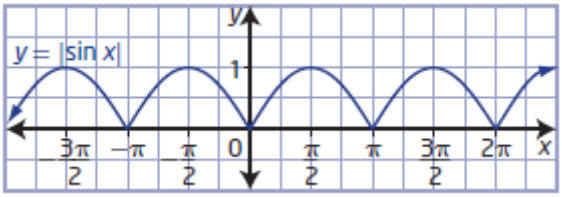
n) This is the graph of $y = \cos x$.

Section 5.1 Page 237 Question C5

a) The graph of $y = |\cos x|$ can be obtained from the graph of $y = \cos x$ by reflecting all parts of the graph below the x-axis in the x-axis.



b) The graph of $y = |\sin x|$ can be obtained from the graph of $y = \sin x$ by reflecting all parts of the graph below the x-axis in the x-axis.

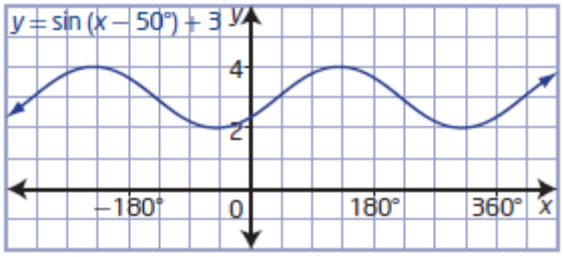


Section 5.2 Transformations of Sinusoidal Functions

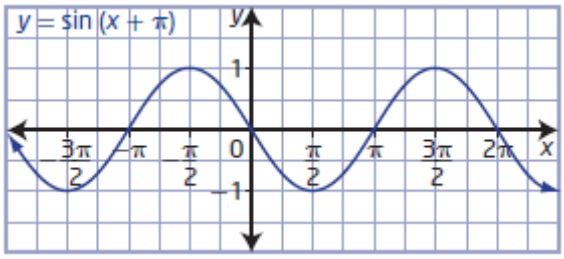
Section 5.2 Page 250 Question 1

Compare each function to the form $y = a \sin b(x - c) + d$. The phase shift is determined by parameter c, while the vertical displacement is determined by parameter d.

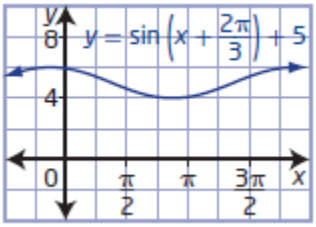
a) For $y = \sin(x - 50°) + 3$, $a = 1$, $b = 1$, $c = 50°$, and $d = 3$. So, the phase shift is 50° to the right and the vertical displacement is 3 units up. To sketch the graph, also use amplitude 1 and period 360°, since there is no vertical or horizontal stretch.



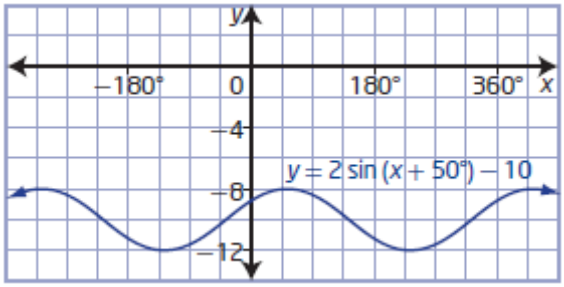
b) For $y = \sin(x + \pi)$, $a = 1$, $b = 1$, $c = -\pi$, and $d = 0$. So, the phase shift is π units to the left and there is no vertical displacement. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



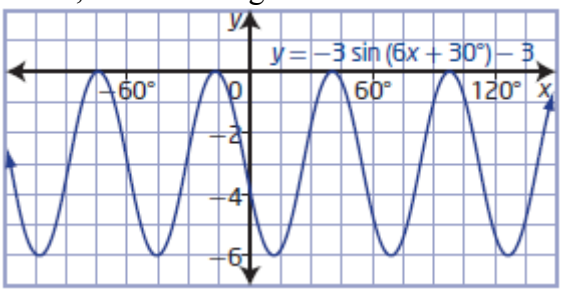
c) For $y = \sin\left(x + \frac{2\pi}{3}\right) + 5$, $a = 1$, $b = 1$, $c = -\frac{2\pi}{3}$, and $d = 5$. So, the phase shift is $\frac{2\pi}{3}$ units to the left and the vertical displacement is 5 units up. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



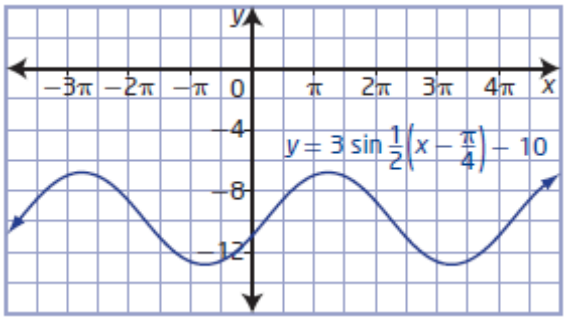
d) For $y = 2\sin(x + 50°) - 10$, $a = 2$, $b = 1$, $c = -50°$, and $d = -10$. So, the phase shift is 50° to the left and the vertical displacement is 10 units down. To sketch the graph, also use amplitude 2 and period 360°, since there is a vertical stretch but no horizontal stretch.



e) First rewrite $y = -3 \sin (6x + 30°) - 3$ as $y = -3 \sin 6(x + 5°) - 3$. Then, $a = -3$, $b = 6$, $c = -5°$, and $d = -3$. So, the phase shift is 5° to the left and the vertical displacement is 3 units down. To sketch the graph, also use amplitude 3 and period 60°, since there is a vertical stretch and a horizontal stretch. Reflect the graph in the x-axis, since a is negative.



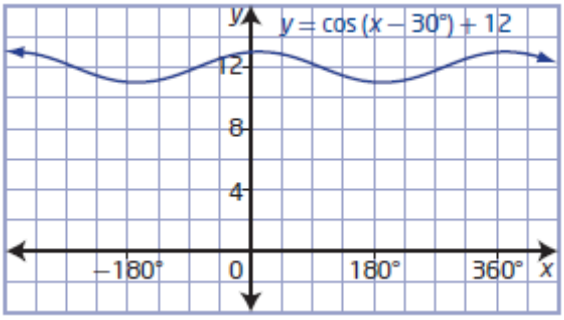
f) For $y = 3 \sin \frac{1}{2}\left(x - \frac{\pi}{4}\right) - 10$, $a = 3$, $b = \frac{1}{2}$, $c = \frac{\pi}{4}$, and $d = -10$. So, the phase shift is $\frac{\pi}{4}$ units to the right and the vertical displacement is 10 units down. To sketch the graph, also use amplitude 3 and period 4π, since there is a vertical stretch and a horizontal stretch.



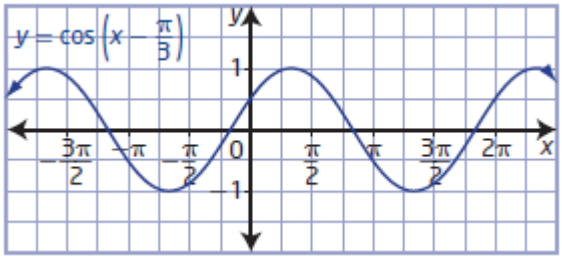
Section 5.2 Page 250 Question 2

Compare each function to the form $y = a \cos b(x - c) + d$. The phase shift is determined by parameter c, while the vertical displacement is determined by parameter d.

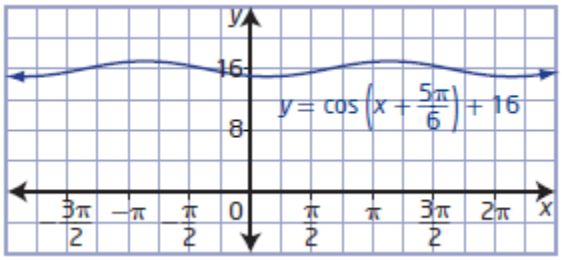
a) For $y = \cos (x - 30°) + 12$, $a = 1$, $b = 1$, $c = 30°$, and $d = 12$. So, the phase shift is 30° to the right and the vertical displacement is 12 units up. To sketch the graph, also use amplitude 1 and period 360°, since there is no vertical or horizontal stretch.



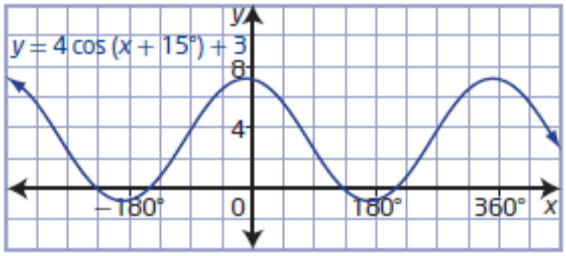
b) For $y = \cos\left(x - \frac{\pi}{3}\right)$, $a = 1$, $b = 1$, $c = \frac{\pi}{3}$, and $d = 0$. So, the phase shift is $\frac{\pi}{3}$ units to the right and there is no vertical displacement. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



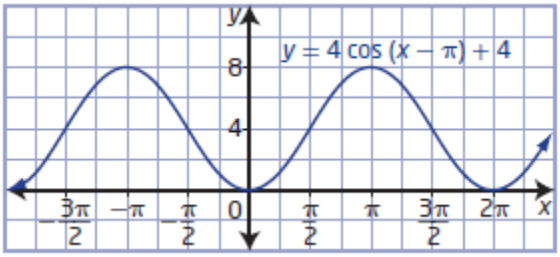
c) For $y = \cos\left(x + \frac{5\pi}{6}\right) + 16$, $a = 1$, $b = 1$, $c = -\frac{5\pi}{6}$, and $d = 16$. So, the phase shift is $\frac{5\pi}{6}$ units to the left and the vertical displacement is 16 units up. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



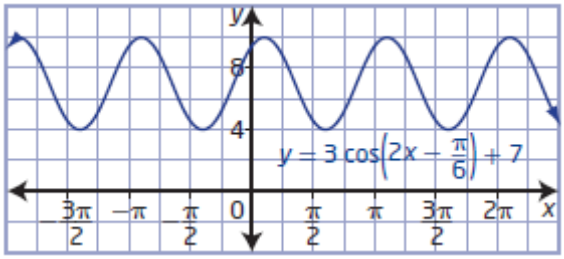
d) For $y = 4\cos(x + 15°) + 3$, $a = 4$, $b = 1$, $c = -15°$, and $d = 3$. So, the phase shift is 15° to the left and the vertical displacement is 3 units up. To sketch the graph, also use amplitude 2 and period 360°, since there is a vertical stretch but no horizontal stretch.



e) For $y = 4\cos(x - \pi) + 4$, $a = 4$, $b = 1$, $c = \pi$, and $d = 4$. So, the phase shift is π to the right and the vertical displacement is 4 units up. To sketch the graph, also use amplitude 4 and period 2π, since there is a vertical stretch but no horizontal stretch.



f) First rewrite $y = 3\cos\left(2x - \frac{\pi}{6}\right) + 7$ as $y = 3\cos 2\left(x - \frac{\pi}{12}\right) + 7$. Then, $a = 3$, $b = 2$, $c = \frac{\pi}{12}$, and $d = 7$. So, the phase shift is $\frac{\pi}{12}$ units to the right and the vertical displacement is 7 units up. To sketch the graph, also use amplitude 3 and period π, since there is a vertical stretch and a horizontal stretch.



Section 5.2 Page 250 Question 3

Use the amplitude, a, and vertical displacement, d, to determine the maximum and minimum values.

a) i) For $y = 3\cos\left(x - \frac{\pi}{2}\right) + 5$, $a = 3$, $b = 1$, $c = \frac{\pi}{2}$, and $d = 5$.

maximum: $d + |a| = 5 + |3|$ $= 8$ minimum: $d - |a| = 5 - |3|$ $= 2$

So, the range of the function is $\{y \mid 2 \le y \le 8, y \in \mathrm{R}\}$.

ii) For $y = -2 \sin(x + \pi) - 3$, $a = -2$, $b = 1$, $c = \pi$, and $d = -3$.

maximum: $d + |a| = -3 + |-2|$ $= -1$ minimum: $d - |a| = -3 - |-2|$ $= -5$

So, the range of the function is $\{y \mid -5 \le y \le -1, y \in \mathrm{R}\}$.

iii) For $y = 1.5 \sin x + 4$, $a = 1.5$, $b = 1$, $c = 0$, and $d = 4$.

maximum: $d + |a| = 4 + |1.5|$ $= 5.5$ minimum: $d - |a| = 4 - |1.5|$ $= 2.5$

So, the range of the function is $\{y \mid 2.5 \le y \le 5.5, y \in \mathrm{R}\}$.

iv) For $y = \frac{2}{3}\cos(x + 50°) + \frac{3}{4}$, $a = \frac{2}{3}$, $b = 1$, $c = 50°$, and $d = \frac{3}{4}$.

maximum: $d + |a| = \frac{3}{4} + \left|\frac{2}{3}\right|$ $= \frac{17}{12}$ minimum: $d - |a| = \frac{3}{4} - \left|\frac{2}{3}\right|$ $= \frac{1}{12}$

So, the range of the function is $\left\{y \mid \frac{1}{12} \le y \le \frac{17}{12}, y \in \mathrm{R}\right\}$.

b) To determine the range of a function of the form $y = a \cos b(x - c) + d$ or $y = a \sin b(x - c) + d$, add and subtract the amplitude to/from the vertical displacement. This gives the maximum and minimum values of the function, and thus the range.

Section 5.2 Page 250 Question 4

a) For $y = -2 \cos 2(x + 4) - 1$, $a = -2$, $b = 2$, $c = -4$, and $d = -1$. So, the amplitude is $|-2|$, or 2, the period is $\frac{2\pi}{|2|}$ or π, the phase shift is 4 units to the left, and the vertical displacement is 1 unit down. Choice **D**.

b) For $y = 2 \sin 2(x - 4) - 1$, $a = 2$, $b = 2$, $c = 4$, and $d = -1$. So, the amplitude is $|2|$, or 2, the period is $\frac{2\pi}{|2|}$ or π, the phase shift is 4 units to the right, and the vertical displacement is 1 unit down. Choice **C**.

c) First, rewrite $y = 2 \sin (2x - 4) - 1$ as $y = 2 \sin 2(x - 2) - 1$. Then, $a = 2$, $b = 2$, $c = 2$, and $d = -1$. So, the amplitude is $|2|$, or 2, the period is $\frac{2\pi}{|2|}$ or π, the phase shift is 2 units to the right, and the vertical displacement is 1 unit down. Choice **B**.

d) First, rewrite $y = 3 \sin (3x - 9) - 1$ as $y = 3 \sin 3(x - 3) - 1$. Then, $a = 3$, $b = 3$, $c = 3$, and $d = -1$. So, the amplitude is $|3|$, or 3, the period is $\frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$, the phase shift is 3 units to the right, and the vertical displacement is 1 unit down. Choice **A**.

e) First, rewrite $y = 3 \sin (3x + \pi) - 1$ as $y = 3\sin 3\left(x + \frac{\pi}{3}\right) - 1$. Then, $a = 3$, $b = 3$, $c = -\frac{\pi}{3}$, and $d = -1$. So, the amplitude is $|3|$, or 3, the period is $\frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$, the phase shift is $\frac{\pi}{3}$ units to the left, and the vertical displacement is 1 unit down. Choice **E**.

Section 5.2 Page 250 Question 5

a) For $y = \sin\left(x - \frac{\pi}{4}\right)$, $a = 1$, $b = 1$, $c = \frac{\pi}{4}$, and $d = 0$. So, the graph will have amplitude 1, period 2π, phase shift of $\frac{\pi}{4}$ units to the right, and no vertical displacement. Choice **D**.

b) For $y = \sin\left(x + \frac{\pi}{4}\right)$, $a = 1$, $b = 1$, $c = -\frac{\pi}{4}$, and $d = 0$. So, the graph will have amplitude 1, period 2π, phase shift of $\frac{\pi}{4}$ units to the left, and no vertical displacement. Choice **B**.

c) For $y = \sin x - 1$, $a = 1$, $b = 1$, $c = 0$, and $d = -1$. So, the graph will have amplitude 1, period 2π, no phase shift, and a vertical displacement of 1 unit down. Choice **C**.

d) For $y = \sin x + 1$, $a = 1$, $b = 1$, $c = 0$, and $d = 1$. So, the graph will have amplitude 1, period 2π, no phase shift, and a vertical displacement of 1 unit up. Choice **A**.

Section 5.2 Page 251 Question 6

a) For amplitude 4, period π, phase shift $\frac{\pi}{2}$ to the right, and vertical displacement 6 units down, $a = 4$, $b = 2$, $c = \frac{\pi}{2}$, and $d = -6$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = 4\sin 2\left(x - \frac{\pi}{2}\right) - 6$.

b) For amplitude 0.5, period 4π, phase shift $\frac{\pi}{6}$ to the left, and vertical displacement 1 unit up, $a = 0.5$, $b = \frac{1}{2}$, $c = -\frac{\pi}{6}$, and $d = 1$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = 0.5\sin\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$.

c) For amplitude $\frac{3}{4}$, period 720°, no phase shift, and vertical displacement 5 units down, $a = \frac{3}{4}$, $b = \frac{1}{2}$, $c = 0$, and $d = -5$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = \frac{3}{4}\sin\frac{1}{2}x - 5$.

Section 5.2 Page 251 Question 7

a) For a vertical stretch by a factor of 3 about the x-axis, horizontal stretch by a factor of 2 about the y-axis, and translated 2 units to the left and 3 units up, $a = 3$, $b = \frac{1}{2}$, $c = -2$, and $d = 3$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = 3\cos\frac{1}{2}(x + 2) + 3$.

b) For a vertical stretch by a factor of $\frac{1}{2}$ about the x-axis, horizontal stretch by a factor of $\frac{1}{4}$ about the y-axis, and translated 3 units to the right and 5 units down, $a = \frac{1}{2}$, $b = 4$, $c = 3$, and $d = -5$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = \frac{1}{2}\cos 4(x - 3) - 5$.

c) For a vertical stretch by a factor of $\frac{3}{2}$ about the x-axis, horizontal stretch by a factor of 3 about the y-axis, reflected in the x-axis, and translated $\frac{\pi}{4}$ units to the right and 1 unit down, $a = -\frac{3}{2}$, $b = \frac{1}{3}$, $c = \frac{\pi}{4}$, and $d = -1$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = -\frac{3}{2}\cos\frac{1}{3}\left(x - \frac{\pi}{4}\right) - 1$.

Section 5.2 Page 251 Question 8

The colours of the wavelengths, in order from greatest to least period are red, orange, yellow, green, blue, indigo, and violet.

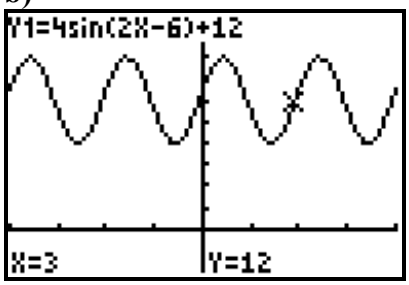
Section 5.2 Page 251 Question 9

For the piston to move faster, the period of the sine curve must be reduced. So, in the equation $y = a \sin b(x - c) + d$, parameter b would be affected.

Section 5.2 Page 252 Question 10

a) Rewrite $f(x) = 4 \sin (2x - 6) + 12$ as $f(x) = 4 \sin 2(x - 3) + 12$. So, the phase shift, c, is 3 units to the right. Stewart is correct.

b)



Section 5.2 Page 252 Question 11

If the range of the original function is $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$, then $a = 3$.
a) For $d = 2$:
maximum: $d + |a| = 2 + |3|$ $= 5$ minimum: $d - |a| = 2 - |3|$ $= -1$
So, the range of the function becomes $\{y \mid -1 \le y \le 5, y \in \mathrm{R}\}$.

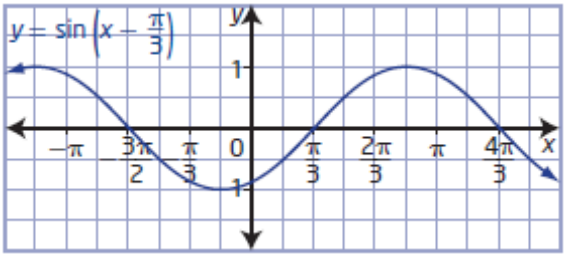
b) For $d = -3$:
maximum: $d + |a| = -3 + |3|$ $= 0$ minimum: $d - |a| = -3 - |3|$ $= -6$
So, the range of the function becomes $\{y \mid -6 \le y \le 0, y \in \mathrm{R}\}$.

c) For $d = -10$:
maximum: $d + |a| = -10 + |3|$ minimum: $d - |a| = -10 - |3|$
$= -7$ $= -13$
So, the range of the function becomes $\{y \mid -13 \le y \le -7, y \in \mathrm{R}\}$.

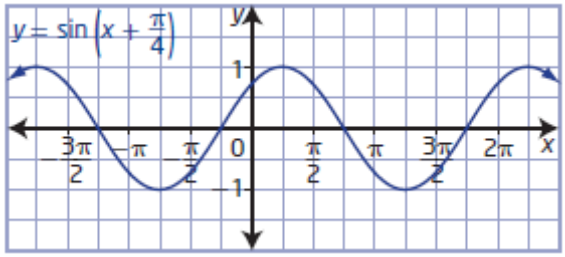
d) For $d = 8$:
maximum: $d + |a| = 8 + |3|$ minimum: $d - |a| = 8 - |3|$
$= 11$ $= 5$
So, the range of the function becomes $\{y \mid 5 \le y \le 11, y \in \mathrm{R}\}$.

Section 5.2 Page 252 Question 12

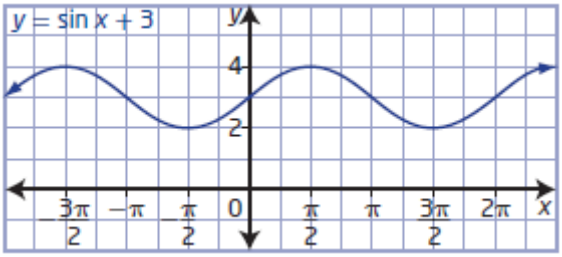
a) For the graph of $f\left(x - \frac{\pi}{3}\right)$, apply a translation of $\frac{\pi}{3}$ units to the right to the graph of $f(x) = \sin x$.



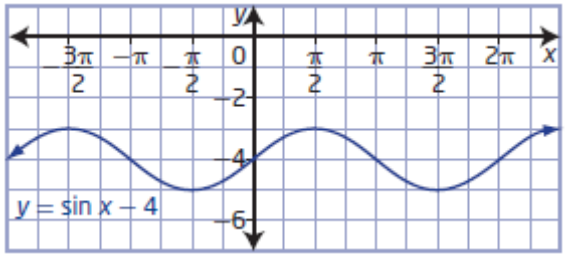
b) For the graph of $f\left(x + \frac{\pi}{4}\right)$, apply a translation of $\frac{\pi}{4}$ units to the left to the graph of $f(x) = \sin x$.



c) For the graph of $f(x) + 3$, apply a translation of 3 units up to the graph of $f(x) = \sin x$.



c) For the graph of $f(x) - 4$, apply a translation of 4 units down to the graph of $f(x) = \sin x$.



Section 5.2 Page 252 Question 13

Given the range of a trigonometric function in the form $y = a \sin b(x - c) + d$ is $\{y \mid -13 \le y \le 5, y \in \mathrm{R}\}$, then the amplitude is

$$\text{amplitude} = \frac{\text{maximum value} - \text{minimum value}}{2}$$

$$a = \frac{5-(-13)}{2}$$

$$a = 9$$

The mid-line is halfway between the maximum and minimum values.

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

$$= \frac{5+(-13)}{2}$$

$$= -4$$

Section 5.2 Page 252 Question 14

a) i) The amplitude is $\frac{3-(-3)}{2}$, or 3.

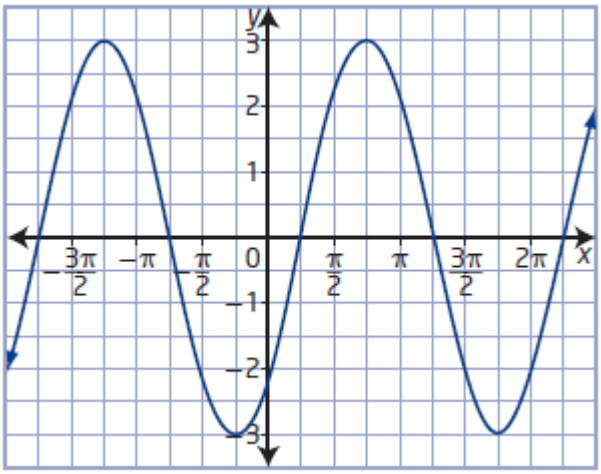
ii) The period is $\frac{9\pi}{4} - \frac{\pi}{4}$, or 2π.

iii) The start of the first cycle of the sine curve is to the right of the y-axis. So, the phase shift is $\frac{\pi}{4}$ units to the right.

iv) There is no vertical displacement.

v) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$.

vi) Over the interval $0 \le x \le 2\pi$, the maximum value of y is 3, which occurs at $x = \frac{3\pi}{4}$.



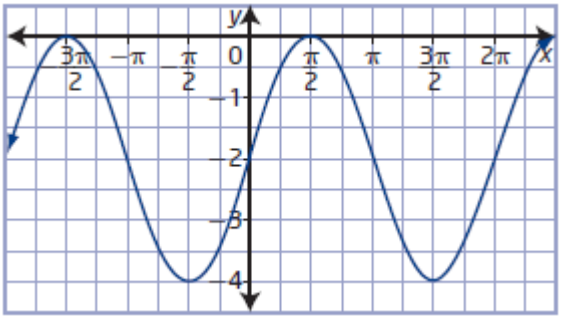
vii) Over the interval $0 \le x \le 2\pi$, the minimum value of y is -3, which occurs at $x = \frac{7\pi}{4}$.

b) i) The amplitude is $\frac{0-(-4)}{2}$, or 2.

ii) The period is $2\pi - 0$, or 2π.

iii) The start of the first cycle of the cosine curve is to the right of the y-axis. So, the phase shift is $\frac{\pi}{2}$ units to the right.

iv) The vertical displacement is 2 units down.

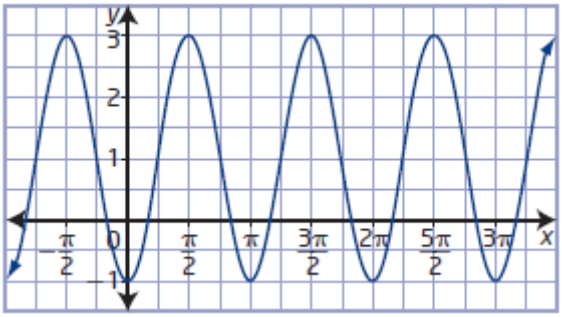


v) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid -4 \le y \le 0, y \in \mathrm{R}\}$.

vi) Over the interval $0 \le x \le 2\pi$, the maximum value of y is 3, which occurs at $x = \frac{\pi}{2}$.

vii) Over the interval $0 \le x \le 2\pi$, the minimum value of y is –3, which occurs at $x = \frac{3\pi}{2}$.

c) i) The amplitude is $\frac{3-(-1)}{2}$, or 2.



ii) The period is $\frac{5\pi}{4} - \frac{\pi}{4}$, or π.

iii) The start of the first cycle of the sine curve is to the right of the y-axis. So, the phase shift is $\frac{\pi}{4}$ units to the right.

iv) The vertical displacement is 1 unit up.

v) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid -1 \le y \le 3, y \in \mathrm{R}\}$.

vi) Over the interval $0 \le x \le 2\pi$, the maximum value of y is 3, which occurs at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

vii) Over the interval $0 \le x \le 2\pi$, the minimum value of y is –1, which occurs at $x = 0$, $x = \pi$, and $x = 2\pi$.

Section 5.2 Page 253 Question 15

a) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.

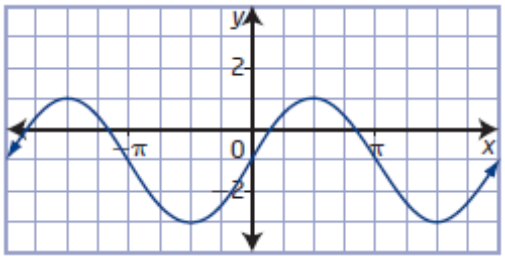
The period is $2\pi - 0$, or 2π. So, $b = 1$.

The start of the first cycle of the sine curve is on the y-axis. So, $c = 0$.

The vertical displacement is 1 unit down.

Locating the mid-line gives $d = -1$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2 \sin x - 1$.



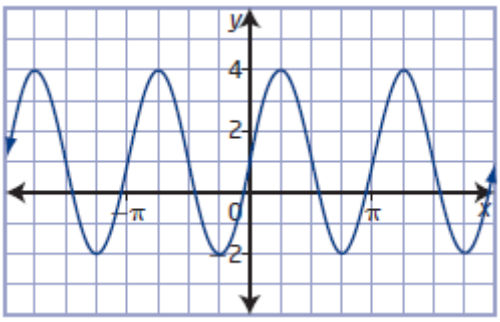
b) The amplitude, a, is $\frac{4-(-2)}{2}$, or 3.

The period is $\pi - 0$, or π. So, $b = 2$.

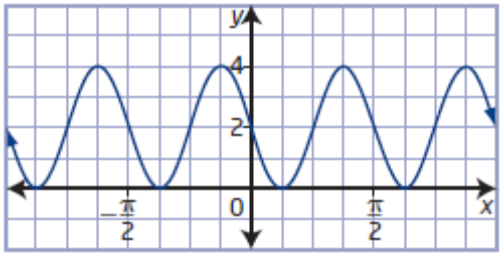
The start of the first cycle of the sine curve is on the y-axis. So, $c = 0$.

The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 3 \sin 2x + 1$.



c) The amplitude, a, is $\frac{4-0}{2}$, or 2.

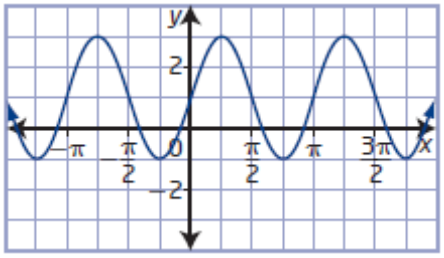


The period is $\frac{\pi}{2} - 0$, or $\frac{\pi}{2}$. So, $b = 4$.
The start of the first cycle of the sine curve is to the right of the y-axis. So, $c = \frac{\pi}{4}$.
The vertical displacement is 2 units up. Locating the mid-line gives $d = 2$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2\sin 4\left(x - \frac{\pi}{4}\right) + 2$.

Section 5.2 Page 253 Question 16

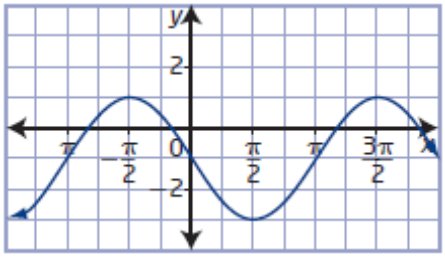
a) The amplitude, a, is $\frac{3-(-1)}{2}$, or 2.



The period is $\pi - 0$, or π. So, $b = 2$.
The start of the first cycle of the cosine curve is to the right of the y-axis. So, $c = \frac{\pi}{4}$.
The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.

So, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 2\cos 2\left(x - \frac{\pi}{4}\right) + 1$.

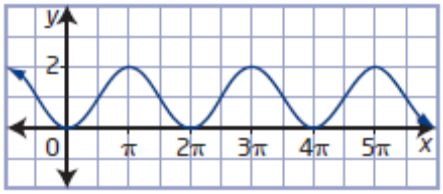
b) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.



The period is $\frac{3\pi}{2} - \left(-\frac{\pi}{2}\right)$, or 2π. So, $b = 1$.
The start of the first cycle of the cosine curve is to the left of the y-axis. So, $c = -\frac{\pi}{2}$.
The vertical displacement is 1 unit down. Locating the mid-line gives $d = -1$.

So, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 2\cos\left(x + \frac{\pi}{2}\right) - 1$.

c) The amplitude, a, is $\frac{2-0}{2}$, or 1.

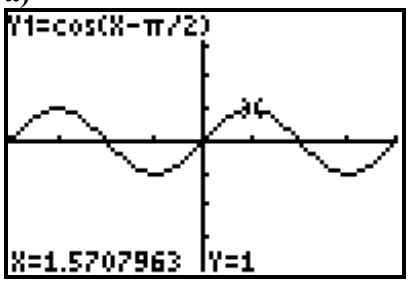


The period is $3\pi - \pi$, or 2π. So, $b = 1$.
The start of the first cycle of the cosine curve is to the right of the y-axis. So, $c = \pi$.

The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.
So, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = \cos(x - \pi) + 1$.

Section 5.2 Page 253 Question 17

a)



b) An equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = \sin x$.

c) The graph of the cosine function shifted $\frac{\pi}{2}$ units to the right is equivalent to the graph of the sine function.

Section 5.2 Page 253 Question 18

The graph of the sine function shifted $\frac{\pi}{2}$ units to the left is equivalent to the graph of the cosine function: $g(x) = \sin\left(x + \frac{\pi}{2}\right)$.

Section 5.2 Page 253 Question 19

a) i) For one cycle $30° \le x \le 390°$ of a cosine function in the form $y = 3 \cos b(x - c)$, the phase shift is 30° to the right, the period is 360°, and the x-intercepts are 120° and 300°.
ii) The minimum is located at (210°, –3) and the maximum is located at (30°, 3) and (390°, 3).

b) i) For one cycle $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$ of a cosine function in the form $y = 3 \cos b(x - c)$, the phase shift is $\frac{\pi}{4}$ to the right, the period is π, and the x-intercepts are $\frac{\pi}{2}$ and π.
ii) The minimum is located at $\left(\frac{3\pi}{4}, -3\right)$ and the maximum is located at $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 3\right)$.

Section 5.2 Page 253 Question 20

The amplitude, a, is $\frac{5100-5000}{2}$, or 50.

The period is 2.75 – 1.75, or 1 mi, which is equivalent to 5280 ft. So, $b = \frac{\pi}{2640}$.

The start of the first cycle of the cosine curve is 1.75 mi, or 9240 ft, to the right of the y-axis. So, $c = 9240$.

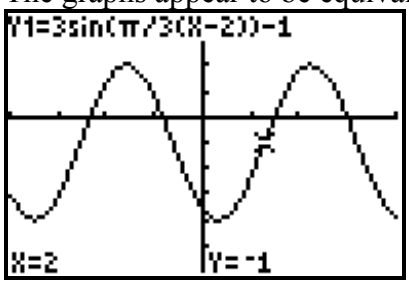
The vertical displacement is $\frac{5100+5000}{2}$, or 5050 units up. So, $d = 5050$.

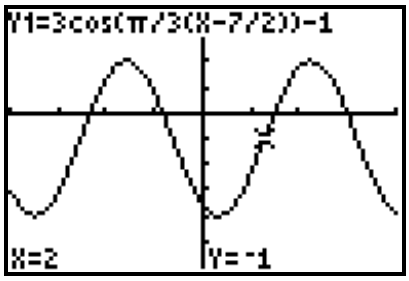
So, an equation in the form $y = a \cos b(x - c) + d$ for the pattern is

$y = 50\cos\frac{\pi}{2640}(x-9240)+5050$.

Section 5.2 Page 254 Question 21

The graphs appear to be equivalent.





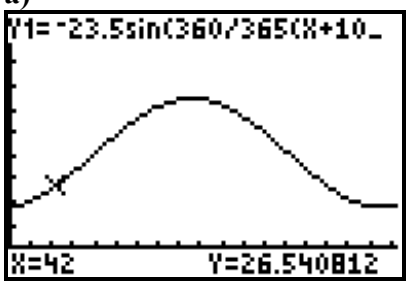
Section 5.2 Page 254 Question 22

An outside noise sound wave with amplitude 4 and period $\frac{\pi}{2}$ has equation $y = 4 \sin 4x$.

To cancel this, the headphones must produce a sound wave that is shifted by π units: $y = 4 \sin 4(x + \pi)$.

Section 5.2 Page 254 Question 23

a)

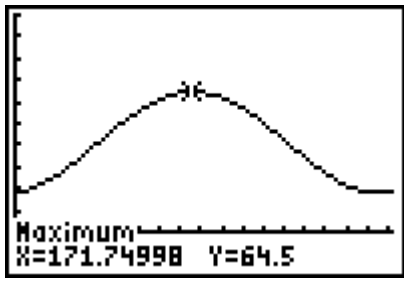


b) February 12 is day 42. Use the graph from part a) or the formula.

$$A = -23.5\sin\frac{360}{365}(x+102)+41$$
$$= -23.5\sin\frac{360}{365}(42+102)+41$$
$$= -23.5\sin\frac{51\,840}{365}+41$$
$$\approx 26.5$$

The sun's angle of inclination on February 12 is approximately 26.5°.

c) Find the maximum value of the graph. The angle of inclination is the greatest in Estevan on approximately day 171 or June 21.

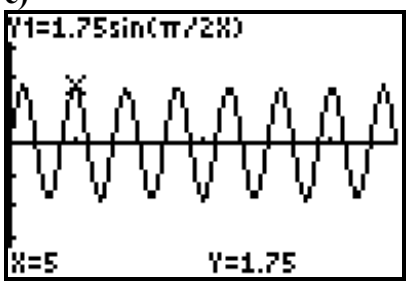


Section 5.2 Page 254 Question 24

a) The period of $r = 1.75\sin\frac{\pi}{2}t$ is $\frac{2\pi}{\left|\frac{\pi}{2}\right|}$, or 4. So, the time for one full respiratory cycle is 4 s.

b) The number of cycles per minute is $\frac{60}{4}$, or 15.

c)



d) Substitute $t = 30$.

$$r = 1.75\sin\frac{\pi}{2}t$$
$$= 1.75\sin\frac{\pi}{2}(30)$$
$$= 1.75\sin 15\pi$$
$$= 0$$

The rate of air flow at a time of 30 s is 0 L/s. In the context of the respiratory cycle, this corresponds to when the lungs are either completely full or completely empty.

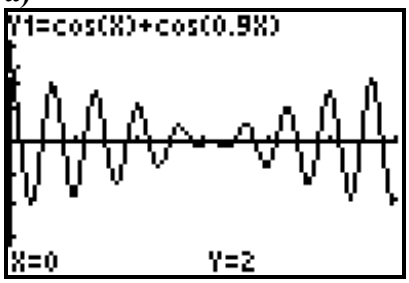
e) Substitute $t = 7.5$.

$$r = 1.75 \sin \frac{\pi}{2} t$$
$$= 1.75 \sin \frac{\pi}{2}(7.5)$$
$$= 1.75 \sin 3.75\pi$$
$$\approx -1.237$$

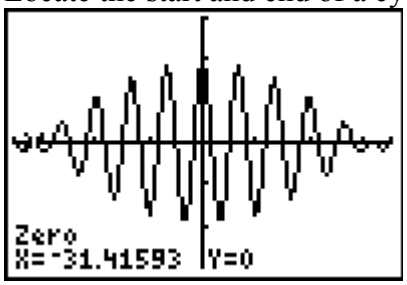
The rate of air flow at a time of 7.5 s is approximately –1.237 L/s. In the context of the respiratory cycle, this corresponds to a portion of the cycle when air is flowing out of the lungs.

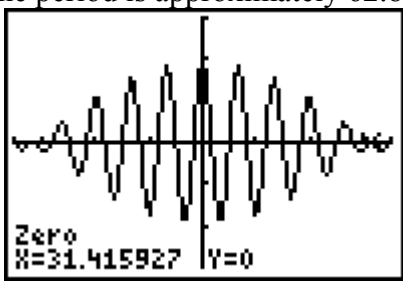
Section 5.2 Page 254 Question 25

a)



b) From the graph, the maximum value is 2 and the minimum value is approximately –2. So, the amplitude is 2.
Locate the start and end of a cycle. The period is approximately 62.8 or 20π.





Section 5.2 Page 254 Question 26

a) From question 17, $\sin x = \cos(x - 90°)$ or $\sin x = \cos\left(x - \frac{\pi}{2}\right)$.

i) Then, $4 \sin(x - 30°) = 4 \cos(x - \mathbf{120°})$.

ii) Then, $2 \sin\left(x - \frac{\pi}{4}\right) = \cos\left(x - \mathbf{\frac{3\pi}{4}}\right)$.

iii) Then, $-3\cos\left(x-\frac{\pi}{2}\right) = 3\sin(x-\pi)$.

iv) First rewrite, $\cos(-2x+6\pi)$ as $\cos(-2(x-3\pi))$. Use the fact that $\cos(-x) = \sin\left(x+\frac{\pi}{2}\right)$. Then, $\cos(-2x) = \sin 2\left(x+\frac{\pi}{4}\right)$.

$$\begin{aligned}\cos(-2(x-3\pi)) &= \sin 2\left(x+\frac{\pi}{4}-3\pi\right)\\ &= \sin 2\left(x-\frac{11\pi}{4}\right)\\ &= \sin 2\left(x-\frac{\pi}{4}\right)\end{aligned}$$

b) See explanation for part a) iv) above.

Section 5.2 Page 255 Question 27

Examples:

a) Given: sine function with amplitude 3, maximum $\left(-\frac{\pi}{2},5\right)$, and nearest maximum to the right at $\left(\frac{3\pi}{2},5\right)$.

$a = 3$

The period is $\frac{3\pi}{2}-\left(-\frac{\pi}{2}\right)$, or 2π. So, $b = 1$.

The maximum for a sine curve appears at one-quarter of the cycle, or $x = \frac{\pi}{2}$. So, $c = -\pi$.

maximum $= d + |a|$
$5 = d + |3|$
$d = 2$

An equation for the sine function is $y = 3\sin(x+\pi)+2$.

Alternatively, the maximum for a sine curve appears at three-quarters of the cycle to the left of the y-axis, or $x = -\frac{3\pi}{2}$. So, $c = \pi$. Then, an equation for the sine function is $y = 3\sin(x-\pi)+2$.

b) Given: sine function with amplitude 3, minimum $\left(\frac{\pi}{4},-2\right)$, and nearest maximum to the right at $\left(\frac{3\pi}{4},4\right)$.

$a = 3$

The distance between the minimum and the maximum of a sine function is one-half the period. The period is $2\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)$, or π. So, $b = 2$.

The minimum for a sine curve appears at three-quarters of the cycle, or $x = \frac{3\pi}{4}$.

So, $c = -\frac{\pi}{2}$.

minimum $= d - |a|$
$-2 = d - |3|$
$d = 1$

An equation for the sine function is $y = 3\sin 2\left(x+\frac{\pi}{2}\right)+1$.

Alternatively, the minimum for a sine curve appears at one-quarter of the cycle to the left of the y-axis, or $x = -\frac{\pi}{4}$. So, $c = \frac{\pi}{2}$. Then, an equation for the sine function is $y = 3\sin 2\left(x-\frac{\pi}{2}\right)+1$.

c) Given: sine function with minimum $(-\pi, 3)$ and nearest maximum to the right at $(0, 7)$.

The amplitude is $\frac{7-3}{2}$, or 2.

The distance between the minimum and the maximum of a sine function is one-half the period. The period is $2(0 - (-\pi))$, or 2π. So, $b = 1$.

The maximum for a sine curve appears at one-quarter of the cycle, or $x = \frac{\pi}{2}$. So, $c = -\frac{\pi}{2}$.

minimum $= d - |a|$
$3 = d - |2|$
$d = 5$

An equation for the sine function is $y = 2\sin\left(x+\frac{\pi}{2}\right)+5$.

Alternatively, the maximum for a sine curve appears at three-quarters of the cycle to the left of the y-axis, or $x = -\frac{3\pi}{2}$. So, $c = \frac{3\pi}{2}$. Then, an equation for the sine function is $y = 2\sin\left(x-\frac{3\pi}{2}\right)+5$.

d) Given: sine function with minimum $(90°, -6)$ and nearest maximum to the right at $(150°, 4)$.

The amplitude is $\frac{4-(-6)}{2}$, or 5.

The distance between the minimum and the maximum of a sine function is one-half the period. The period is $2(150° - 90°)$, or $120°$. So, $b = 3$.

The maximum for a sine curve appears at one-quarter of the cycle, or $x = 30°$. So, $c = 120°$.

minimum $= d - |a|$

$-6 = d - |5|$

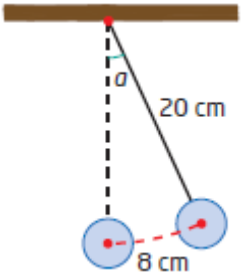
$d = -1$

An equation for the sine function is $y = 5 \sin 3(x - 120°) - 1$.

Alternatively, since the period is 120°, the first maximum to the right of the y-axis occurs at $x = 30°$. So, $c = 0$. Then, an equation for the sine function is $y = 5 \sin 3x - 1$.

Section 5.2 Page 255 Question 28

a) Given: $P = a \cos bt$, where a represents the maximum distance from its equilibrium and b is the horizontal stretch factor.



Use the formula for the length of an arc to determine a.

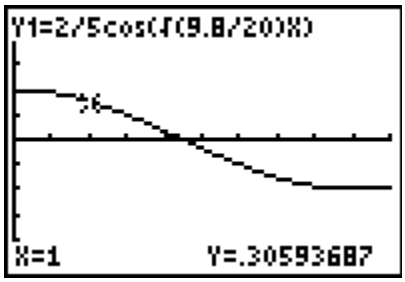
arc length = (central angle)(radius)

$8 = a(20)$

$a = \frac{8}{20}$

$a = \frac{2}{5}$

The period is given by $2\pi\sqrt{\frac{20}{9.8}}$.



So, $b = \frac{2\pi}{2\pi\sqrt{\frac{20}{9.8}}}$ or $\sqrt{\frac{9.8}{20}}$.

So, graph $P = \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}t$ for $0 \le t \le 5$.

b) Substitute $t = 6$.

$P = \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}t$

$= \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}(6)$

$= -0.196...$

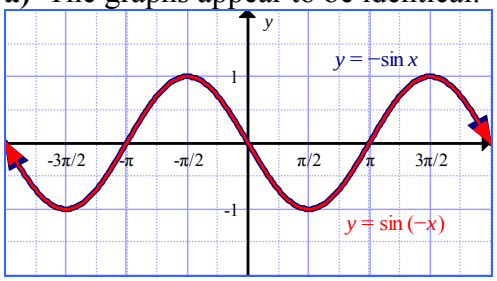
The position of the pendulum is approximately –0.20 radians, or 3.9 cm along the arc to the left of the vertical.

Section 5.2 Page 255 Question C1

For the graph of a sinusoidal function of the form $y = a \sin b(x - c) + d$,
• the parameter a changes the amplitude to $|a|$
• the parameter b changes the period to $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$
• the parameter c changes the phase shift
• the parameter d changes the vertical displacement
These parameters can be related to function transformations from Chapter 1:
• vertical stretch by a factor of $|a|$ and reflected in the x-axis if $a < 0$
• horizontal stretch by a factor of $\frac{1}{|b|}$ and reflected in the y-axis if $b < 0$
• horizontal translation of c units, to right if $c > 0$ and to left if $c < 0$
• vertical translation of d units, up if $d > 0$ and down if $d < 0$

Section 5.2 Page 255 Question C2

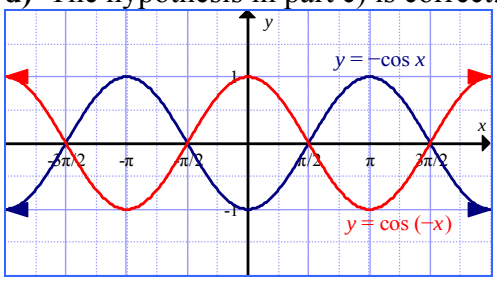
a) The graphs appear to be identical.



b) The graph of $y = -\sin x$ is a reflection in the x-axis of the graph of $y = \sin x$. The graph of $y = \sin(-x)$ is a reflection in the y-axis of the graph of $y = \sin x$. These result in the same graph.

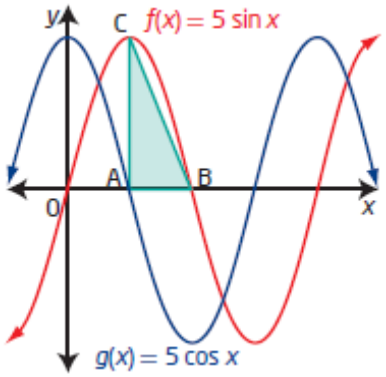
c) The graph of $y = -\cos x$ is a reflection in the x-axis of the graph of $y = \cos x$. The graph of $y = \cos(-x)$ is a reflection in the y-axis of the graph of $y = \cos x$. The two graphs will be reflections of each in the x-axis because the graph $y = \cos(-x)$ is the same as the graph $y = \cos x$.

d) The hypothesis in part c) is correct.



Section 5.2 Page 255 Question C3

Determine the height and base of the triangle.
The height, CA, is the amplitude of $f(x) = 5 \sin x$, or 5.
The base, AB, is the distance between the x-intercepts of the two graphs, $\pi - \frac{\pi}{2}$, or $\frac{\pi}{2}$.



Now, find the area of the triangle.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}\left(\frac{\pi}{2}\right)(5)$$
$$= \frac{5\pi}{4}$$

The area of $\triangle ABC$ is $\frac{5\pi}{4}$ units^2.

Section 5.2 Page 255 Question C4

Given: equation of a sine function $y = a \sin b(x - c) + d$, where $a > 0$ and $b > 0$.
a) For the period to be greater than 2π, $0 < b < 1$. The other parameters do not affect the period.

b) For the amplitude to be greater than 1 unit, $a > 1$. The other parameters do not affect the amplitude.

c) For the graph to pass through the origin, $c = 0$ and $d = 0$.

d) For the graph to have no x-intercepts, the entire graph must lie above or below the x-axis. Either $d - a > 0$ or $d + a < 0$, respectively. The other parameters do not affect the x-intercepts.

e) For the graph to have a y-intercept at a, $c = -\frac{\pi}{2}$ and $d = 0$.

f) For the length of one cycle to be 120°, $b = 3$. The other parameters do not affect the period.

Section 5.3 The Tangent Function

Section 5.3 Page 262 Question 1

a) $\tan\theta = \frac{1}{1}$ in quadrant I

$\tan\theta = 1$

$\theta = \tan^{-1} 1$

$\theta = 45°$

b) $\tan\theta = \frac{-1.7}{1}$ in quadrant II

$\tan\theta = -1.7$

$\theta = \tan^{-1}(-1.7)$

$\theta \approx 120.5°$

c) $\tan\theta = \frac{-1.7}{1}$ in quadrant IV

$\tan\theta = -1.7$

$\theta = \tan^{-1}(-1.7)$

$\theta \approx 300.5°$

d) $\tan\theta = \frac{1}{1}$ in quadrant III

$\tan\theta = 1$

$\theta = \tan^{-1} 1$

$\theta = 225°$

Section 5.3 Page 263 Question 2

a) $\tan\frac{\pi}{2}$ is undefined

b) $\tan\frac{3\pi}{4} = -1$

c) $\tan\left(-\frac{7\pi}{4}\right) = 1$

d) $\tan 0 = 0$

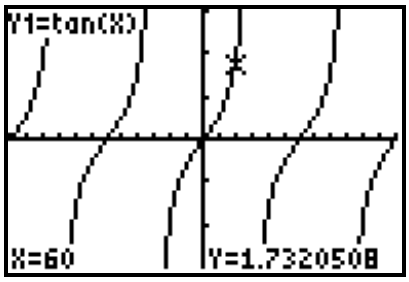
e) $\tan\pi = 0$

f) $\tan\frac{5\pi}{4} = 1$

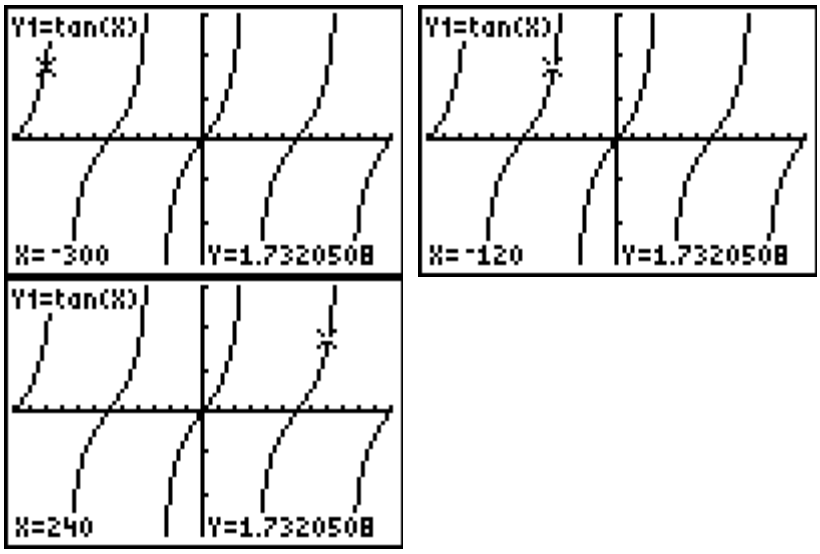
Section 5.3 Page 263 Question 3

No, $y = \tan x$ does not have an amplitude because it has no maximum or minimum values.

Section 5.3 Page 263 Question 4

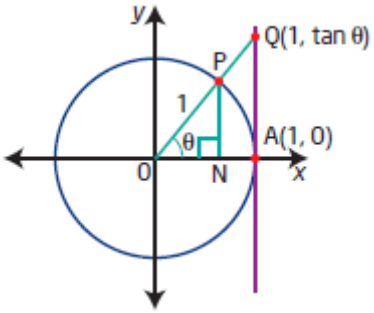


In the domain [–360°, 360°], I predict the other values of x with the same value as tan 60° will be –300°, –120°, and 240°.



Section 5.3 Page 263 Question 5

Given: △PON and △QOA are similar triangles.



Any point on the unit circle has coordinates (cos θ, sin θ).
Then, from properties of similar triangles

$$\frac{QA}{PN} = \frac{AO}{NO}$$

$$\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Section 5.3 Page 263 Question 6

a) For a terminal arm of angle θ that intersects the unit circle at P(x, y), slope = $\frac{y}{x}$.

b) Any point on the unit circle also has coordinates (cos θ, sin θ). So, the slope of the terminal arm can be expressed as slope = tan θ.

c) In terms of sine and cosine, slope = $\frac{\sin\theta}{\cos\theta}$.

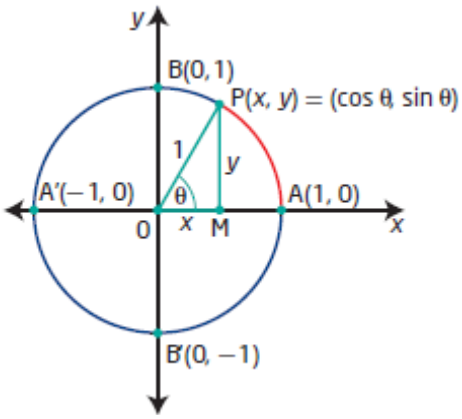
d) You can determine tan θ when the coordinates of point P are known by tan θ = $\frac{y}{x}$.

Section 5.3 Page 263 Question 7

a) In $\triangle POM$, $\tan\theta = \frac{y}{x}$.

b) Using $\cos\theta$ and $\sin\theta$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

c) From parts a) and b), $y = \sin\theta$ and $x = \cos\theta$.



Section 5.3 Page 264 Question 8

a)

θ	tan θ
89.5°	114.59
89.9°	572.96
89.999°	57 295.78
89.999 999°	57 295 779.51

b) As θ approaches 90°, tan θ approaches infinity.

c) I predict a similar result as θ approaches 90° from the other direction.

θ	tan θ
90.5°	−114.59
90.01°	−5729.58
90.001°	−57 295.78
90.000 001°	−57 295 779.51

As θ approaches 90° from the other direction, tan θ approaches negative infinity.

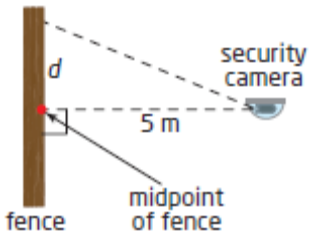
Section 5.3 Page 264 Question 9

a) From the diagram, $\tan\theta = \frac{d}{5}$.

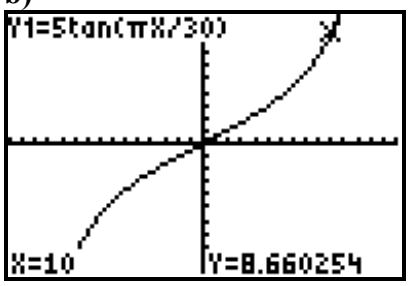
The period is 60 s. So, $\theta = \frac{2\pi}{60}t$ or $\frac{\pi}{30}t$. Then,

$\tan\frac{\pi}{30}t = \frac{d}{5}$

$d = 5\tan\frac{\pi}{30}t$



b)



c) Substitute $t = 10$.

$$d = 5 \tan \frac{\pi}{30} t$$
$$= 5 \tan \frac{\pi}{30} (10)$$
$$= 5 \tan \frac{\pi}{3}$$
$$= 5\sqrt{3}$$
$$\approx 8.7$$

The distance from the midpoint of the fence at $t = 10$ s is 8.7 m, to the nearest tenth of a metre.

d) When $t = 15$, $d = 5 \tan \frac{\pi}{2}$, which is undefined. The camera is pointing along a line parallel to the wall and is turning away from the wall.

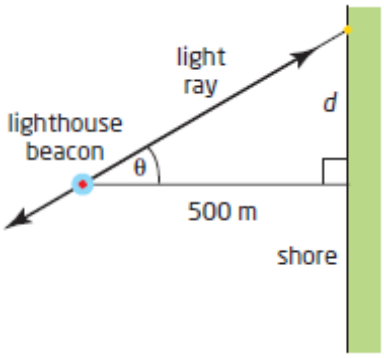
Section 5.3 Page 264 Question 10

a) From the diagram, $\tan \theta = \frac{d}{500}$.

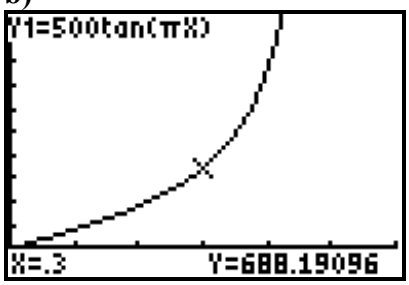
The period is 2 min. So, $\theta = \frac{2\pi}{2} t$ or πt. Then,

$$\tan \pi t = \frac{d}{500}$$
$$d = 500 \tan \pi t$$

The equation that expresses the distance, d, in metres, as a function of time, t, in minutes is $d = 500 \tan \pi t$.



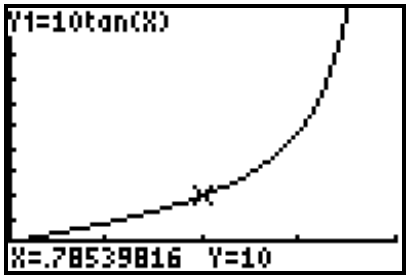
b)

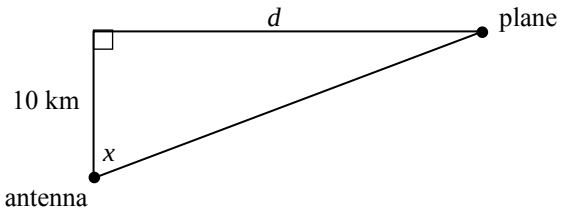


c) The asymptote in the graph at $\theta = 90°$, or $t = \frac{90}{\pi}$ represents the moment when the ray of light shines along a line that is parallel to the shore.

Section 5.3 Page 265 Question 11

$$\tan x = \frac{d}{10}$$
$$d = 10 \tan x$$





Section 5.3 Page 265 Question 12

a) Since Andrea measures the length of the shadow of a pole and calculates the slope of the string from the top of the pole to the ground, I would expect the graph representing her data to show a tangent function.

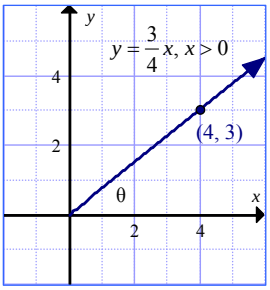
b) When the Sun is directly overhead and no shadow results, the slope of the string is undefined. This is represented by a vertical asymptote in the graph.

Section 5.3 Page 265 Question 13

a) Choose the integral point (4, 3).

b) From the diagram, $\tan \theta = \frac{3}{4}$.

c) The slope of the line is $\tan \theta$.



Section 5.3 Page 265 Question 14

a) Substitute $x = 0.5$.

Left Side

$\tan x$
$= \tan 0.5$
$= 0.546\,302...$
≈ 0.5463

Right Side

$x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315}$
$= 0.5 + \frac{0.5^3}{3} + \frac{2(0.5)^5}{15} + \frac{17(0.5)^7}{315}$
$= 0.546\,254...$
≈ 0.5463

Left Side = Right Side

b) Substitute $x = 0.5$.

Left Side	Right Side
$\sin x$	$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$
$= \sin 0.5$	$= 0.5 - \frac{0.5^3}{6} + \frac{0.5^5}{120} - \frac{0.5^7}{5040}$
$= 0.479\,425...$	$= 0.479\,525...$
≈ 0.4794	≈ 0.4795

Left Side ≈ Right Side

c) Substitute $x = 0.5$.

Left Side	Right Side
$\cos x$	$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$
$= \cos 0.5$	$= 1 - \frac{0.5^2}{2} + \frac{0.5^4}{24} - \frac{0.5^6}{720}$
$= 0.877\,582...$	$= 0.877\,582...$
≈ 0.8776	≈ 0.8776

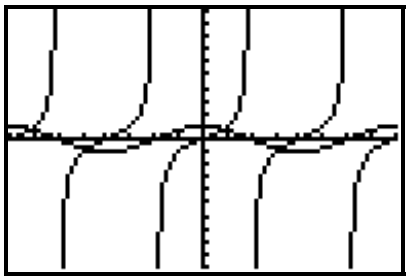
Left Side = Right Side

Section 5.3 Page 265 Question C1

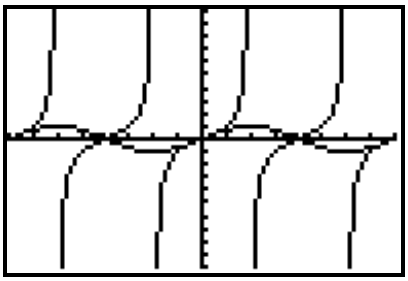
While the domain of both $\sin x$ and $\cos x$ is $\{x \mid x \in \mathrm{R}\}$, the domain of $\tan x$ is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$. The domains differ because $\tan x$ can be expressed as $\frac{\sin x}{\cos x}$. In this form, you can see that the domain is restricted to non-zero values of $\cos x$.

Section 5.3 Page 265 Question C2

a) The graph of the tangent function has asymptotes at the same locations as the x-intercepts of the graph of the cosine function.



b) The graph of the tangent function has x-intercepts at the same locations as the x-intercepts of the graph of the sine function.



Section 5.3 Page 265 Question C3

Example: A circular or periodic function repeats its values over a specific period. In the case of $y = \tan x$, the period is π. So, the equation $\tan (x + \pi) = \tan x$ is true for all values of x in the domain of $\tan x$.

Section 5.4 Equations and Graphs of Trigonometric Functions

Section 5.4 Page 275 Question 1

a) From the graph, the solutions to the equation $\sin x = 0$ in the interval $0 \le x \le 2\pi$ are $x = 0, \pi$, and 2π.

b) The general solution for $\sin x = 0$ is $x = n\pi$, $n \in \mathrm{I}$.

c) $\sin 3x = 0$
$\sin^{-1} 0 = 3x$
$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots$
For the interval $0 \le x \le 2\pi$,

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$$

Section 5.4 Page 275 Question 2

Examples:
a) From the graph, $x \approx 1.3$ and $x \approx 4.6$.

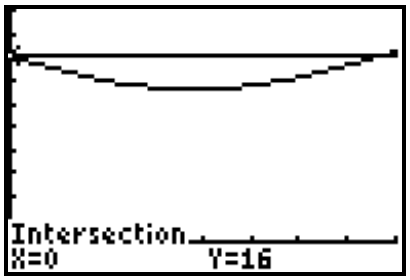
b) From the graph, $x \approx -3, -2, 0.1, 1.1, 3.2, 4.2, 6.3,$ and 7.3.

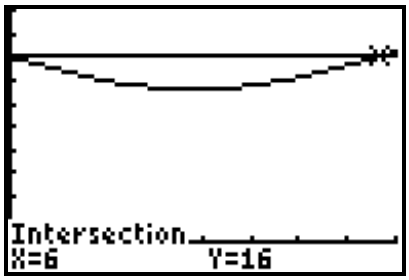
Section 5.4 Page 275 Question 3

From the graph, the solutions to the equation $4 \cos (2(x - 60°)) + 6 = 3$ are $x \approx -50°, -10°, 130°, 170°, 310°$, and $350°$.

Section 5.4 Page 275 Question 4

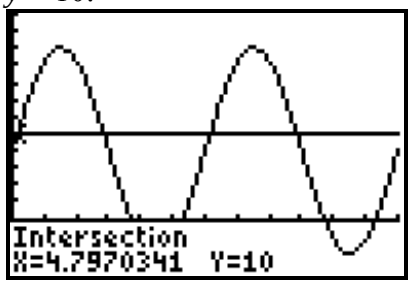
a) Determine the points of intersection for the graphs of $y = -2.8 \sin\left(\frac{\pi}{6}(x - 12)\right) + 16$ and $y = 16$.

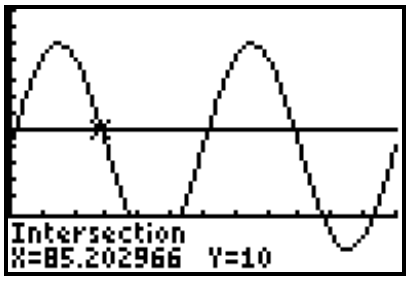


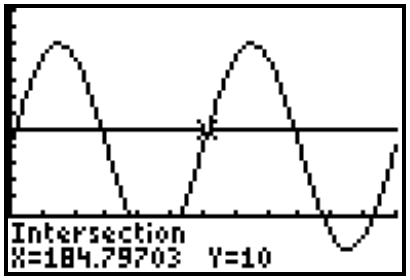


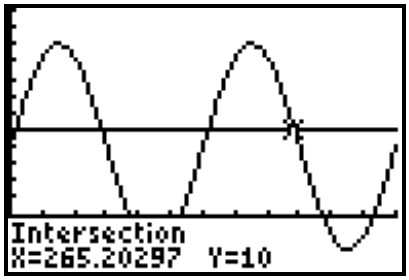
In the interval $0 \le x \le 2\pi$, the solutions are $x = 0$ and $x = 6$.

b) Determine the points of intersection for the graphs of $y = 12 \cos (2(x - 45°)) + 8$ and $y = 10$.



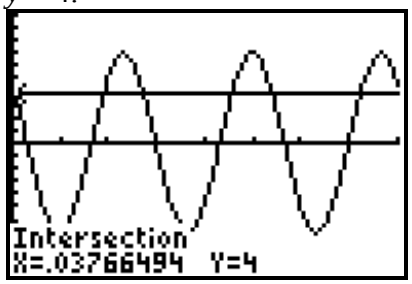


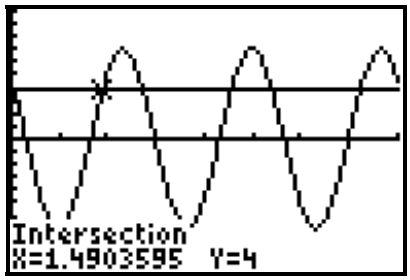


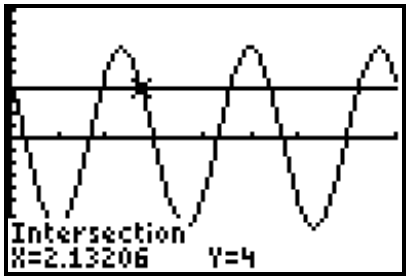


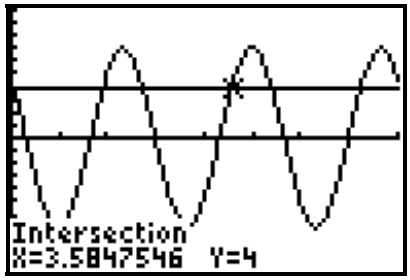
In the interval $0 \le x \le 360°$, the solutions are $x \approx 4.8°$, 85.2°, 184.8°, and 265.2°.

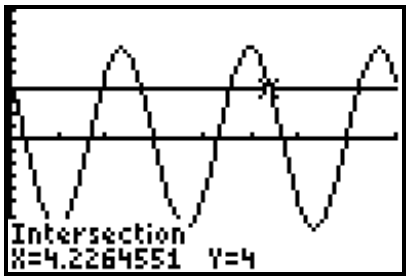
c) Determine the points of intersection for the graphs of $y = 7 \cos (3x - 18)$ and $y = 4$.

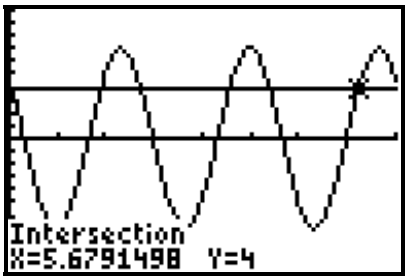






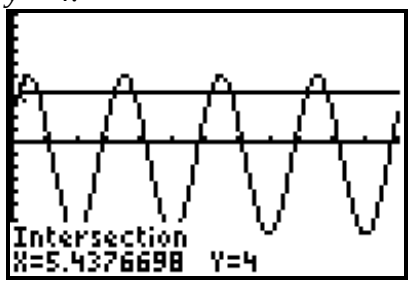


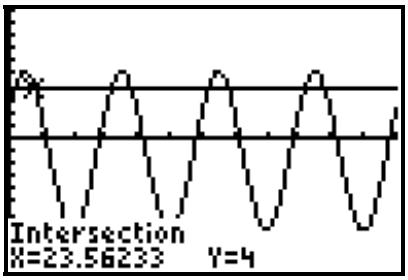


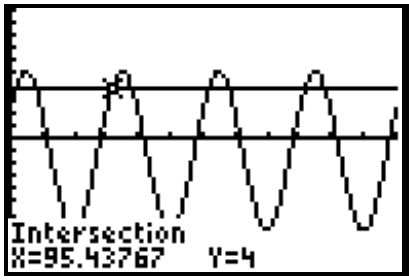


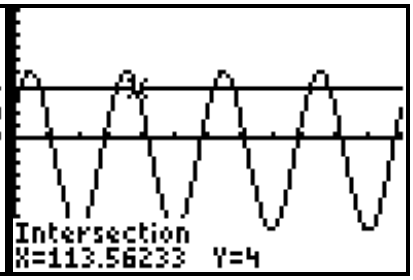
In the interval $0 \le x \le 2\pi$, the solutions are $x \approx 0.04, 1.49, 2.13, 3.58, 4.23$, and 5.68.

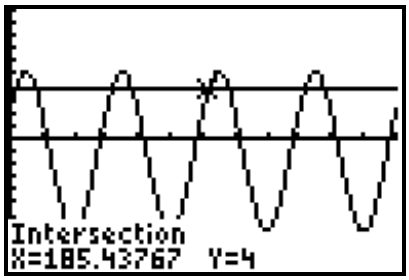
d) Determine the points of intersection for the graphs of $y = 6.2 \sin(4(x + 8°)) - 1$ and $y = 4$.

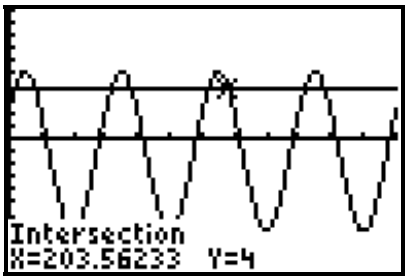


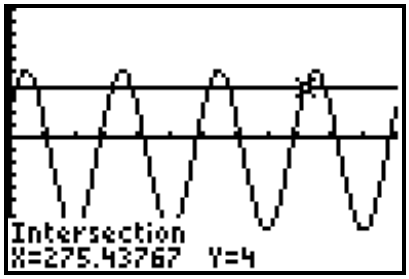


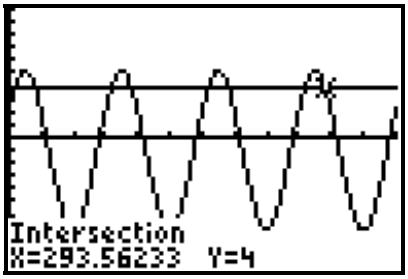








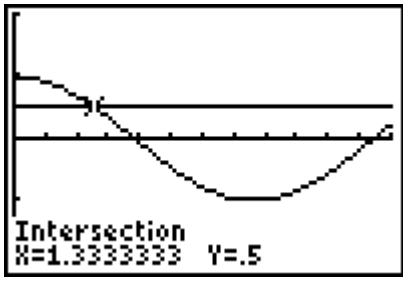




In the interval $0 \le x \le 360°$, the solutions are $x \approx 5.44°, 23.56°, 95.44°, 113.56°, 185.44°, 203.56°, 275.44°$, and $293.56°$.

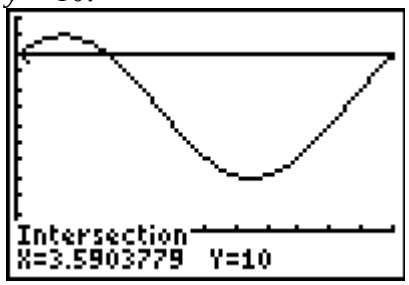
Section 5.4 Page 275 Question 5

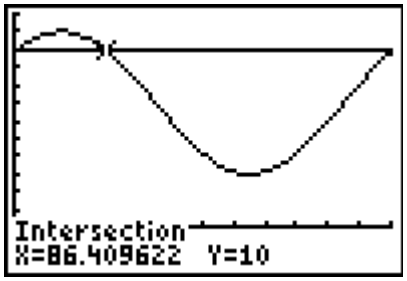
a) Determine the points of intersection for the graphs of $y = \sin\left(\frac{\pi}{4}(x-6)\right)$ and $y = 0.5$.



In the interval $0 \le x \le 2\pi$, the solution is $x = \frac{4}{3}$ or approximately 1.33.

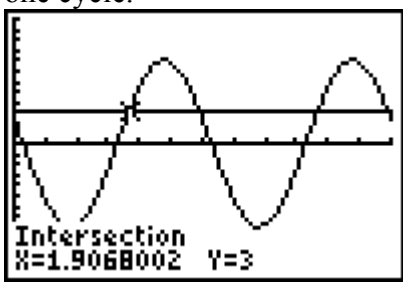
b) Determine the points of intersection for the graphs of $y = 4\cos(x - 45°) + 7$ and $y = 10$.

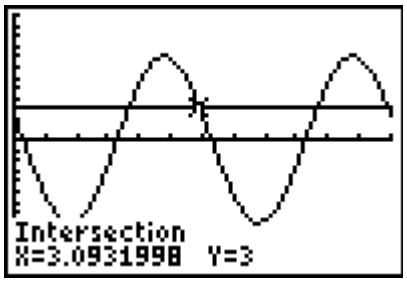




In the interval $0 \le x \le 360°$, the solutions are $x \approx 3.59°$ and $86.41°$.

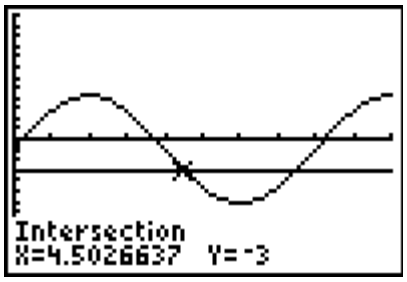
c) Determine the points of intersection for the graphs of $y = 8\cos(2x - 5)$ and $y = 3$ in one cycle.

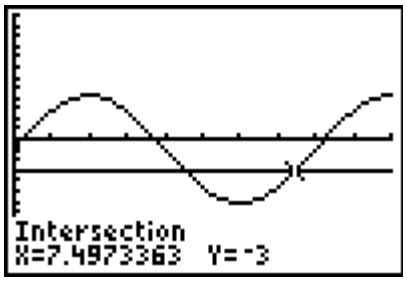




The general solution is $x \approx 1.91 + \pi n$ and $x \approx 3.09 + \pi n$, where $n \in \mathrm{I}$.

d) Determine the points of intersection for the graphs of $y = 5.2\sin(45(x + 8°)) - 1$ and $y = -3$ in one cycle.





The general solution is $x \approx 4.5° + (8°)n$ and $x \approx 7.5° + (8°)n$, where $n \in \text{I}$.

Section 5.4 Page 276 Question 6

a) For the population of a lakeside town with large numbers of seasonal residents that is modelled by the function $P(t) = 6000 \sin (t - 8) + 8000$, a possible domain is $\{t \mid t \geq 0, t \in \text{R}\}$ and a possible range is $\{P \mid 2000 \leq P \leq 14\,000, \text{P} \in \text{N}\}$.

b) For the height of the tide on a given day that is modelled using the function $h(t) = 6 \sin (t - 5) + 7$, a possible domain is $\{t \mid t \geq 0, t \in \text{R}\}$ and a possible range is $\{h \mid 1 \leq h \leq 13, h \in \text{R}\}$.

c) For the height above the ground of a rider on a Ferris wheel that can be modelled by $h(t) = 6 \sin 3(t - 30) + 12$, a possible domain is $\{t \mid t \geq 0, t \in \text{R}\}$ and a possible range is $\{h \mid 6 \leq h \leq 18, h \in \text{R}\}$.

d) For the average daily temperature that may be modelled by the function $h(t) = 9 \cos \frac{2\pi}{365}(t - 200) + 14$, a possible domain is $\{t \mid t \geq 0, t \in \text{R}\}$ and a possible range is $\{h \mid 5 \leq h \leq 23, h \in \text{R}\}$.

Section 5.4 Page 276 Question 7

If the fly's wings flap 200 times in one second, the period of the musical note is $\frac{1}{200}$ s.

Section 5.4 Page 276 Question 8

a) Given: sine function with first maximum at (30°, 24) and first minimum to the right of the maximum at (80°, 6)

period: $2(80° - 30°) = 100°$ amplitude: $\frac{24 - 6}{2} = 9$ sinusoidal axis: $y = 6 + 9 = 15$

b) Given: cosine function with first maximum at (0, 4) and first minimum to the right of the maximum at $\left(\frac{2\pi}{3}, -16\right)$

period: $2\left(\frac{2\pi}{3}-0\right)=\frac{4\pi}{3}$ amplitude: $\frac{4-(-16)}{2}=10$

sinusoidal axis: $y = -16 + 10 = -6$

c) Given: electron oscillates back and forth 50 times per second, the maximum occurs at +10, and the minimum occurs at –10

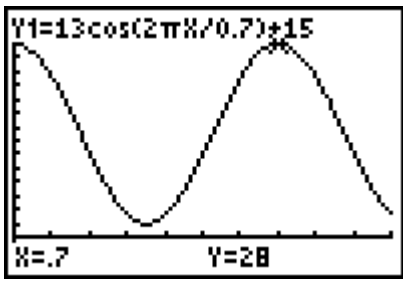
period: $\frac{1}{50}$ s amplitude: $\frac{10-(-10)}{2}=10$ sinusoidal axis: $y = -10 + 10 = 0$

Section 5.4 Page 276 Question 9

Given: $h(t) = 13\cos\left(\frac{2\pi}{0.7}t\right) + 15$

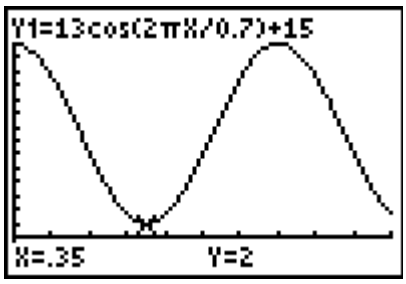
a) The maximum height of the point is 15 + 13, or 28 m.

b) The maximum height is reached at 0 min, 0.7 min, 1.4 min ….

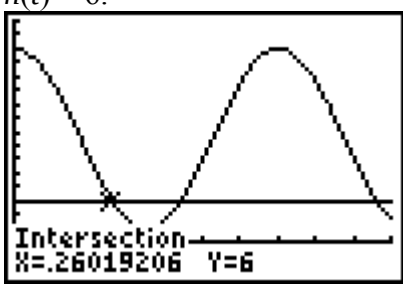


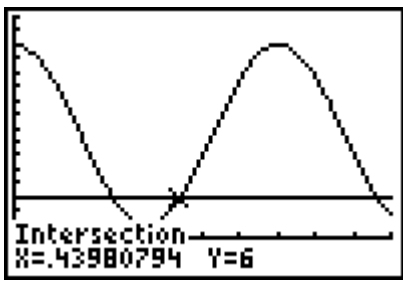
c) The minimum height of the point is 15 – 13, or 2 m.

d) The minimum height is reached at 0.35 min, 1.05 min, 1.75 min ….



e) Determine the points of intersection for the graphs of $h(t) = 13\cos\left(\frac{2\pi}{0.7}t\right) + 15$ and $h(t) = 6$.





Within one cycle, the point is less than 6 m above the ground for approximately 0.44 – 0.26, or 0.18 min.

f) Determine the height at 1 h 12 min, or 72 min.

$$h(t) = 13 \cos\left(\frac{2\pi}{0.7}t\right) + 15$$

$$h(72) = 13 \cos\left(\frac{2\pi}{0.7}(72)\right) + 15$$

$h(72) = 23.105\ldots$

The height of the point if the wheel is allowed to turn for 1 h 12 min is approximately 23.1 m.

Section 5.4 Page 276 Question 10

Substitute $t = 17.5$.

$$h(t) = 59 + 24 \sin 125t$$

$$h(17.5) = 59 + 24 \sin 125(17.5)$$

$h(17.5) = 78.54\ldots$

The height of the mark, to the nearest tenth of a centimetre, when $t = 17.5$ s, is 78.5 cm.

Section 5.4 Page 276 Question 11

For a sinusoidal function that oscillates between –155 V and +155 V and makes 60 complete cycles each second, $a = 155$, $b = 120\pi$, $c = 0$, and $d = 0$. An equation for the voltage as a function of time, t is $V = 155 \sin 120\pi t$.

Section 5.4 Page 277 Question 12

a) The period of the satellite is $\frac{2\pi}{28\pi}$, or $\frac{1}{14}$ days.

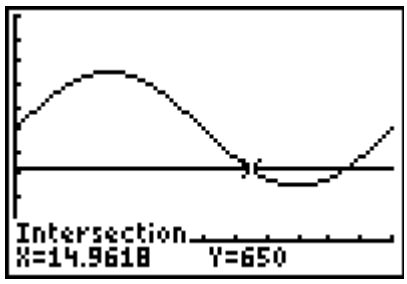
b) It will take the satellite $\frac{1}{14}(24)(60)$, or approximately 102.9 min to orbit Earth.

c) The satellite will make $\frac{60(24)}{102.9}$, or approximately 14 orbits per day.

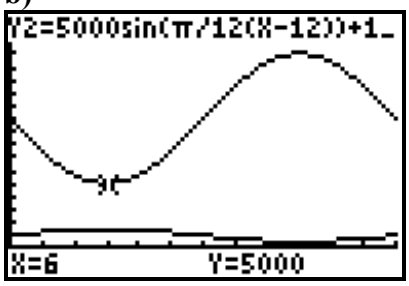
Section 5.4 Page 277 Question 13

a) Determine the point of intersection of the graphs of $F(t) = 500 \sin \frac{\pi}{12}t + 1000$ and $F(t) = 650$.

It takes approximately 15 months for the fox population to drop to 650.



b)



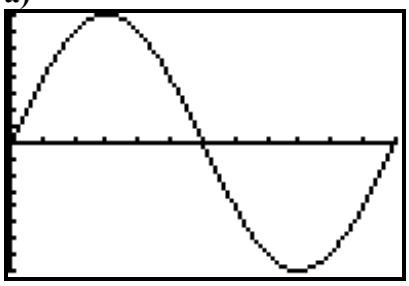
c)

	Arctic Fox	Lemming
Maximum Population	1500	15 000
Month	6	18
Minimum Population	500	5000
Month	18	6

d) Example: The maximum for the predator occurs at a minimum for the prey and vice versa. The predators population depends on the prey, so every time the lemming population changes the arctic fox population changes in accordance.

Section 5.4 Page 277 Question 14

a)



b) Substitute $t = 2.034$.

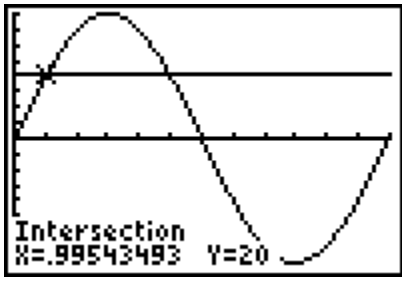
$h = 40 \sin 0.526t$

$= 40 \sin 0.526(2.034)$

$= 35.085\ldots$

The guest will have swayed approximately 35.1 cm after 2.034 s.

c) Determine the point of intersection of the graphs $h = 40 \sin 0.526t$ and $h = 20$. It will take approximately 1 s for the guest to sway 20 cm.



Section 5.4 Page 278 Question 15

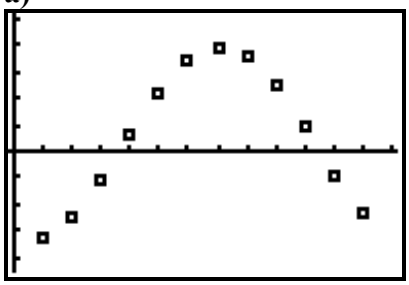
a) From the graph, the maximum height of the Sun is approximately 7.5 Sun widths and the minimum height of the Sun is approximately 1 Sun width.

b) The period is 24 h.

c) Since the amplitude is 3.25, $a = -3.25$. Since the period is 24, $b = \frac{\pi}{12}$. The sinusoidal axis is 1+ 3.25, or $d = 4.25$. Then, the sinusoidal equation that models the midnight Sun is $y = -3.25 \sin \frac{\pi}{12}x + 4.25$.

Section 5.4 Page 278 Question 16

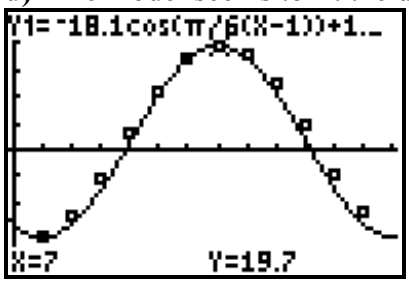
a)



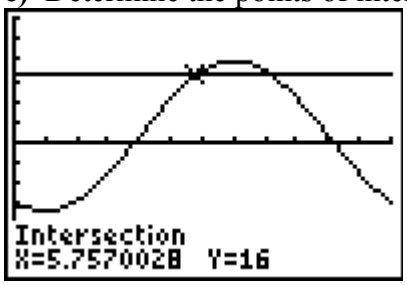
b) The temperature that is halfway between the maximum and minimum average monthly temperatures is $\frac{19.7 + (-16.5)}{2}$, or 1.6 °C.

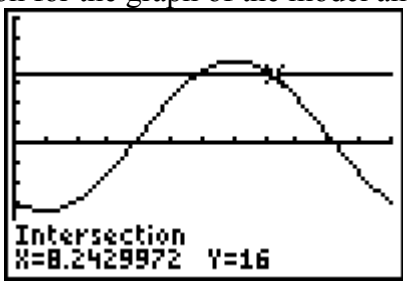
c) The best fit curve is of the form $y = -\cos x$. Since the amplitude is $\frac{19.7 - (-16.5)}{2} = 18.1$, $a = 18.1$. Since the period is 12 months, $b = \frac{\pi}{6}$. Since the minimum occurs in January, $c = 1$. From part b), $d = 1.6$. Then, a sinusoidal function to model the temperature for Winnipeg is $T = -18.1 \cos \frac{\pi}{6}(M - 1) + 1.6$, where T is the average monthly temperature, in degrees Celsius, and M is the time, in months.

d) The model seems to fit the data relatively well.



e) Determine the points of intersection for the graph of the model and $T = 16$.





The average monthly temperature in Winnipeg is greater than or equal to 16 °C for approximately 8.24 – 5.78, or about 2.5 months.

Section 5.4 Page 278 Question 17

a) The best fit curve is of the form $y = -\cos x$. Use the turn on and off temperatures to determine $a = \frac{43-34}{2}$, or 4.5. The period is 60 min, so $b = \frac{\pi}{30}$. The sinusoidal axis is $d = \frac{43+34}{2}$, or 38.5. Then, the equation that expresses temperature as a function of time is $T = -4.5 \cos \frac{\pi}{15} t + 38.5$.

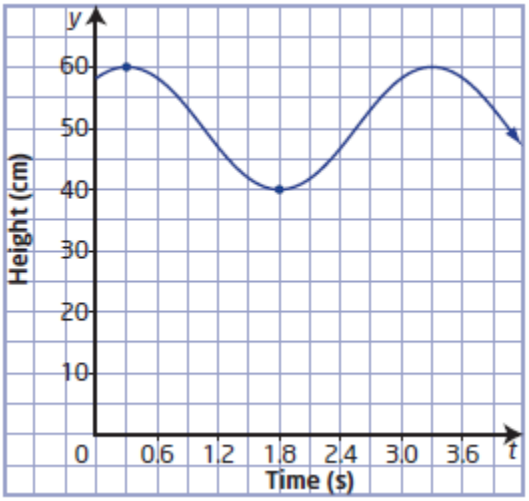
b) Substitute $t = 10$.

$$\begin{aligned} T &= -4.5 \cos \frac{\pi}{30} t + 38.5 \\ &= -4.5 \cos \frac{\pi}{30}(10) + 38.5 \\ &= -4.5 \cos \frac{\pi}{3} + 38.5 \\ &= 36.25 \end{aligned}$$

The temperature 10 min after the heater turns on is 36.25 °C.

Section 5.4 Page 279 Question 18

a)



b) Example: From the graph, $a = 10$, $b = \frac{2\pi}{3}$, $c = 0.3$, and $d = 50$. Then, the cosine equation for the distance, d, in centimetres, from the floor as a function of time, t, in seconds, is $d = 10 \cos \frac{2\pi}{3}(t - 0.3) + 50$.

c) Substitute $t = 17.2$.

$$d = 10 \cos \frac{2\pi}{3}(t - 0.3) + 50$$

$$= 10 \cos \frac{2\pi}{3}(17.2 - 0.3) + 50$$

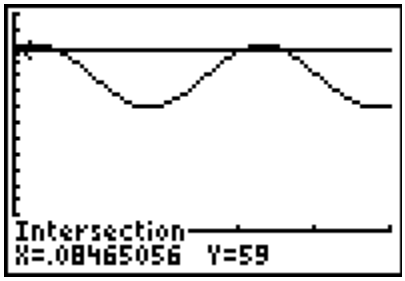
$$= 43.308\ldots$$

The distance from the floor when the stopwatch reads 17.2 s is approximately 43.3 cm.

d) Determine the first point of intersection of the graphs of

$d = 10 \cos \frac{2\pi}{3}(t - 0.3) + 50$ and $d = 59$.

The first time when the mass is 59 cm above the floor is approximately 0.085 s.



Section 5.4 Page 279 Question 19

a) The best fit curve is of the form $y = -\cos x$. Since the radius is 10 m, $a = 10$. The period is 60 s, so $b = \frac{\pi}{30}$. Since the minimum height is 2 m, $d = 2 + 10$, or 12. Then, an equation to model the path of a passenger on the Ferris wheel, where the height, h, in metres, is a function of time, t, in seconds is $h = -10 \cos \frac{\pi}{30}t + 12$.

b) 2.3 min = 138 s

Substitute $t = 138$.

$$h = -10 \cos \frac{\pi}{30}t + 12$$

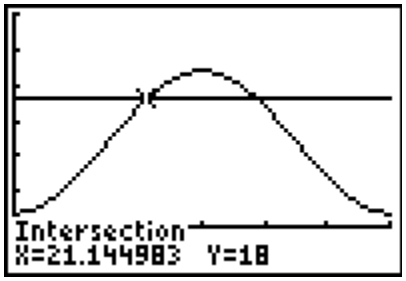
$$= -10 \cos \frac{\pi}{30}(138) + 12$$

$$= 15.090\ldots$$

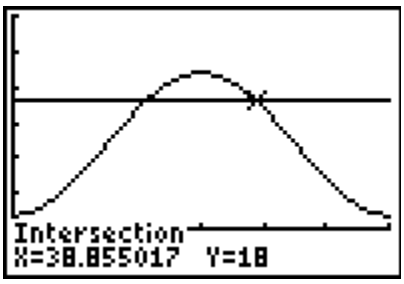
Emily's height above the ground, to the nearest tenth of a metre, when the wheel has been in motion for 2.3 min is 15.1 m.

c) Determine the first point of intersection of the graphs of $h = -10 \cos \frac{\pi}{30}t + 12$ and $h = 18$.

The amount of time that passes before a rider reaches a height of 18 m for the first time is approximately 21.1 s.



Another time when the rider is at that same height within the first cycle is approximately 38.9 s.



Section 5.4 Page 279 Question 20

a) Use a cosine function. From the maximum and minimum heights, $a = \frac{22-8}{2}$, or 7.

The period is 5 s, so $b = \frac{2\pi}{5}$. The minimum is reached at 2 s, but the minimum of a cosine curve typically occurs at the halfway point in the period, or 2.5, then $c = -0.5$. Since the minimum height is 8 m, $d = 8 + 7$, or 15. Then, an equation to model the rotation of the tip of the blade, where the height, h, in metres, is a function of time, t, in seconds is $h = 7 \cos \frac{2\pi}{5}(t + 0.5) + 15$.

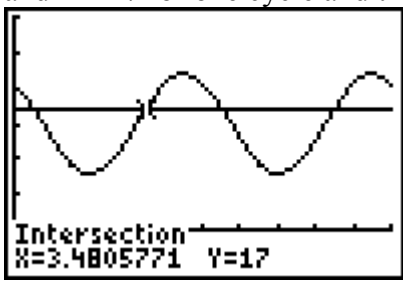
An equivalent sine function is $h = 7 \sin \frac{2\pi}{5}(t + 1.75) + 15$.

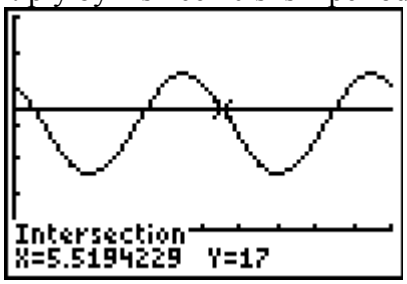
b) Substitute $t = 4$.

$$h = 7 \cos \frac{2\pi}{5}(t + 0.5) + 15$$
$$= 7 \cos \frac{2\pi}{5}(4 + 0.5) + 15$$
$$= 20.663\ldots$$

The height of the tip of the blade after 4 s is approximately 20.7 m.

c) Determine the points of intersection of the graphs of $h = 7 \cos \frac{2\pi}{5}(t + 0.5) + 15$ and $h = 17$ for one cycle and then multiply by 2 since 10 s is 2 periods.





$2(5.5194 - 3.4806) = 4.0776$

The tip of the blade above a height of 17 m in the first 10 s is approximately 4.08 s.

Section 5.4 Page 280 Question 21

a) The best fit curve is of the form $y = -\cos x$. From the maximum and minimum temperatures, $a = \frac{23.6 - 4.2}{2}$, or 9.7. The period is 366 days, so $b = \frac{\pi}{183}$. The minimum is reached on day 26, but the minimum of a reflected cosine curve typically occurs at the start point in the period, so $c = 26$. Since the minimum temperature is 4.2 °C, $d = 4.2 + 9.7$, or 13.9. Then, an equation to model the temperature, T, in degrees Celsius, is a function of time, t, in days, is

$$T = -9.7 \cos \frac{\pi}{183} (t - 26) + 13.9.$$

b) Substitute $t = 147$.

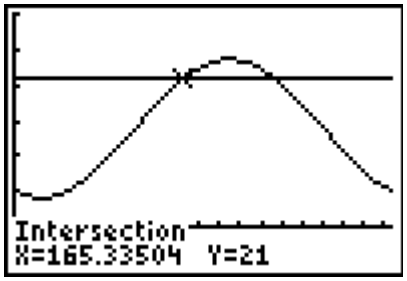
$$T = -9.7 \cos \frac{\pi}{183} (t - 26) + 13.9$$

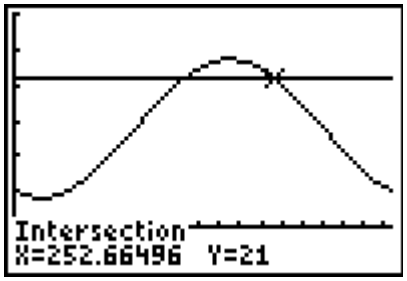
$$= -9.7 \cos \frac{\pi}{183} (147 - 26) + 13.9$$

$$= 18.605\ldots$$

The temperature for day 147, May 26, is approximately 18.6 °C.

c) Determine the points of intersection for the graphs of $T = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$ and $T = 21.0$.



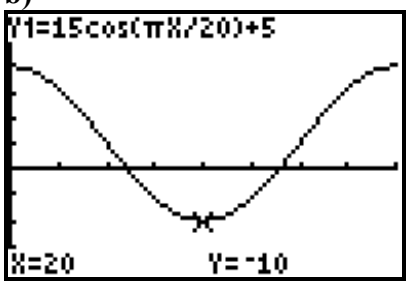


A maximum temperature of 21.0 °C or higher is expected for approximately 253 – 165, or 88 days.

Section 5.4 Page 280 Question 22

a) Use a cosine function. From the maximum and minimum cash flow percents, $a = \frac{20 - (-10)}{2}$, or 15. The period is 40 years, so $b = \frac{\pi}{20}$. Since the minimum percent is –10, $d = -10 + 15$, or 5. Then, an equation to model the cash flow, C, in percent of total assets, is a function of time, t, in years, is $C = 15 \cos \frac{\pi}{20} t + 5$.

b)



c) For 2008, substitute $t = 88$.

$$C = 15 \cos \frac{\pi}{20} t + 5$$

$$= 15 \cos \frac{\pi}{20} (88) + 5$$

$$= 9.635\ldots$$

The cash flow for the company in 2008 is approximately 9.6% of total assets.

d) Example: No. I would not invest with this company, because too great a portion of the cycle has losses.

Section 5.4 Page 280 Question 23

a) Use a sine function. Since the distance of the turns from the midline is 1.2 m, $a = 1.2$. For 10 turns in 20 s, the period is 4 s (2 turns per cycle) and $b = \frac{\pi}{2}$. Then, the function that models the path of the skier is $y = 1.2 \sin \frac{\pi}{2} t$, where t represents the time, in seconds, and y represents the distance for one turn, in metres, from the midline.

b) If the skier made only eight turns in 20 s, then the period would change to 5 s and equation becomes $y = 1.2 \sin \frac{2\pi}{5} t$.

Section 5.4 Page 280 Question C1

Examples:

a) Use a sine function as a model when the curve or data begins at or near the intersection of the vertical axis and the sinusoidal axis.

b) Use a cosine function as a model when the curve or data has a maximum or minimum near or at the vertical axis.

Section 5.4 Page 280 Question C2

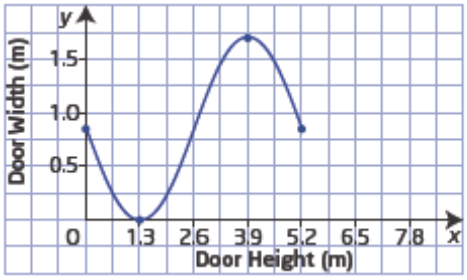
Examples:
a) $y = a \sin b(x - c) + d$
The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

b) $y = a \cos b(x - c) + d$
The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

Section 5.4 Page 281 Question C3

a) The best fit curve is of the form $y = -\sin x$. Since the amplitude is half the width of the door, $a = 0.85$. Since the period is the height of the door, $b = \frac{2\pi}{5.2}$. Then, a sinusoidal function that could model the shape of the open door is $y = -0.85 \sin \frac{2\pi}{5.2}x + 0.85$, where x represents the height of the door, in metres, and y represents the width of the door, in metres.

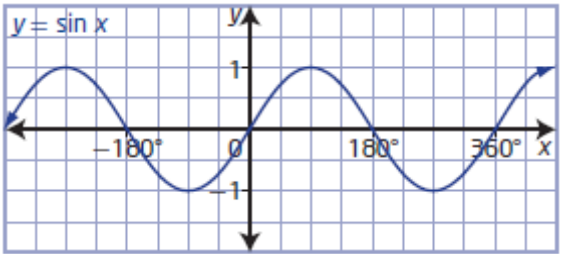
b)



Chapter 5 Review

Chapter 5 Review Page 282 Question 1

a) The x-intercepts in the interval $-360° \le x \le 360°$ are $-360°$, $-180°$, $0°$, $180°$, and $360°$.



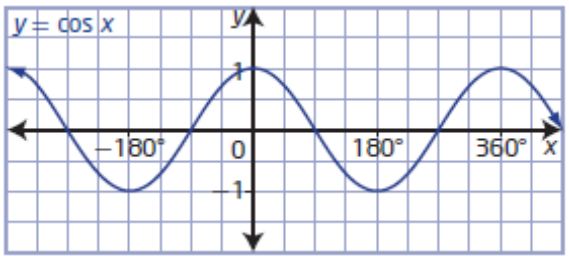
b) The y-intercept is 0.

c) The domain is $\{x \mid x \in \mathrm{R}\}$. The range is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$. The period is 360°.

d) The greatest value of $y = \sin x$ is 1.

Chapter 5 Review Page 282 Question 2

a) The x-intercepts in the interval $-360° \le x \le 360°$ are $-270°$, $-90°$, $0°$, $90°$, and $270°$.



b) The y-intercept is 1.

c) The domain is $\{x \mid x \in \mathrm{R}\}$. The range is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$. The period is 360°.

d) The greatest value of $y = \cos x$ is 1.

Chapter 5 Review Page 282 Question 3

a) For $y = \sin x$, the amplitude is 1 and the period is 2π. This matches graph **A**.

b) For $y = \sin 2x$, the amplitude is 1 and the period is π. This matches graph **D**.

c) For $y = -\sin x$, the amplitude is 1, the period is 2π, and the graph is reflected in the x-axis. This matches graph **B**.

d) For $y = \frac{1}{2}\sin x$, the amplitude is $\frac{1}{2}$ and the period is 2π. This matches graph **C**.

Chapter 5 Review Page 282 Question 4

a) For $y = -3 \sin 2x$, $a = -3$ and $b = 2$. So, the amplitude is $|-3| = 3$ and the period is $\frac{2\pi}{2} = \pi$ or $\frac{360°}{2} = 180°$.

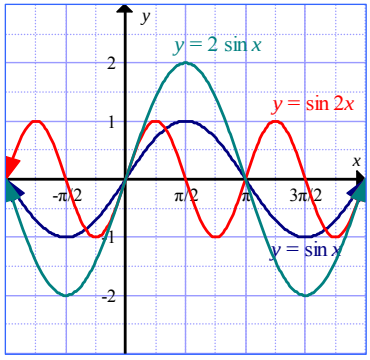
b) For $y = 4 \cos 0.5x$, $a = 4$ and $b = 0.5$. So, the amplitude is $|4| = 4$ and the period is $\frac{2\pi}{0.5} = 4\pi$ or $\frac{360°}{0.5} = 720°$.

c) For $y = \frac{1}{3}\sin\frac{5}{6}x$, $a = \frac{1}{3}$ and $b = \frac{5}{6}$. So, the amplitude is $\left|\frac{1}{3}\right| = \frac{1}{3}$ and the period is $\frac{2\pi}{\frac{5}{6}} = \frac{12\pi}{5}$ or $\frac{360°}{\frac{5}{6}} = 432°$.

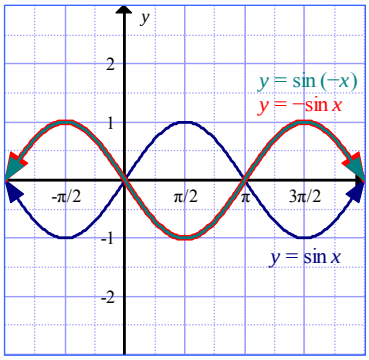
d) For $y = -5\cos\frac{3}{2}x$, $a = -5$ and $b = \frac{3}{2}$. So, the amplitude is $|-5| = 5$ and the period is $\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$ or $\frac{360°}{\frac{3}{2}} = 240°$.

Chapter 5 Review Page 282 Question 5

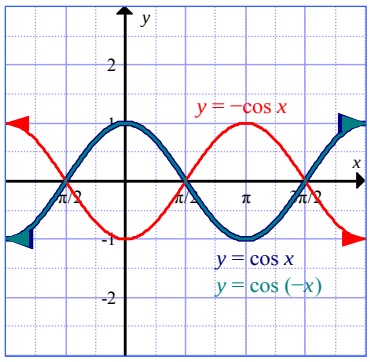
a) The graph of $y = \sin x$ has an amplitude of 1 and a period of 2π. The graph of $y = \sin 2x$ has the same amplitude as the graph of $y = \sin x$, but a period of π. The graph of $y = 2\sin x$ has an amplitude of 2 and the same period as the graph of $y = \sin x$.



b) The graph of $y = \sin x$ has an amplitude of 1 and a period of 2π. The graph of $y = -\sin x$ has the same amplitude and period as the graph of $y = \sin x$, but is reflected in the x-axis. The graph of $y = \sin(-x)$ has the same amplitude and period as the graph of $y = \sin x$, but is reflected in the y-axis.



c) The graph of $y = \cos x$ has an amplitude of 1 and a period of 2π. The graph of $y = -\cos x$ has the same amplitude and period as the graph of $y = \cos x$, but is reflected in the x-axis. The graph of $y = \cos(-x)$ has the same amplitude and period as the graph of $y = \cos x$, but is reflected in the y-axis.



Chapter 5 Review Page 282 Question 6

a) For a cosine function with an amplitude of 3 and a period of π, $a = 3$ and $b = 2$. Then, the equation of the function in the form $y = a \cos bx$ is $y = 3 \cos 2x$.

b) For a cosine function with an amplitude of 4, $a = 4$.

For a period of 150°, $b = \frac{360°}{150°}$.

$= \frac{12}{5}$

Then, the equation of the function in the form $y = a \cos bx$ is $y = 4 \cos \frac{12}{5}x$.

c) For a cosine function with an amplitude of $\frac{1}{2}$ and a period of 720°, $a = \frac{1}{2}$ and $b = \frac{1}{2}$.

Then, the equation of the function in the form $y = a \cos bx$ is $y = \frac{1}{2} \cos \frac{1}{2}x$.

d) For a cosine function with an amplitude of $\frac{3}{4}$, $a = \frac{3}{4}$.

For a period of $\frac{\pi}{6}$, $b = \frac{2\pi}{\frac{\pi}{6}}$.

$= 12$

Then, the equation of the function in the form $y = a \cos bx$ is $y = \frac{3}{4} \cos 12x$.

Chapter 5 Review Page 283 Question 7

a) For a sine function with an amplitude of 8 and a period of 180°, $a = 8$ and $b = 2$. Then, the equation of the function in the form $y = a \sin bx$ is $y = 8 \sin 2x$.

b) For a sine function with an amplitude of 0.4 and a period of 60°, $a = 0.4$ and $b = 6$. Then, the equation of the function in the form $y = a \sin bx$ is $y = 0.4 \sin 6x$.

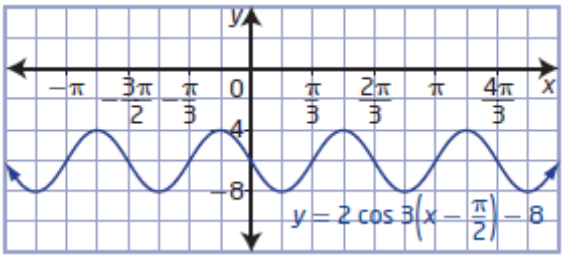
c) For a sine function with an amplitude of $\frac{3}{2}$ and a period of 4π, $a = \frac{3}{2}$ and $b = \frac{1}{2}$.

Then, the equation of the function in the form $y = a \sin bx$ is $y = \frac{3}{2} \sin \frac{1}{2}x$.

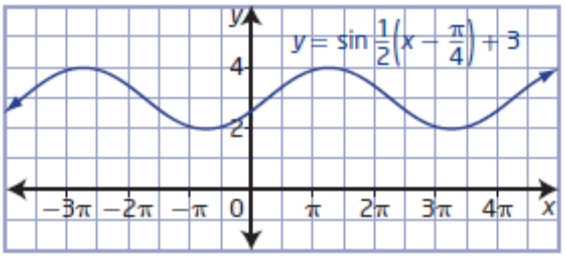
d) For a sine function with an amplitude of 2 and a period of $\frac{2\pi}{3}$, $a = 2$ and $b = 3$. Then, the equation of the function in the form $y = a \sin bx$ is $y = 2 \cos 3x$.

Chapter 5 Review Page 283 Question 8

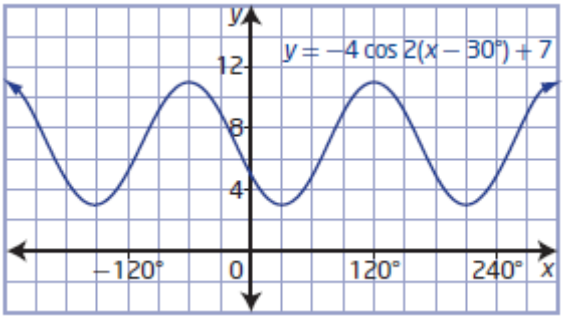
a) For $y = 2\cos 3\left(x - \frac{\pi}{2}\right) - 8$, $a = 2$, $b = 3$, $c = \frac{\pi}{2}$, and $d = -8$. The amplitude is $|2| = 2$, the period is $\frac{2\pi}{3}$, the phase shift is $\frac{\pi}{2}$ units to the right, and the vertical displacement is 8 units down.



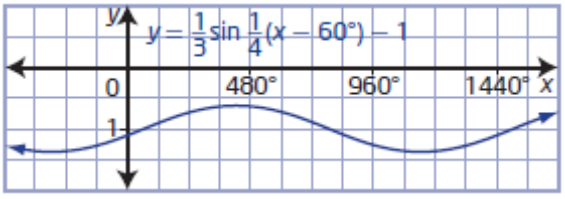
b) For $y = \sin\frac{1}{2}\left(x - \frac{\pi}{4}\right) + 3$, $a = 1$, $b = \frac{1}{2}$, $c = \frac{\pi}{4}$, and $d = 3$. The amplitude is $|1| = 1$, the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, the phase shift is $\frac{\pi}{4}$ units to the right, and the vertical displacement is 3 units up.



c) For $y = -4\cos 2(x - 30°) + 7$, $a = -4$, $b = 2$, $c = 30°$, and $d = 7$. The amplitude is $|-4| = 4$, the period is $\frac{360°}{2} = 180°$, the phase shift is 30° to the right, and the vertical displacement is 7 units up.

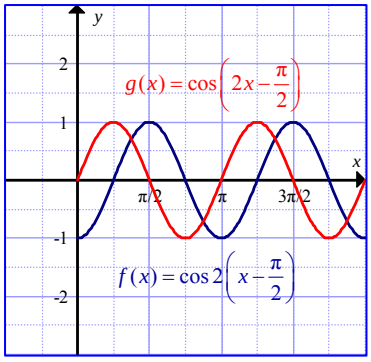


d) For $y = \frac{1}{3}\sin\frac{1}{4}(x - 60°) - 1$, $a = \frac{1}{3}$, $b = \frac{1}{4}$, $c = 60°$, and $d = -1$. The amplitude is $\left|\frac{1}{3}\right| = \frac{1}{3}$, the period is $\frac{360°}{\frac{1}{4}} = 1440°$, the phase shift is 60° to the right, and the vertical displacement is 1 unit down.



Chapter 5 Review Page 283 Question 9

a) The period of both functions is π.



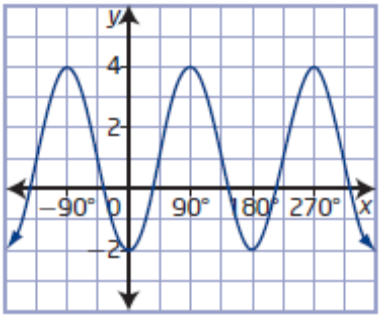
b) The phase shift for $f(x)$ is $\frac{\pi}{2}$ units to the right, while the phase shift for $g(x)$ is $\frac{\pi}{4}$ units to the right.

c) The phase shift of the function $y = b(x - \pi)$ is π units to the right.

d) The phase shift of the function $y = (bx - \pi)$ is $\frac{\pi}{b}$ units to the right.

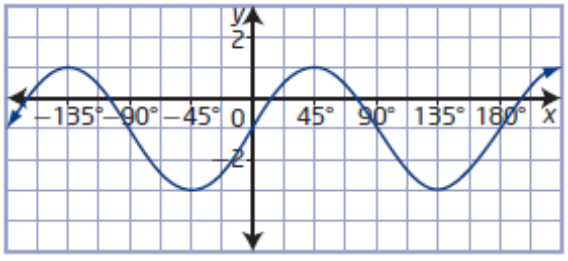
Chapter 5 Review Page 283 Question 10

a) The amplitude, a, is $\frac{4-(-2)}{2}$, or 3.



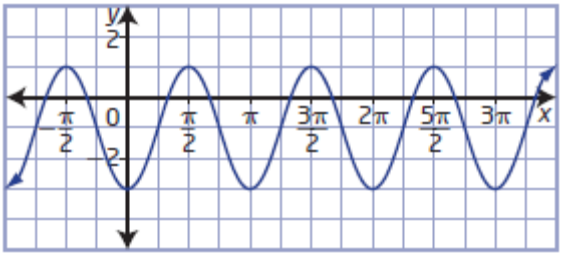
The period is 180° – 0°, or 180°. So, $b = 2$.
The start of the first cycle of the sine curve to the right of the y-axis is at 45°. So, $c = 45°$.
The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 3 \sin 2(x - 45°) + 1$. Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = -3 \cos 2x + 1$.

b) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.



The period is 180° – 0°, or 180°. So, $b = 2$.
The start of the first cycle of the sine curve is at the y-axis. So, $c = 0°$.
The vertical displacement is 1 unit down. Locating the mid-line gives $d = -1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2 \sin 2x - 1$. Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 2 \cos 2(x - 45°) - 1$.

c) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.



The period is π – 0, or π. So, $b = 2$.

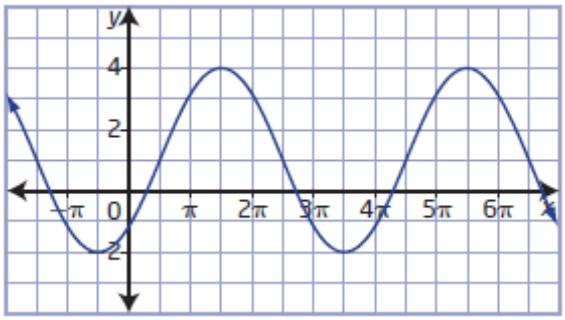
The start of the first cycle of the sine curve is to the right of the y-axis. So, $c = \frac{\pi}{4}$.

The vertical displacement is 1 unit down. Locating the mid-line gives $d = -1$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2\sin 2\left(x - \frac{\pi}{4}\right) - 1$.

Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = -2 \cos 2x - 1$.

d) The amplitude, a, is $\frac{4 - (-2)}{2}$, or 3.



The period is $4\pi - 0$, or 4π. So, $b = 0.5$.

The start of the first cycle of the sine curve is to the right of the y-axis. So, $c = \frac{\pi}{2}$.

The vertical displacement is 1 unit up.

Locating the mid-line gives $d = 1$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 3\sin 0.5\left(x - \frac{\pi}{2}\right) + 1$. Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 3\cos 0.5\left(x - \frac{3\pi}{2}\right) + 1$.

Chapter 5 Review Page 283 Question 11

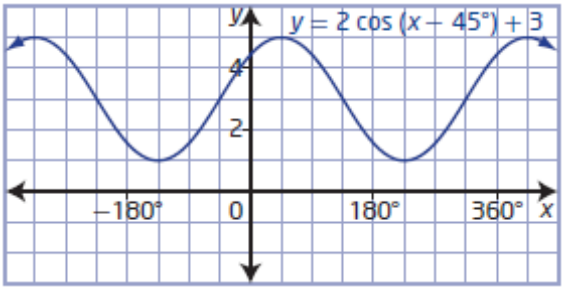
a) For amplitude 4, period π, phase shift $\frac{\pi}{3}$ to the right, and vertical displacement 5 units down, $a = 4$, $b = 2$, $c = \frac{\pi}{3}$, and $d = -5$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = 4\sin 2\left(x - \frac{\pi}{3}\right) - 5$.

b) For amplitude 0.5, period 4π, phase shift $\frac{\pi}{6}$ to the left, and vertical displacement 1 unit up, $a = 0.5$, $b = \frac{1}{2}$, $c = -\frac{\pi}{6}$, and $d = 1$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = 0.5\cos\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$.

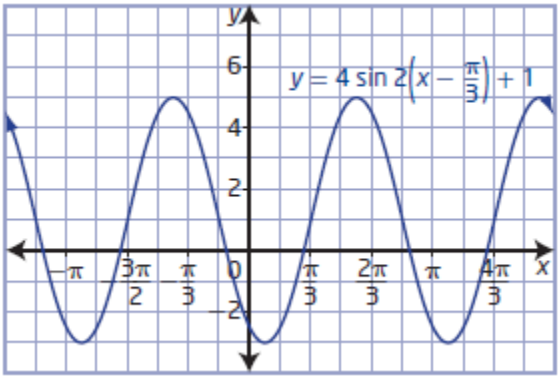
c) For amplitude $\frac{2}{3}$, period 540°, no phase shift, and vertical displacement 5 units down, $a = \frac{2}{3}$, $b = \frac{2}{3}$, $c = 0$, and $d = -5$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = \frac{2}{3}\sin\frac{2}{3}x - 5$.

Chapter 5 Review Page 284 Question 12

a) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range $\{y \mid 1 \le y \le 5, y \in \mathrm{R}\}$. The maximum value is 5 and the minimum value is 1. There are no x-intercepts. The y-intercept is $\sqrt{2} + 3$, or approximately 4.41.



b) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range $\{y \mid -3 \le y \le 5, y \in \mathrm{R}\}$. The maximum value is 5 and the minimum value is –3. The x-intercepts are approximately $0.9 + n\pi$ and $2.7 + n\pi$, $n \in \mathrm{I}$. The y-intercept is $-2\sqrt{3} + 1$, or approximately –2.5.



Chapter 5 Review Page 284 Question 13

a) For $y = 3 \sin 2\left(x - \frac{\pi}{3}\right) + 6$, $a = 3$, $b = 2$, $c = \frac{\pi}{3}$, and $d = 6$. Compared to the graph of $y = \sin x$, the graph of $y = 3 \sin 2\left(x - \frac{\pi}{3}\right) + 6$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, stretched vertically by a factor of 3 about the x-axis, and translated $\frac{\pi}{3}$ units to the right and 6 units up.

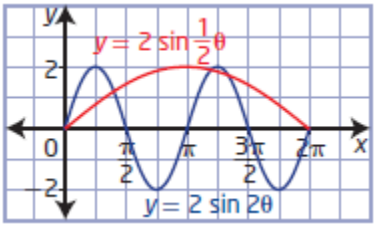
b) For $y = -2\cos\frac{1}{2}\left(x+\frac{\pi}{4}\right) - 3$, $a = -2$, $b = \frac{1}{2}$, $c = -\frac{\pi}{4}$, and $d = -3$. Compared to the graph of $y = \cos x$, the graph of $y = -2\cos\frac{1}{2}\left(x+\frac{\pi}{4}\right) - 3$ is stretched horizontally by a factor of 2 about the y-axis, stretched vertically by a factor of 2 about the x-axis, reflected in the x-axis, and translated $\frac{\pi}{4}$ units to the left and 3 units down.

c) For $y = \frac{3}{4}\cos 2(x - 30°) + 10$, $a = \frac{3}{4}$, $b = 2$, $c = 30°$, and $d = 10$. Compared to the graph of $y = \cos x$, the graph of $y = \frac{3}{4}\cos 2(x - 30°) + 10$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, stretched vertically by a factor of $\frac{3}{4}$ about the x-axis, and translated 30° to the right and 10 units up.

d) For $y = -\sin 2(x + 45°) - 8$, $a = -1$, $b = 2$, $c = -45°$, and $d = -8$. Compared to the graph of $y = \sin x$, the graph of $y = -\sin 2(x + 45°) - 8$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, reflected in the x-axis, and translated 45° to the left and 8 units down.

Chapter 5 Review Page 284 Question 14

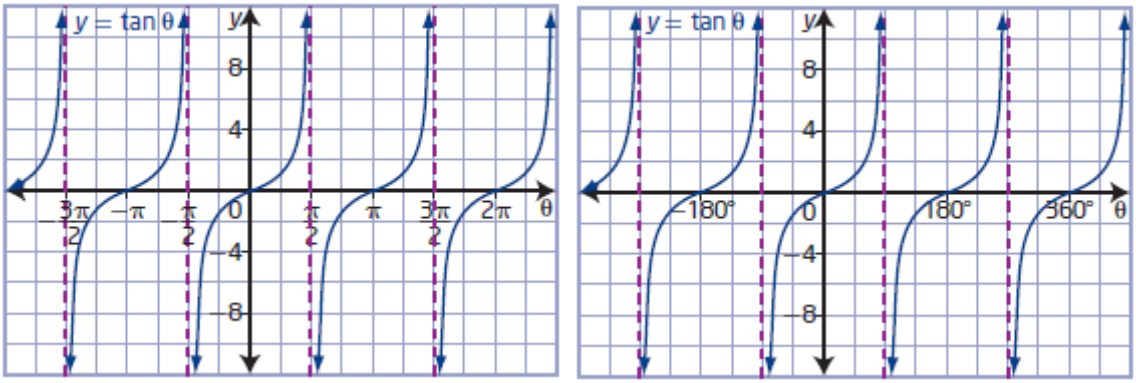
a)



b) Compared to the graph of $y = \sin\theta$, the graph of $y = 2\sin 2\theta$ is vertically stretched by a factor of 2 about the x-axis and half the period. Compared to the graph of $y = \sin\theta$, the graph of $y = 2\sin\frac{1}{2}\theta$ is vertically stretched by a factor of 2 about the x-axis and double the period.

Chapter 5 Review Page 284 Question 15

a)



b) i) For the interval $-2\pi \le \theta \le 2\pi$, the domain is $\left\{x \mid x \ne -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, x \in \mathrm{R}\right\}$. For the interval $-360° \le \theta \le 360°$, the domain is $\{x \mid x \ne -270°, -90°, 90°, 270°, x \in \mathrm{R}\}$.

ii) The range is $\{y \mid y \in \mathrm{R}\}$.

iii) The y-intercept is 0.

iv) For the interval $-2\pi \le \theta \le 2\pi$, the x-intercepts are -2π, $-\pi$, 0, π, and 2π. For the interval $-360° \le \theta \le 360°$, the x-intercepts are $-360°$, $-180°$, 0°, 180°, and 360°.

v) For the interval $-2\pi \le \theta \le 2\pi$, the equations of the asymptotes are $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$. For the interval $-360° \le \theta \le 360°$, the equations of the asymptotes are $x = -270°, -90°, 90°, 270°$.

Chapter 5 Review Page 284 Question 16

a) The exact coordinates of point Q are (1, tan θ) or $\left(1, \frac{1}{\sqrt{3}}\right)$.

b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

c) As θ approaches 90°, tan θ approaches infinity.

d) When θ = 90°, cos 90° = 0 and thus tan 90° is undefined.

Chapter 5 Review Page 284 Question 17

a) Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, the asymptotes of the graph of $y = \tan\theta$ occur when $\cos\theta = 0$.

b) Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, the x-intercepts of the graph of $y = \tan\theta$ occur when $\sin\theta = 0$.

Chapter 5 Review Page 284 Question 18

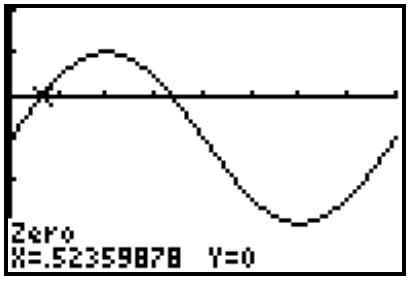
When the Sun is directly overhead, the shadow has no length which means $\tan\theta$ is undefined. This relates to the asymptote of the graph of $y = \tan\theta$ at $\theta = 90°$.

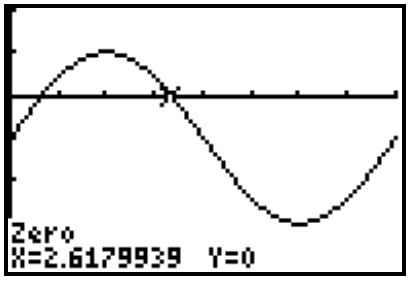
Chapter 5 Review Page 284 Question 19

A vertical asymptote is an imaginary line that the graph approaches. If a trigonometric function is represented by a quotient, such as the tangent function, asymptotes occur at values for which the function is undefined; that is, when the function in the denominator is equal to zero.

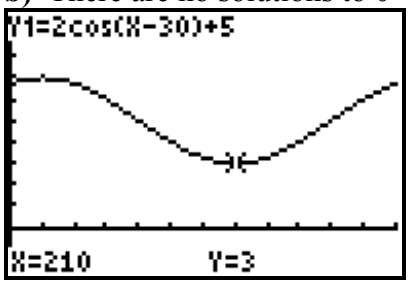
Chapter 5 Review Page 284 Question 20

a) The solutions to $2\sin x - 1 = 1$ in the interval $0 \le x \le 2\pi$ are approximately $x = 0.52$ and $x = 2.62$.

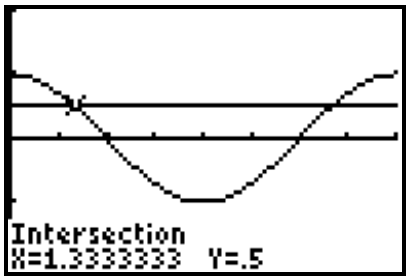


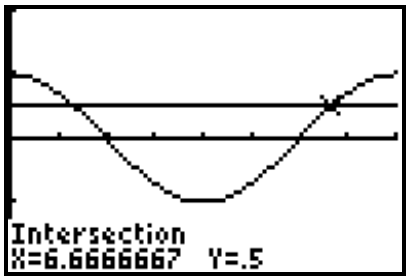


b) There are no solutions to $0 = 2\cos(x - 30°) + 5$ in the interval $0° \le x \le 360°$.



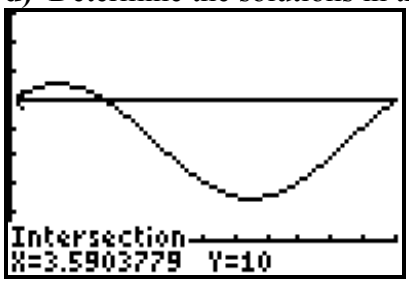
c) Determine the solutions in the first cycle.

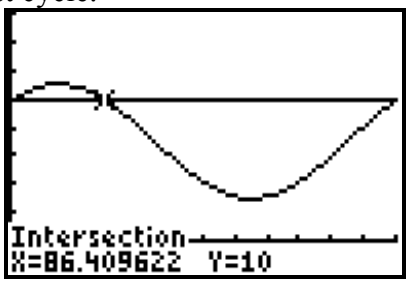




The general solution to $\sin\left(\frac{\pi}{4}(x-6)\right) = 0.5$ is approximately 1.33 + 8n and 6.67 + 8n, where $n \in$ I.

d) Determine the solutions in the first cycle.

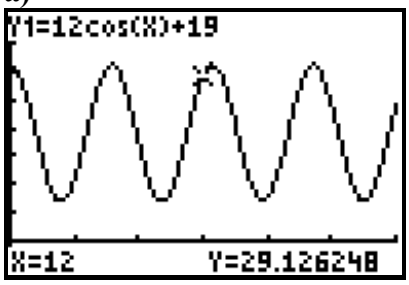




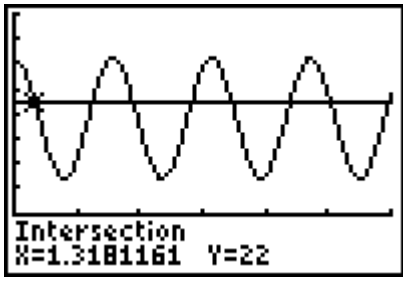
The general solution to 4 cos (x – 45°) + 7 = 10 is approximately 3.59° + (360°)n and 86.41° + (360°)n, where $n \in$ I.

Chapter 5 Review Page 285 Question 21

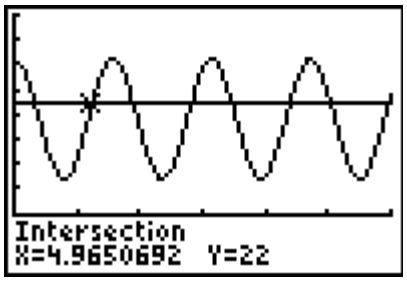
a)

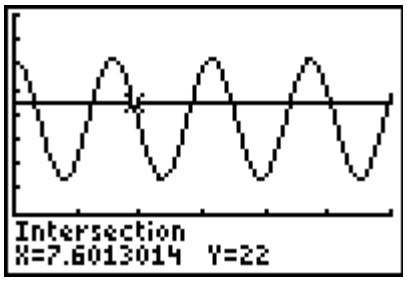


b) Determine several points of intersection between the graphs of T = 12 cos t + 19 and T = 22.

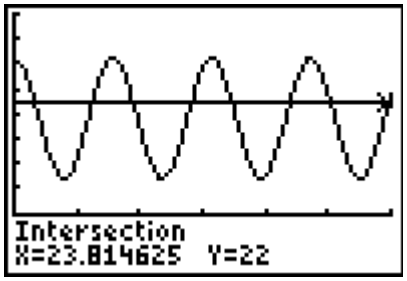


The first point of intersection gives approximately 1.32 h.





Using the approximate second and third points of intersection gives 7.60 – 4.97, or 2.63 h. So, in one day 3(2.63), or 7.89 h.

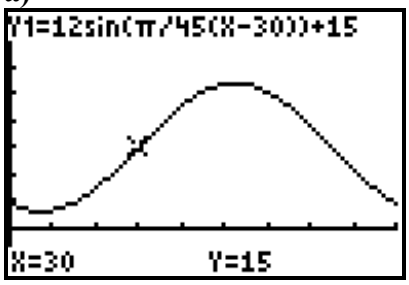


The last point of intersection gives approximately 24 – 23.81, or 0.19 h.
To the nearest tenth of an hour, the amount of time in one day that the air conditioning will operate is 1.32 + 7.89 + 0.19, or 9.4 h.

c) Example: A model for temperature variance is important for maintaining constant temperatures to preserve artifacts.

Chapter 5 Review Page 285 Question 22

a)



b) The maximum height is 12 + 15, or 27 m. The minimum height is 15 – 12, or 3 m.

c) The time required for the Ferris wheel to complete one revolution is $\frac{2\pi}{\frac{\pi}{45}}$, or 90 s.

d) Substitute $t = 45$.

$h(t) = 12 \sin \frac{\pi}{45}(t - 30) + 15$

$h(45) = 12 \sin \frac{\pi}{45}(45 - 30) + 15$

$h(45) = 12 \sin \frac{\pi}{3} + 15$

$h(45) = 12\left(\frac{\sqrt{3}}{2}\right) + 15$

$h(45) = 6\sqrt{3} + 15$

$h(45) = 25.392\ldots$

The height of the rider above the ground after 45 s is approximately 25.4 m.

Chapter 5 Review Page 285 Question 23

a) The best fit curve is of the form $y = -\cos x$. Use the hours of daylight to determine $a = \frac{15.7 - 8.3}{2}$, or 3.7. The period is 365 days, so $b = \frac{2\pi}{365}$. The minimum is reached on day 355, but the minimum of a reflected cosine curve typically occurs at the start point in the period, so $c = -10$. The sinusoidal axis is $d = \frac{15.7 + 8.3}{2}$, or 12. Then, the equation that expresses the number of hours of daylight, L, as a function of time, t, in days, is $L = -3.7 \cos \frac{2\pi}{365}(t + 10) + 12$.

Chapter 5 Review Page 285 Question 24

a) The best fit curve is of the form $y = \cos x$. Use the number of sunspots to determine $a = \frac{110 - 10}{2}$, or 50. The period is $\frac{1948 - 1750}{18}$, or 11 years. So, $b = \frac{2\pi}{11}$. The sinusoidal axis is $d = \frac{110 + 10}{2}$, or 60. Then, the equation that expresses the sunspots, S, as a function of time, t, in years, is $S = 50 \cos \frac{2\pi}{11}t + 60$.

For the year 2000, substitute $t = 250$.

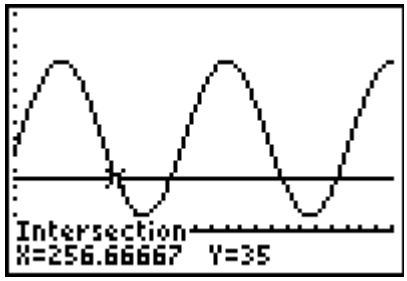
$S = 50 \cos \frac{2\pi}{11}t + 60$

$= 50 \cos \frac{2\pi}{11}(250) + 60$

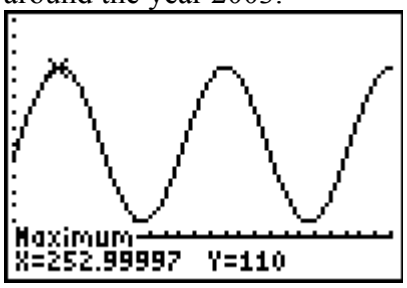
$= 52.884\ldots$

You would expect about 53 sunspots in the year 2000.

b) Determine the point of intersection of the graphs of $S = 50 \cos \frac{2\pi}{11} t + 60$ and $S = 35$. The first year after 2000 in which the number of sunspots will be about 35 is around the year 2007.



c) The first year after 2000 in which the number of sunspots will be a maximum is around the year 2003.



Chapter 5 Practice Test

Chapter 5 Practice Test Page 286 Question 1

For the function $y = 2 \sin x + 1$, $a = 2$ and $d = 1$. So, the range is $\{y \mid -1 \leq y \leq 3, y \in \mathrm{R}\}$: Choice **A**.

Chapter 5 Practice Test Page 286 Question 2

For $f(x) = 3 \sin 2\left(x - \frac{\pi}{3}\right) + 1$, $a = 3$, $b = 2$, $c = \frac{\pi}{3}$, and $d = 1$. The phase shift is $\frac{\pi}{3}$ units to the right. The period is $\frac{2\pi}{2}$ or π. The amplitude is $|3|$, or 3. Choice **D**.

Chapter 5 Practice Test Page 286 Question 3

Since $\sin x = \cos\left(x - \frac{\pi}{2}\right)$, then $\sin\left(x - \frac{\pi}{4}\right) = \cos\left(x - \frac{3\pi}{4}\right)$. Choice **C**.

Chapter 5 Practice Test Page 286 Question 4

Given: cosine function with maximum (3, 14) and nearest minimum to the right at (8, 2).

The amplitude is $\frac{14 - 2}{2}$, or 6.

The distance between the minimum and the maximum of a cosine function is one-half the period. The period is 2(8 – 3), or 10. So, $b = \frac{\pi}{5}$.

The maximum for a cosine curve appears at the start of the cycle, or $x = 3$. So, $c = 3$.

minimum = $d - |a|$

$2 = d - |6|$

$d = 8$

An equation for the cosine function is $y = 6\sin\frac{\pi}{5}(x-3)+8$. Choice **D**.

Chapter 5 Practice Test Page 286 Question 5

From the graph, the amplitude is 2 and the period is π. So, $a = 2$ and $b = 2$. A possible equation for the function is $y = 2 \sin 2\theta$. Choice **B**.

Chapter 5 Practice Test Page 286 Question 6

For a sine function of the form $y = a \sin b(x - c) + d$,
• the values of a and d affect the range of the function
• the values of c and d determine the horizontal and vertical translations, respectively
• the value of b determines the number of cycles within the distance of 2π
• the values of a and b are vertical and horizontal stretches
All of Monique's statements are true. Choice **A**.

Chapter 5 Practice Test Page 286 Question 7

If the bicycle were pedalled at a greater constant speed, the period of the graph would decrease. Choice **C**.

Chapter 5 Practice Test Page 287 Question 8

For the function $f(x) = \sin 2x$, the period is $\frac{2\pi}{2}$ or π. The horizontal distance between two consecutive zeros is half of the period, $\frac{\pi}{2}$.

Chapter 5 Practice Test Page 287 Question 9

For $y = \tan\theta$, the asymptotes occur at $x = \frac{\pi}{2} + n\pi$, $n \in \text{I}$, the domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \text{R}, n \in I\right\}$, the range is $\{y \mid y \in \text{R}\}$, and the period is π.

Chapter 5 Practice Test Page 287 Question 10

For $f(x) = -4 \sin x$, $a = -4$, $b = 1$, $c = 0$, and $d = 0$.

For $g(x) = -4 \cos \frac{1}{2}x$, $a = -4$, $b = \frac{1}{2}$, $c = 0$, and $d = 0$.

Functions $f(x)$ and $g(x)$ have the same amplitude of 4, the same phase shift of 0, and the same vertical displacement of 0.

Chapter 5 Practice Test Page 287 Question 11

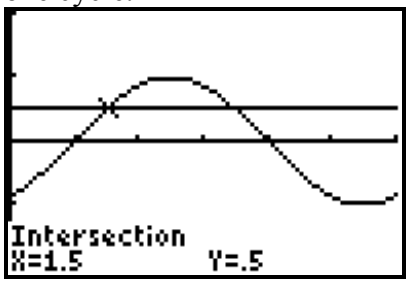
For $V(t) = 120 \sin 2513t$, $a = 120$ and $b = 2513$. So, the amplitude is $|120|$, or 120, and the period is $\frac{2\pi}{2513}$, or approximately 0.0025 s.

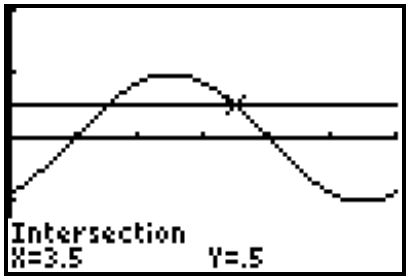
Chapter 5 Practice Test Page 287 Question 12

For $d(t) = -3 \cos \frac{\pi}{6}t + 5$, $a = -3$, $b = \frac{\pi}{6}$, and $d = 5$. So, the amplitude is $|-3|$, or 3, the period is $\frac{2\pi}{\frac{\pi}{6}}$, or 12, the maximum is 5 + 3, or 8, and the minimum is 5 – 3, or 2. The minimum of this cosine function is 2 m; it occurs at the start/end of the cycle, 00:00, 12:00, and 24:00. The maximum of this cosine function is 8 m; it occurs halfway into a cycle, 6:00 and 18:00.

Chapter 5 Practice Test Page 287 Question 13

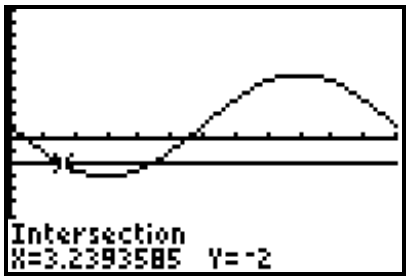
a) Determine the points of intersection for the graphs of $y = \sin \frac{\pi}{3}(x-1)$ and $y = 0.5$ in one cycle.

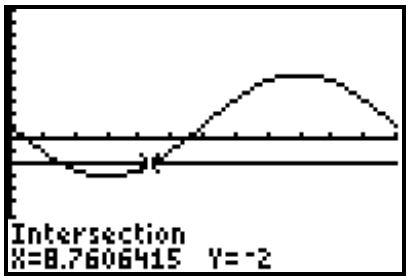




The general solution is $x = 1.5 + 6n$ and $x = 3.5 + 6n$, where $n \in \mathrm{I}$.

b) Determine the points of intersection for the graphs of $y = 4 \cos 15(x + 30°) + 1$ and $y = -2$ in one cycle.





The general solution is $x \approx 3.24° + (24°)n$ and $x \approx 8.76° + (24°)n$, where $n \in \text{I}$.

Chapter 5 Practice Test Page 287 Question 14

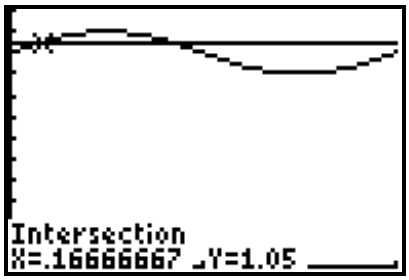
Graph I shows a cosine function with amplitude 2 and period 1 radian. Graph II shows a sine function reflected in the x-axis, amplitude 2, and period 0.5 radians.

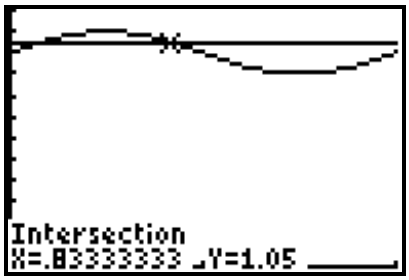
Example: Graph II has half the period of graph I. Graph I represents a cosine curve with no phase shift. Graph II represents a sine curve with no phase shift. Graph I and II have the same amplitude and both graphs have no vertical translations.

Chapter 5 Practice Test Page 287 Question 15

a) The best fit curve is of the form $y = \sin x$. Use the maximum and rest displacements to determine $a = 0.1$. The period is 2 s, so $b = \frac{2\pi}{2}$, or π. The sinusoidal axis is given as $d = 1$. Then, the equation that models the displacement, d, in metres, as a function of time, t, in seconds, is $d = 0.1 \sin \pi t + 1$.

b) The mass is 1.05 m above the floor in the first cycle at approximately 0.17 s and 0.83 s.





c) Verify $x = 0.17$ and $x = 0.83$.

For $x = 0.17$,
$d = 0.1 \sin \pi t + 1$
$= 0.1 \sin \pi(0.17) + 1$
$= 1.050\ldots$

For $x = 0.83$,
$d = 0.1 \sin \pi t + 1$
$= 0.1 \sin \pi(0.83) + 1$
$= 1.050\ldots$

Chapter 5 Practice Test Page 287 Question 16

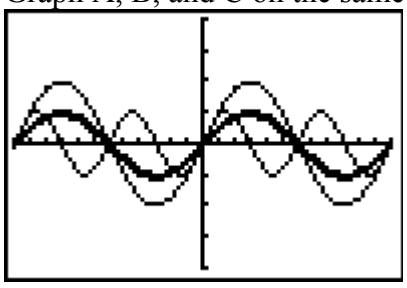
From the graph, the amplitude is 3, the period is π, the sinusoidal axis is $y = -1$.

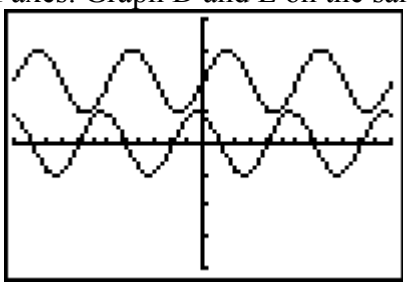
a) For a sine function, the cycle starts at $\frac{\pi}{4}$. So, $a = 3$, $b = 2$, $c = \frac{\pi}{4}$, $d = -1$. Then, the equation in the form $y = a \sin b(x - c) + d$ is $y = 3 \sin 2\left(x - \frac{\pi}{4}\right) - 1$.

b) For a cosine function, the curve is reflected in the x-axis. So, $a = -3$, $b = 2$, $c = 0$, $d = -1$. Then, the equation in the form $y = a \cos b(x - c) + d$ is $y = -3 \cos 2x - 1$.

Chapter 5 Practice Test Page 287 Question 17

Graph A, B, and C on the same set of axes. Graph D and E on the same set of axes.





a) The graphs of $f(x) = \sin x$ (A) and $g(x) = 2 \sin x$ (B) have the same x-intercepts.

b) The graphs of $f(x) = \sin x$ (A) and $g(x) = 2 \sin x$ (B) have the same period. The graphs of $h(x) = \sin 2x$ (C), $k(x) = \sin (2x + 2)$ (D), and $m(x) = \sin 2x + 2$ (E) have the same period.

c) The graph of $g(x) = 2 \sin x$ (B) has a different amplitude than the others.

Chapter 6 Trigonometric Identities

Section 6.1 Reciprocal, Quotient, and Pythagorean Identities

Section 6.1 Page 296 Question 1

a) For $\frac{\cos x}{\sin x}$, non-permissible values occur when $\sin x = 0$.
$\sin x = 0$ at $x = 0, \pi, 2\pi, \ldots$
Therefore, $x \neq \pi n$, where $n \in \text{I}$.

b) For $\frac{\cos x}{\tan x}$, non-permissible values occur when $\tan x = 0$. Since $\tan x = \frac{\sin x}{\cos x}$ the non-permissible values occur when $\cos x = 0$ and when $\sin x = 0$.
$\cos x = 0$ at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ $\sin x = 0$ at $x = 0, \pi, 2\pi, \ldots$
Therefore, $x \neq \frac{\pi}{2} + \pi n$ and $x \neq \pi n$, where $n \in \text{I}$.
These two sets of restrictions can be combined as $x \neq \left(\frac{\pi}{2}\right) n$, where $n \in \text{I}$.

c) For $\frac{\cot x}{1 - \sin x}$, non-permissible values occur, for the denominator, when $1 - \sin x = 0$ and, for the numerator, when $\sin x = 0$.
For the first restriction, $\sin x = 1$ at $x = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$. In general, $x \neq \frac{\pi}{2} + 2\pi n$, where $n \in \text{I}$.
For the other restriction, $\sin x = 0$ at $x = 0, \pi, 2\pi, \ldots$. In general, $x \neq \pi n$, where $n \in \text{I}$..

d) For $\frac{\tan x}{\cos x + 1}$, non-permissible values occur, for the denominator, when $\cos x + 1 = 0$ and, for the numerator, when $\cos x = 0$.
For the first restriction, $\cos x = -1$ at $x = \pi, 3\pi, \ldots$. In general, $x \neq \pi + 2\pi n$, , where $n \in \text{I}$.
For the other restriction, $\cos x = 0$ at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$. In general, $x \neq \frac{\pi}{2} + \pi n$, where $n \in \text{I}$.

Section 6.1 Page 296 Question 2

Some identities have non-permissible values because they involve rational expressions and some values of the variable would make the denominator zero. This is not permitted. For example, an identity involving $\tan x$ has non-permissible values when $\cos x = 0$.

Section 6.1 Page 296 Question 3

a) $\sec x \sin x = \left(\frac{1}{\cos x}\right)\sin x$

$= \tan x$

b) $\sec x \cot x \sin^2 x = \left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)\sin^2 x$

$= \sin x$

c) $\frac{\cos x}{\cot x} = \cos x\left(\frac{\sin x}{\cos x}\right)$

$= \sin x$

Section 6.1 Page 296 Question 4

a) $\left(\frac{\cos x}{\tan x}\right)\left(\frac{\tan x}{\sin x}\right) = \frac{\cos x}{\sin x}$

$= \cot x$

b) $\csc x \cot x \sec x \sin x = \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)\sin x$

$= \frac{1}{\sin x}$

$= \csc x$

c) $\frac{\cos x}{1-\sin^2 x} = \frac{\cos x}{\cos^2 x}$

$= \frac{1}{\cos x}$

$= \sec x$

Section 6.1 Page 296 Question 5

a) For $x = 30°$:

$$\text{Left Side} = \frac{\sec x}{\tan x + \cot x}$$
$$= \frac{\sec 30°}{\tan 30° + \cot 30°}$$
$$= \frac{\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}}$$
$$= \frac{2}{\sqrt{3}} \div \frac{1+3}{\sqrt{3}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

$$\text{Right Side} = \sin x$$
$$= \sin 30°$$
$$= \frac{1}{2}$$

For $x = \frac{\pi}{4}$:

$$\text{Left Side} = \frac{\sec x}{\tan x + \cot x}$$
$$= \frac{\sec \frac{\pi}{4}}{\tan \frac{\pi}{4} + \cot \frac{\pi}{4}}$$
$$= \frac{\sqrt{2}}{1+1}$$
$$= \frac{\sqrt{2}}{2}$$

$$\text{Right Side} = \sin x$$
$$= \sin \frac{\pi}{4}$$
$$= \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

The equation checks for both values.

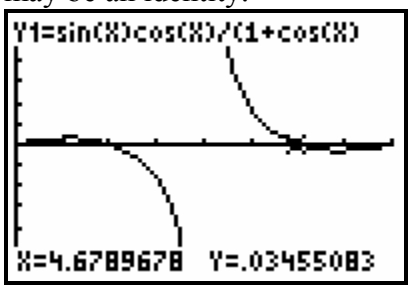
b) The non-permissible values occur for the numerator when $\cos x = 0$ and, for the denominator, when $\sin x = 0$. So, in the domain $0° \le x < 360°$, $x \ne 0°$, 90°, 180°, and 270°.

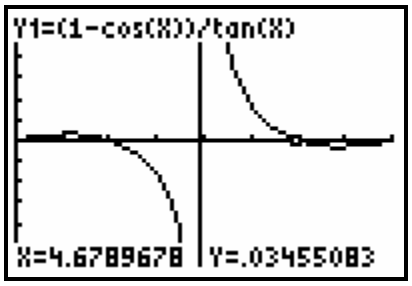
Section 6.1 Page 297 Question 6

a) The non-permissible values occur for the left side of the equation when $1 + \cos x = 0$, or $\cos x = -1$, and, for the right side of the equation, when $\tan x = 0$.

So, in radians, $x \ne \pi + 2\pi n,\ n \in \mathrm{I}$ and $x \ne \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

b) The graph of the left side looks the same as the graph of the right side, so the equation may be an identity.





c) For $\frac{\pi}{4}$:

$$\text{Left Side} = \frac{\sin x \cos x}{1+\cos x}$$
$$= \frac{\sin\frac{\pi}{4}\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}$$
$$= \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)}{1+\left(\frac{1}{\sqrt{2}}\right)}$$
$$= \left(\frac{1}{2}\right) \div \left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$$
$$= \left(\frac{1}{2}\right)\left(\frac{2-\sqrt{2}}{1}\right)$$
$$= \frac{2-\sqrt{2}}{2}$$

$$\text{Right Side} = \frac{1-\cos x}{\tan x}$$
$$= \frac{1-\cos\frac{\pi}{4}}{\tan\frac{\pi}{4}}$$
$$= 1-\frac{1}{\sqrt{2}}$$
$$= 1-\frac{\sqrt{2}}{2}$$
$$= \frac{2-\sqrt{2}}{2}$$

Both sides have the same value, so the equation is true when $x = \frac{\pi}{4}$.

Section 6.1 Page 297 Question 7

a) Using the Pythagorean identity, $\sin^2\theta + \cos^2\theta = 1$, an equivalent expression for $1 - \sin^2\theta$ is $\cos^2\theta$.

b) When $\theta = \frac{\pi}{6}$, the fraction of light lost is given by

$$\cos^2\theta = \cos^2\frac{\pi}{6}$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{3}{4}$$

c) When $\theta = 60°$, the fraction of light lost is given by
$$\cos^2\theta = \cos^2 60°$$
$$= \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{4}$$
As a percent, the amount lost is 25%.

Section 6.1 Page 297 Question 8

a) When $x = \frac{\pi}{3}$:
$$\sin x = \sin\frac{\pi}{3}$$
$$= \frac{\sqrt{3}}{2}$$

$$\sqrt{1-\cos^2 x} = \sqrt{1-\cos^2\left(\frac{\pi}{3}\right)}$$
$$= \sqrt{1-\left(\frac{1}{2}\right)^2}$$
$$= \sqrt{\frac{3}{4}}$$
$$= \frac{\sqrt{3}}{2}$$

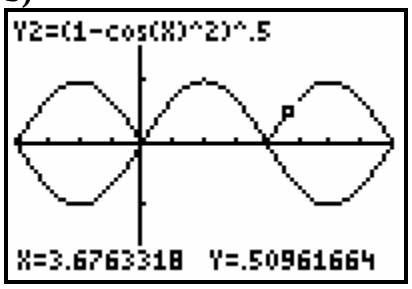
So, for $x = \frac{\pi}{3}$, $\sin x = \sqrt{1-\cos^2 x}$.

When $x = \frac{5\pi}{6}$:
$$\sin x = \sin\frac{5\pi}{6}$$
$$= \frac{1}{2}$$

$$\sqrt{1-\cos^2 x} = \sqrt{1-\cos^2\left(\frac{5\pi}{6}\right)}$$
$$= \sqrt{1-\left(-\frac{\sqrt{3}}{2}\right)^2}$$
$$= \sqrt{\frac{1}{4}}$$
$$= \frac{1}{2}$$

So, for $x = \frac{5\pi}{6}$, $\sin x = \sqrt{1-\cos^2 x}$.

b)



c)

The equation is not an identity. As the graph shows, $y = \sqrt{1-\cos^2 x}$ only has positive values in its range, whereas $y = \sin x$ has all values from –1 to 1 in its range.

Section 6.1 Page 297 Question 9

a)

$$\sec\theta = \frac{I}{ER^2}$$

$$\frac{1}{\cos\theta} = \frac{I}{ER^2}$$

$$E = \frac{I\cos\theta}{R^2}$$

b)

$$E = \frac{I\cot\theta}{R^2\csc\theta}$$

$$E = \frac{I\left(\frac{\cos\theta}{\sin\theta}\right)}{R^2\left(\frac{1}{\sin\theta}\right)}$$

$$E = \left(\frac{I\cos\theta}{\sin\theta}\right)\left(\frac{\sin\theta}{R^2}\right)$$

$$E = \frac{I\cos\theta}{R^2}$$

Section 6.1 Page 297 Question 10

$$\frac{\csc x}{\tan x + \cot x} = \left(\frac{1}{\sin x}\right) \div \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \left(\frac{1}{\sin x}\right) \div \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$$

$$= \left(\frac{1}{\sin x}\right)\left(\frac{\cos x \sin x}{1}\right)$$

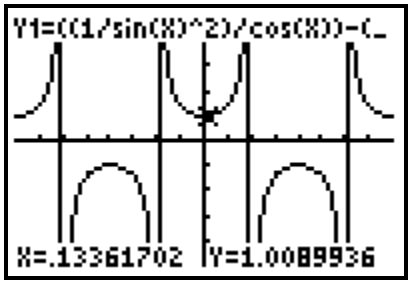
$$= \cos x$$

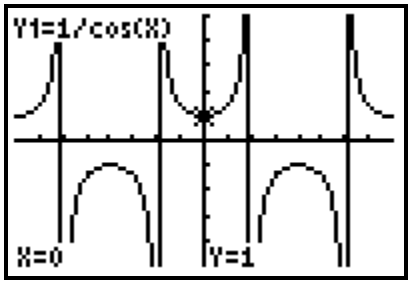
Division by $\sin x$ and by $\cos x$ occurs, so $\sin x \neq 0$ and $\cos x \neq 0$.

In the domain $0 \leq \theta < 2\pi$, $x \neq 0$, $\frac{\pi}{2}$, π, and $\frac{3\pi}{2}$.

Section 6.1 Page 298 Question 11

a) The graph of $y = \dfrac{\csc^2 x - \cot^2 x}{\cos x}$ appears to be equivalent to the graph of $y = \sec x$.





b) Division by $\sin x$ and by $\cos x$ occurs, so $\sin x \neq 0$ and $\cos x \neq 0$. So in general, in radians, $x \neq \dfrac{\pi}{2} + \pi n,\ n \in \text{I}$.

c)
$$\begin{aligned}\frac{\csc^2 x - \cot^2 x}{\cos x} &= \frac{\dfrac{1}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x}}{\cos x} \\ &= \frac{\dfrac{1-\cos^2 x}{\sin^2 x}}{\cos x} \\ &= \frac{\dfrac{\sin^2 x}{\sin^2 x}}{\cos x} \\ &= \frac{1}{\cos x} \\ &= \sec x\end{aligned}$$

Section 6.1 Page 298 Question 12

a) Substitute $x = \dfrac{\pi}{4}$:

$$\begin{aligned}\text{Left Side} &= \frac{\cot x}{\sec x} + \sin x \\ &= \frac{\cot \frac{\pi}{4}}{\sec \frac{\pi}{4}} + \sin \frac{\pi}{4} \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ &= \frac{2}{\sqrt{2}} \text{ or } \sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= \csc x \\ &= \csc \frac{\pi}{4} \\ &= \sqrt{2}\end{aligned}$$

Since the right side equals the left side for this one value, $\frac{\cot x}{\sec x} + \sin x = \csc x$ may be an identity.

b) $\frac{\cot x}{\sec x} + \sin x = \frac{\cos x}{\sin x} \div \frac{1}{\cos x} + \sin x$

$= \frac{\cos^2 x}{\sin x} + \sin x$

$= \frac{\cos^2 x + \sin^2 x}{\sin x}$

$= \csc x$

Section 6.1 Page 298 Question 13

a) Substitute $x = 0$:

Left Side $= \sin x + \cos x$
$= \sin 0 + \cos 0$
$= 0 + 1$
$= 1$

Right Side $= \tan x + 1$
$= \tan 0 + 1$
$= 0 + 1$
$= 1$

For $x = 0$, Left Side = Right Side = 1.

b) Substitute $x = \frac{\pi}{2}$:

Left Side $= \sin x + \cos x$
$= \sin \frac{\pi}{2} + \cos \frac{\pi}{2}$
$= 1 + 0$
$= 1$

Right Side $= \tan x + 1$
$= \tan \frac{\pi}{2} + 1$
$=$ undefined $+ 1$

For $x = \frac{\pi}{2}$, Left Side = 1 but the right side is undefined.

c) Lisa's choice for x is not permissible because $\cos \frac{\pi}{2} = 0$, so in the right side of the equation, in $\tan x$, you would be dividing by 0 which is not permitted.

d) Substitute $x = \frac{\pi}{4}$:

$$\text{Left Side} = \sin x + \cos x \qquad \text{Right Side} = \tan x + 1$$
$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} \qquad = \tan\frac{\pi}{4} + 1$$
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \qquad = 1 + 1$$
$$= \frac{2}{\sqrt{2}} \qquad = 2$$

For $x = \frac{\pi}{4}$, the left side = $\frac{2}{\sqrt{2}}$ but the right side = 2.

e) Yes, the three students have enough information to conclude that the equation in not an identity. Giselle has found a permissible value of x for which the left and right sides do not have the same value.

Section 6.1 Page 298 Question 14

$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2$$
$$= \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x$$
$$= 2\sin^2 x + 2\cos^2 x$$
$$= 2(\sin^2 x + \cos^2 x)$$
$$= 2$$

Section 6.1 Page 298 Question 15

Given $\csc^2 x + \sin^2 x = 7.89$.

$$\frac{1}{\csc^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\frac{1}{\sin^2 x}} + \csc^2 x$$
$$= \sin^2 x + \csc^2 x$$
$$= 7.89$$

Section 6.1 Page 298 Question 16

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$$
$$= \frac{2}{(1-\sin^2\theta)}$$
$$= 2\sec^2\theta$$

Section 6.1 Page 298 Question 17

$$\begin{aligned}\frac{2-\cos^2 x}{\sin x} &= \frac{1+1-\cos^2 x}{\sin x}\\ &= \frac{1+\sin^2 x}{\sin x}\\ &= \frac{1}{\sin x}+\sin x\\ &= \csc x+\sin x\end{aligned}$$

So, $\frac{2-\cos^2 x}{\sin x} = m + \sin x$ is an identity if $m = \csc x$.

Section 6.1 Page 298 Question C1

$$\begin{aligned}\cot^2 x+1 &= \frac{\cos^2 x}{\sin^2 x}+\frac{\sin^2 x}{\sin^2 x}\\ &= \frac{\cos^2 x+\sin^2 x}{\sin^2 x}\\ &= \frac{1}{\sin^2 x}\\ &= \csc^2 x\end{aligned}$$

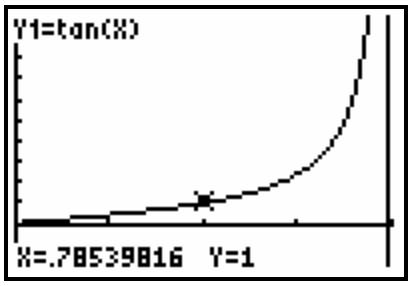
Section 6.1 Page 298 Question C2

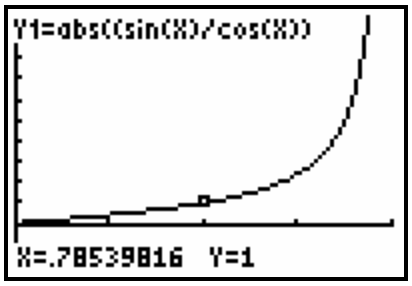
$$\begin{aligned}\left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right) &= \frac{\sin\theta-\sin\theta\cos\theta}{1-\cos^2\theta}\\ &= \frac{\sin\theta-\sin\theta\cos\theta}{\sin^2\theta}\\ &= \frac{1-\cos\theta}{\sin\theta}\end{aligned}$$

It helps to simplify by creating an opportunity to use the Pythagorean identity.

Section 6.1 Page 298 Question C3

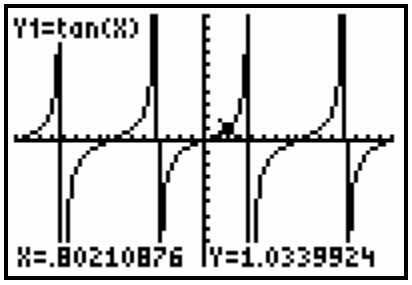
Step 1 For the domain $0 \le x < \frac{\pi}{2}$, the graphs $y = \tan x$ and $y = \left|\frac{\sin x}{\cos x}\right|$ are the same. In this domain, $\tan x = \left|\frac{\sin x}{\cos x}\right|$ is an identity.

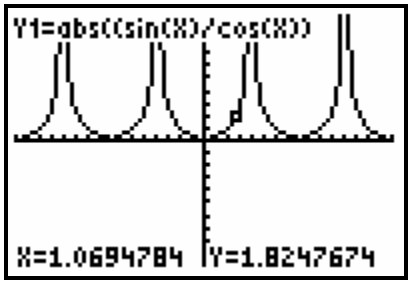




Step 2 For the domain $-2\pi < x \leq 2\pi$, the graphs $y = \tan x$ and $y = \left|\frac{\sin x}{\cos x}\right|$ are not the same.

In this domain, $\tan x = \left|\frac{\sin x}{\cos x}\right|$ is not an identity.





Step 3 Example: $y = \cot\theta$ and $y = \left|\frac{\cos\theta}{\sin\theta}\right|$ are identities over the domain $0 \leq \theta \leq \frac{\pi}{2}$ but not over the domain $-2\pi \leq \theta \leq 2\pi$.

Step 4 The weakness of using a graphical or numerical approach is that for some equations you may think it is an identity when really it is only an identity over a restricted domain.

Section 6.2 Sum, Difference, and Double-Angle Identities

Section 6.2 Page 306 Question 1

a) $\cos 43° \cos 27° - \sin 43° \sin 27° = \cos (43° + 27°)$
$= \cos 70°$

b) $\sin 15° \cos 20° + \cos 15° \sin 20° = \sin (15° + 20°)$
$= \sin 35°$

c) $\cos^2 19° - \sin^2 19° = \cos 2(19°)$
$= \cos 38°$

d) $\sin\frac{3\pi}{2}\cos\frac{5\pi}{4}-\cos\frac{3\pi}{2}\sin\frac{5\pi}{4}=\sin\left(\frac{3\pi}{2}-\frac{5\pi}{4}\right)$

$=\sin\frac{\pi}{4}$

e) $8\sin\frac{\pi}{3}\cos\frac{\pi}{3}=4\sin 2\left(\frac{\pi}{3}\right)$

$=4\sin\frac{2\pi}{3}$

Section 6.2 Page 306 Question 2

a) $\cos 40° \cos 20° - \sin 40° \sin 20° = \cos (40° + 20°)$

$= \cos 60°$

$= 0.5$

b) $\sin 20° \cos 25° + \cos 20° \sin 25° = \sin (20° + 25°)$

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

c) $\cos^2\frac{\pi}{6}-\sin^2\frac{\pi}{6}=\cos 2\left(\frac{\pi}{6}\right)$

$=\cos\frac{\pi}{3}$

$=0.5$

d) $\cos\frac{\pi}{2}\cos\frac{\pi}{3}-\sin\frac{\pi}{2}\sin\frac{\pi}{3}=\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

$=\cos\frac{5\pi}{6}$

$=-\frac{\sqrt{3}}{2}$

Section 6.2 Page 306 Question 3

Substitute $\cos 2x = 1 - 2 \sin^2 x$.

$1 - \cos 2x = 1 - (1 - 2 \sin^2 x)$

$= 1 - 1 + 2 \sin^2 x$

$= 2 \sin^2 x$

Section 6.2 Page 306 Question 4

a) $2\sin\frac{\pi}{4}\cos\frac{\pi}{4} = \sin 2\left(\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{2}$

b) $(6\cos^2 24° - 6\sin^2 24°)\tan 48° = 6\cos 2(24°)\tan 48°$

$= 6\cos 48°\tan 48°$

$= 6\cos 48°\left(\frac{\sin 48°}{\cos 48°}\right)$

$= 6\sin 48°$

c) $\frac{2\tan 76°}{1-\tan^2 76°} = \tan 2(76°)$

$= \tan 152°$

d) $2\cos^2\frac{\pi}{6} - 1 = \cos 2\left(\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{3}$

e) $1 - 2\cos^2\frac{\pi}{12} = -\left(2\cos^2\frac{\pi}{12} - 1\right)$

$= -\cos 2\left(\frac{\pi}{12}\right)$

$= -\cos\frac{\pi}{6}$

Section 6.2 Page 306 Question 5

a) $\frac{\sin 2\theta}{2\cos\theta} = \frac{2\sin\theta\cos\theta}{2\cos\theta}$

$= \sin\theta$

b) $\cos 2x\cos x + \sin 2x\sin x = (\cos^2 x - \sin^2 x)\cos x + 2\sin x\cos x\sin x$

$= \cos^3 x - \sin^2 x\cos x + 2\sin^2 x\cos x$

$= \cos^3 x + \sin^2 x\cos x$

$= \cos x(\cos^2 x + \sin^2 x)$

$= \cos x$

c) $\dfrac{\cos 2\theta + 1}{2\cos\theta} = \dfrac{2\cos^2\theta - 1 + 1}{2\cos\theta}$

$= \dfrac{2\cos^2\theta}{2\cos\theta}$

$= \cos\theta$

d) $\dfrac{\cos^3 x}{\cos 2x + \sin^2 x} = \dfrac{\cos x(1 - \sin^2 x)}{(1 - 2\sin^2 x) + \sin^2 x}$

$= \dfrac{\cos x(1 - \sin^2 x)}{1 - \sin^2 x}$

$= \cos x$

Section 6.2 Page 306 Question 6

Use a counterexample to show that $\sin(x - y) \neq \sin x - \sin y$.
Substitute $x = 60°$ and $y = 30°$:

Left Side $= \sin(60° - 30°)$
$= \sin 30°$
$= 0.5$

Right Side $= \sin 60° - \sin 30°$
$= \dfrac{\sqrt{3}}{2} - 0.5$
≈ 0.366

Left Side ≠ Right Side

Section 6.2 Page 306 Question 7

$\cos(90° - x) = \cos 90° \cos x + \sin 90° \sin x$
$= 0 (\cos x) + 1 \sin x$
$= \sin x$

Section 6.2 Page 306 Question 8

a) $\cos 75° = \cos(45° + 30°)$
$= \cos 45° \cos 30° - \sin 45° \sin 30°$
$= \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}\right) - \dfrac{1}{\sqrt{2}}\left(\dfrac{1}{2}\right)$
$= \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

b) $\tan 165° = \tan(120° + 45°)$
$= \dfrac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$
$= \dfrac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$
$= \dfrac{-\sqrt{3} + 1}{1 + \sqrt{3}}$ or $\dfrac{(1 - \sqrt{3})(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} = \dfrac{4 - 2\sqrt{3}}{-2}$ or $\sqrt{3} - 2$

c) $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$

d) $\cos 195° = \cos(60° + 135°)$

$= \cos 60° \cos 135° - \cos 60° \sin 135°$

$= \frac{1}{2}\left(-\frac{1}{\sqrt{2}}\right)-\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

e)

$\csc\frac{\pi}{12} = \csc\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)$

$= \csc\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$= 1 \div \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$= 1 \div \left[\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)\right]$

$= 1 \div \left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)\right]$

$= 1 \div \left[\frac{\sqrt{3}-1}{2\sqrt{2}}\right]$

$= \frac{2\sqrt{2}}{\sqrt{3}-1}$ or $\frac{\left(2\sqrt{2}\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)} = \sqrt{2}\left(\sqrt{3}+1\right)$

f) $\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

Section 6.2 Page 306 Question 9

a) $P = 1000 (\sin x \cos 113.5° + \cos 113.5° \sin x)$
$= 1000 \sin (x + 113.5°)$

b) i) For Whitehorse, Yukon, 60.7° N, substitute $x = 60.7°$.
$P = 1000 \sin (60.7° + 113.5°)$
$= 1000 \sin 174.2°$
≈ 101.056
The amount of power received from the sun on the winter solstice at Whitehorse is approximately 101.056 W/m^2.

ii) For Victoria, British Columbia, 48.4° N, substitute $x = 48.4°$.
$P = 1000 \sin (48.4° + 113.5°)$
$= 1000 \sin 161.9°$
≈ 310.676
The amount of power received from the sun on the winter solstice at Victoria is approximately 310.676 W/m^2.

iii) For Igloolik, Nunavut, 69.4° N, substitute $x = 69.4°$.
$P = 1000 \sin (69.4° + 113.5°)$
$= 1000 \sin 182.9°$
≈ -50.593
The amount of power received from the sun on the winter solstice at Iglooik is approximately –50.593 W/m^2.

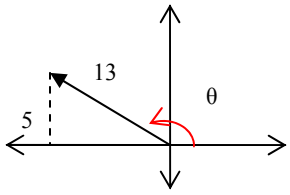
c) On the winter solstice, Igloolik receives no sunlight, so no warmth from the sun. The land is losing heat.
When $x = 66.5°$
$P = 1000 \sin (66.5° + 113.5°)$
$= 1000 \sin 180°$
$= 0$
At latitute 66.5° N, the power received is 0 W/m^2.

Section 6.2 Page 307 Question 10

$\cos(\pi + x) + \cos(\pi - x)$
$= \cos\pi\cos x - \sin\pi\sin x + \cos\pi\cos x + \sin\pi\sin x$
$= 2\cos\pi\cos x$
$= 2(-1)\cos x$
$= -2\cos x$

Section 6.2 Page 307 Question 11

$\sin\theta = \frac{5}{13}$ and θ is in quadrant II.
Using the Pythagorean theorem, $x = -12$.



a) $\cos 2\theta = 1 - 2\sin^2\theta$

$$= 1 - 2\left(\frac{5}{13}\right)^2$$
$$= 1 - 2\left(\frac{25}{169}\right)$$
$$= \frac{119}{169}$$

b) $\sin 2\theta = 2\cos\theta\sin\theta$

$$= 2\left(\frac{-12}{13}\right)\left(\frac{5}{13}\right)$$
$$= -\frac{120}{169}$$

c) $\sin\left(\theta + \frac{\pi}{2}\right) = \sin\theta\cos\left(\frac{\pi}{2}\right) + \cos\theta\sin\left(\frac{\pi}{2}\right)$

$$= \left(\frac{5}{13}\right)0 + \left(\frac{-12}{13}\right)1$$
$$= -\frac{12}{13}$$

Section 6.2 Page 307 Question 12

a) Substitute $x = \frac{\pi}{6}$.
Left Side $= \tan 2x$

$$= \tan 2\left(\frac{\pi}{6}\right)$$
$$= \tan\left(\frac{\pi}{3}\right)$$
$$= \sqrt{3}$$

$$\text{Right Side} = \frac{2\tan x}{1-\tan^2 x}$$
$$= \frac{2\tan\left(\frac{\pi}{6}\right)}{1-\tan^2\left(\frac{\pi}{6}\right)}$$
$$= 2\left(\frac{1}{\sqrt{3}}\right) \div \left(1-\left(\frac{1}{\sqrt{3}}\right)^2\right)$$
$$= \left(\frac{2}{\sqrt{3}}\right)\left(\frac{3}{2}\right)$$
$$= \sqrt{3}$$

Since Left Side = Right Side, the equation $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ is true for $x = \frac{\pi}{6}$.

b) Substitute $x = \frac{\pi}{6}$.

From part a),
Left Side = $\tan 2x$
$= \sqrt{3}$

$$\text{Right Side} = \frac{\sin 2x}{\cos 2x}$$
$$= \frac{\sin 2\left(\frac{\pi}{6}\right)}{\cos 2\left(\frac{\pi}{6}\right)}$$
$$= \frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}$$
$$= \left(\frac{\sqrt{3}}{2}\right) \div \left(\frac{1}{2}\right)$$
$$= \sqrt{3}$$

Since Left Side = Right Side, the equation $\tan 2x = \frac{\sin 2x}{\cos 2x}$ is true for $x = \frac{\pi}{6}$.

c) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$$= \frac{2\tan x}{1-\tan^2 x}\left(\frac{\cos^2 x}{\cos^2 x}\right)$$

$$= \frac{2\left(\dfrac{\sin x}{\cos x}\right)(\cos^2 x)}{\left(1-\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x}$$

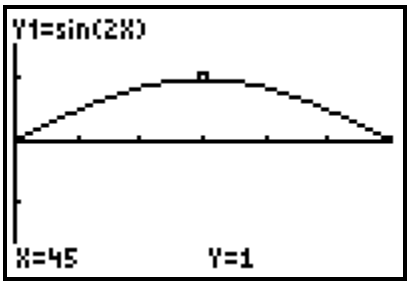
$$= \frac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

Section 6.2 Page 307 Question 13

a) $d = \dfrac{2v^2 \sin\theta\cos\theta}{g}$

$$d = \frac{v^2 \sin 2\theta}{g}$$

b) Since the values of sine increases from 0 to 1 as θ increases from 0° to 90°, it is reasonable that the maximum distance occurs when θ = 45°, or when the value of sin 2x is its maximum value, 1.
Graphing the function $y = \sin 2x$ confirms this.
The maximum distance would be $\dfrac{v_0}{9.8}$ metres.



c) It is easier after applying the double-angle identity since there is only one trigonometric function to consider.

Section 6.2 Page 307 Question 14

$$(\sin x + \cos x)^2 = k$$
$$\sin^2 x + 2\sin x\cos x + \cos^2 x = k$$
$$1 + \sin 2x = k$$

Therefore, $\sin 2x = k - 1$.

Section 6.2 Page 307 Question 15

a) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
$= \cos^2 x - \sin^2 x$
$= \cos 2x$

b)
$$\frac{\csc^2 x - 2}{\csc^2 x} = 1 - \frac{2}{\csc^2 x}$$
$$= 1 - 2\sin^2 x$$
$$= \cos 2x$$

Section 6.2 Page 307 Question 16

a) $$\frac{1-\cos 2x}{2} = \frac{1-(1-2\sin^2 x)}{2}$$
$$= \sin^2 x$$

b) $$\frac{4-8\sin^2 x}{2\sin x\cos x} = \frac{4\cos 2x}{\sin 2x}$$
$$= \frac{4}{\tan 2x}$$

Section 6.2 Page 307 Question 17

For the point (2, 5), $x = 2$, $y = 5$ and so $r = \sqrt{29}$.
$\cos(\pi + x) = \cos\pi\cos x - \sin\pi\sin x$
$$= -1\left(\frac{2}{\sqrt{29}}\right) - 0\left(\frac{5}{\sqrt{29}}\right)$$
$$= -\frac{2}{\sqrt{29}}$$
This answer can also be obtained by reasoning that the value of $\cos(\pi + x)$ will be numerically the same as the value of $\cos x$, but negative because the angle is in quadrant III.

Section 6.2 Page 307 Question 18

$\sin 5x \cos x + \cos 5x \sin x = \sin(5x + x)$
$= \sin 6x$
$= 2 \sin 3x \cos 3x$
The equation $\sin 5x \cos x + \cos 5x \sin x = 2 \sin kx \cos kx$ is true when $k = 3$.

Section 6.2 Page 307 Question 19

a) Given $\cos\theta = \frac{3}{5}$ and $0 < \theta < 2\pi$, θ may be in quadrant I or quadrant IV.

In quadrant I: $\sin\theta = \frac{4}{5}$

$$\sin\left(\theta + \frac{\pi}{6}\right) = \sin\theta\cos\left(\frac{\pi}{6}\right) + \cos\theta\sin\left(\frac{\pi}{6}\right)$$
$$= \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)$$
$$= \frac{4\sqrt{3}+3}{10} \approx 0.9928$$

In quadrant IV: $\sin\theta = \frac{-4}{5}$

$$\sin\left(\theta + \frac{\pi}{6}\right) = \sin\theta\cos\left(\frac{\pi}{6}\right) + \cos\theta\sin\left(\frac{\pi}{6}\right)$$
$$= \left(\frac{-4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)$$
$$= \frac{-4\sqrt{3}+3}{10} \approx -0.3928$$

b) Given $\sin\theta = -\frac{2}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$, θ must be in quadrant IV.

In quadrant IV, $\cos\theta = \frac{\sqrt{5}}{3}$,

$$\cos\left(\theta + \frac{\pi}{3}\right) = \cos\theta\cos\left(\frac{\pi}{3}\right) - \sin\theta\sin\left(\frac{\pi}{3}\right)$$
$$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{2}\right) - \left(-\frac{2}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{5}+2\sqrt{3}}{6} \approx 0.9500$$

Section 6.2 Page 307 Question 20

Given $\sin A = \frac{4}{5}$, then $\cos A = \frac{3}{5}, \tan A = \frac{4}{3}$

$\cos B = \frac{12}{13}$, then $\sin B = \frac{5}{13}, \tan A = \frac{5}{12}$

a) $\cos (A - B) = \cos A \cos B + \sin A \sin B$

$$= \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$$

$$= \frac{36 + 20}{65}$$

$$= \frac{56}{65}$$

b) $\sin (A + B) = \sin A \cos B + \cos A \sin B$

$$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$= \frac{48 + 15}{65}$$

$$= \frac{63}{65}$$

c) $\cos 2A = \cos^2 A - \sin^2 A$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$$

$$= \frac{9}{25} - \frac{16}{25}$$

$$= \frac{-7}{25}$$

d) $\sin 2A = 2 \sin A \cos A$

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

Section 6.2 Page 307 Question 21

a)

$$\frac{?\sin 2x}{2-2\cos^2 x} = \cos x$$

$$\frac{?2\sin x\cos x}{2(1-\cos^2 x)} = \cos x$$

$$\frac{?2\sin x\cos x}{2(\sin^2 x)} = \cos x$$

$$\frac{?2\cos x}{2\sin x} = \cos x$$

The equation will be true if the missing ratio is $\sin x$.

Section 6.2 Page 307 Question 22

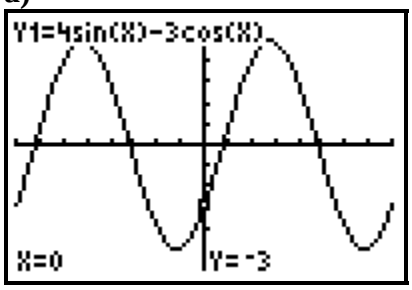
$$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$$

$$\frac{\cos x + 1}{2} = \cos^2\left(\frac{x}{2}\right)$$

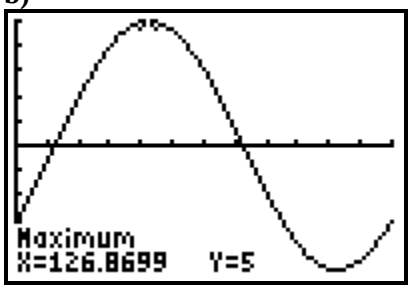
$$\pm\sqrt{\frac{\cos x + 1}{2}} = \cos\frac{x}{2}$$

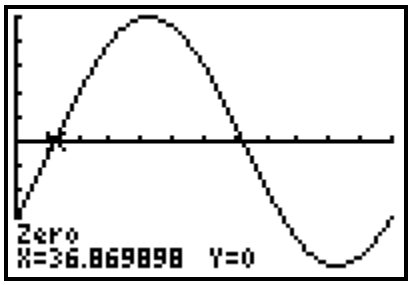
Section 6.2 Page 308 Question 23

a)



b)





By analysing the graph using technology, the maximum is 5 and the horizontal shift is approximately 37. For the curve in the form $y = a\sin(x - c)$, $a = 5$ and $c \approx 37°$.

c) $4\sin x - 3\cos x = 5\left[\sin x\left(\frac{4}{5}\right) - \cos x\left(\frac{3}{5}\right)\right]$

$= 5\left[\sin\left(x - \cos^{-1}\left(\frac{4}{5}\right)\right)\right]$

$\approx 5\sin(x - 36.87)$

Section 6.2 Page 308 Question 24

$y = 6\sin x\cos^3 x + 6\sin^3 x\cos x - 3$
$y = 6\sin x\cos x(\cos^2 x + \sin^2 x) - 3$
$y = 3(2\sin x\cos x) - 3$
$y = 3\sin 2x - 3$

Section 6.2 Page 308 Question C1

a) If $\cos x = -\frac{5}{13}$ and $\pi < x < \frac{3\pi}{2}$ then $\sin x = -\frac{12}{13}$

i) Since $\cos x = -\frac{5}{13}$ and x in in quadrant III, $x \approx 4.3176$. Then,

$\sin x = -\cos\left(x + \frac{\pi}{2}\right)$

$\sin 2x = -\cos\left(2x + \frac{\pi}{2}\right)$

$\sin 2x \approx -\cos\left(2(4.3176) + \frac{\pi}{2}\right)$

$\sin 2x \approx 0.7101$

ii) $\sin 2x = 2\sin x\cos x$

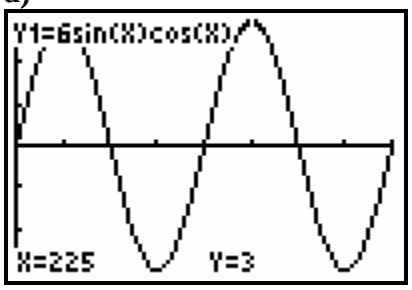
$= 2\left(-\frac{12}{13}\right)\left(-\frac{5}{13}\right)$

$= \frac{120}{169} \approx 0.7101$

b) Using the double-angle identity is more straightforward.

Section 6.2 Page 308 Question C2

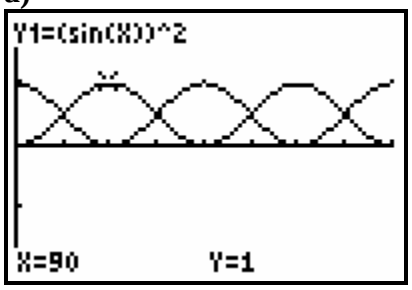
a)



b) To find the sine function from the graph you compare the amplitude and the period to that of a base sine curve. The alternative equation is $y = 3 \sin 2x$. This equation is found directly from the given equation $y = 6 \sin x \cos x$ using the the double-angle identity.

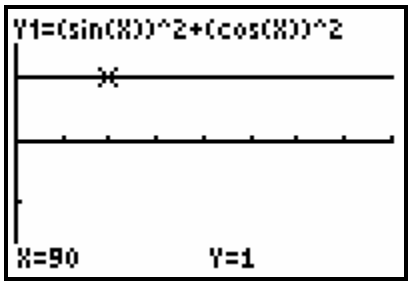
Section 6.2 Page 308 Question C3

a)

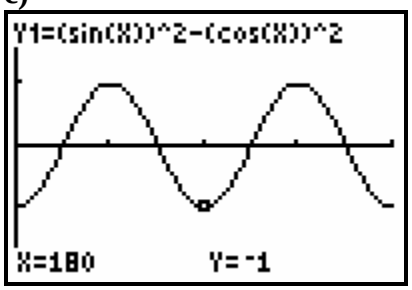


The graphs have the same shape and may be related by a reflection in the line $y = 0.5$ or by a translation of 90° to the right.

b) I predict that $y_1 + y_2$ will be a horizontal line passing through (0, 1), because the two functions increase and decrease from 0 to 1 relative to each other, and their sum is always 1.



c)



The resulting graph is a cosine function reflected over the x-axis and the period becomes π.

d) Using trigonometric identities,

$$\begin{aligned}\sin^2 x - \cos^2 x &= 1 - \cos^2 x - \cos^2 x \\ &= 1 - 2\cos^2 x \\ &= -\cos 2x\end{aligned}$$

So in the form $f(x) = a \cos bx$, the function is $f(x) = -\cos 2x$.

Section 6.3 Proving Identities

Section 6.3 Page 314 Question 1

a) $\dfrac{\sin x - \sin x \cos^2 x}{\sin^2 x} = \dfrac{\sin x(1-\cos^2 x)}{\sin^2 x}$

$= \dfrac{\sin x(\sin^2 x)}{\sin^2 x}$

$= \sin x$

b) $\dfrac{\cos^2 x - \cos x - 2}{6\cos x - 12} = \dfrac{(\cos x - 2)(\cos x + 1)}{6(\cos x - 2)}$

$= \dfrac{\cos x + 1}{6}$

c) $\dfrac{\sin x \cos x - \sin x}{\cos^2 x - 1} = \dfrac{\sin x(\cos x - 1)}{(\cos x - 1)(\cos x + 1)}$

$= \dfrac{\sin x}{\cos x + 1}$

d) $\dfrac{\tan^2 x - 3\tan x - 4}{\sin x \tan x + \sin x} = \dfrac{(\tan x - 4)(\tan x + 1)}{\sin x(\tan x + 1)}$

$= \dfrac{\tan x - 4}{\sin x}$ or $\sec x - 4\csc x$

Section 6.3 Page 314 Question 2

a) Left Side $= \cos x + \cos x \tan^2 x$

$= \cos x + \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x} + \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{1}{\cos x}$

$= \sec x$

$=$ Right Side

b) Left Side $= \dfrac{\sin^2 x - \cos^2 x}{\sin x + \cos x}$

$= \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{\sin x + \cos x}$

$= \sin x - \cos x$

$=$ Right Side

c) Left Side $= \dfrac{\sin x \cos x - \sin x}{\cos^2 x - 1}$

$= \dfrac{\sin x \cos x - \sin x}{-\sin^2 x}$

$= \dfrac{-\sin x(1 - \cos x)}{-\sin^2 x}$

$= \dfrac{1 - \cos x}{\sin x}$

= Right Side

d) Left Side $= \dfrac{1 - \sin^2 x}{1 + 2\sin x - 3\sin^2 x}$

$= \dfrac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)(1 + 3\sin x)}$

$= \dfrac{1 + \sin x}{1 + 3\sin x}$

= Right Side

Section 6.3 Page 314 Question 3

a) $\dfrac{\sin x}{\cos x} + \sec x = \dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x}$

$= \dfrac{\sin x + 1}{\cos x}$

b) $\dfrac{1}{\sin x - 1} + \dfrac{1}{\sin x + 1} = \dfrac{\sin x + 1 + \sin x - 1}{(\sin x - 1)(\sin x + 1)}$

$= \dfrac{2\sin x}{\sin^2 x - 1}$

$= \dfrac{2\sin x}{-\cos^2 x}$

$= \dfrac{-2\tan x}{\cos x}$

c) $\dfrac{\sin x}{1 + \cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos x(1 + \cos x)}{(1 + \cos x)\sin x}$

$= \dfrac{\sin^2 x + \cos x + \cos^2 x}{(1 + \cos x)\sin x}$

$= \dfrac{1 + \cos x}{(1 + \cos x)\sin x}$

$= \dfrac{1}{\sin x}$

$= \csc x$

d) $\dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1} = \dfrac{\cos x(\sec x + 1) + \cos x(\sec x - 1)}{(\sec x - 1)(\sec x + 1)}$

$$= \frac{\cos x\left(\frac{1}{\cos x} + 1\right) + \cos x\left(\frac{1}{\cos x} - 1\right)}{\sec^2 x - 1}$$

$$= \frac{1 + \cos x + 1 - \cos x}{\tan^2 x}$$

$$= \frac{2}{\tan^2 x}$$

$$= 2\cot^2 x$$

Section 6.3 Page 314 Question 4

a) $\dfrac{\sec x - \cos x}{\tan x} = \dfrac{\sec x}{\tan x} - \dfrac{\cos x}{\tan x}$

$$= \left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right) - \cos x\left(\frac{\cos x}{\sin x}\right)$$

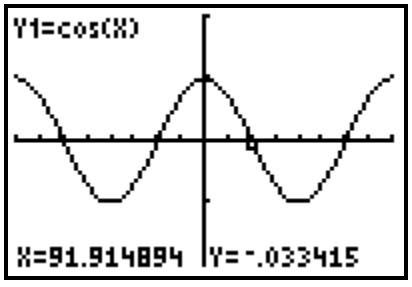
$$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

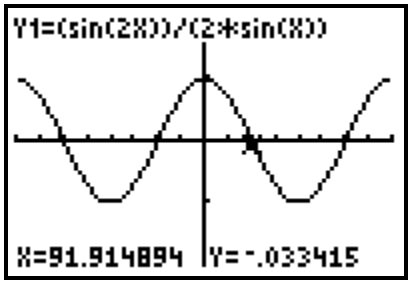
b) $\dfrac{1}{\sin x} - \dfrac{\cos^2 x}{\sin x} = \dfrac{1 - \cos^2 x}{\sin x}$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x$$

Section 6.3 Page 314 Question 5





From the graphs, $\cos x = \dfrac{\sin 2x}{2\sin x}$ appears to be an identity.

$$\frac{\sin 2x}{2\sin x} = \frac{2\sin x\cos}{2\sin x}$$

$$= \cos x$$

To allow division by $\sin x$, $x \neq \pi n;\ n \in \mathrm{I}$.

Section 6.3 Page 314 Question 6

$$\begin{aligned}(\sec x - \tan x)(\sin x + 1) &= \sec x \sin x + \sec x - \tan x \sin x - \tan x \\ &= \left(\frac{1}{\cos x}\right)\sin x + \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x}\right)\sin x - \frac{\sin x}{\cos x} \\ &= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \\ &= \frac{1-\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x}{\cos x} \\ &= \cos x\end{aligned}$$

Section 6.3 Page 314 Question 7

a)
$$\begin{aligned}\frac{\csc x}{2\cos x} &= \frac{1}{2\sin x \cos x} \\ &= \frac{1}{\sin 2x} \\ &= \csc 2x\end{aligned}$$

b)
$$\begin{aligned}\sin x + \cos x \cot x &= \sin x + \cos x\left(\frac{\cos x}{\sin x}\right) \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x} \\ &= \frac{1}{\sin x} \\ &= \csc x\end{aligned}$$

Section 6.3 Page 314 Question 8

Hanna's Method:

$$\begin{aligned}\text{Left Side} &= \frac{\cos 2x - 1}{\sin 2x} \\ &= \frac{1 - 2\sin^2 x - 1}{2\sin x \cos x} \\ &= \frac{-2\sin^2 x}{2\sin x \cos x} \\ &= \frac{-\sin x}{\cos x} \\ &= -\tan x \\ &= \text{ Right Side}\end{aligned}$$

Chloe's Method:

$$\begin{aligned}\text{Left Side} &= \frac{\cos 2x - 1}{\sin 2x} \\ &= \frac{2\cos^2 x - 1 - 1}{2\sin x \cos x} \\ &= \frac{2(\cos^2 x - 1)}{2\sin x \cos x} \\ &= \frac{2(-\sin^2 x)}{2\sin x \cos x} \\ &= \frac{-\sin x}{\cos x} \\ &= -\tan x \\ &= \text{ Right Side}\end{aligned}$$

Hanna's method is a bit simpler and leads to a shorter proof.

Section 6.3 Page 314 Question 9

a) Substitute $v_0 = 21$, $\theta = 55°$, and $g = 9.8$.

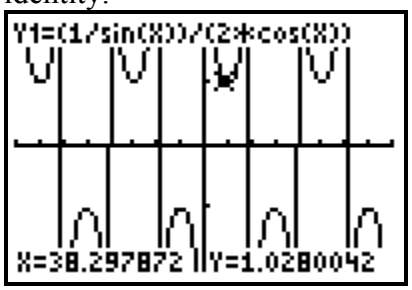
$$d = \frac{v_0^2 \sin 2\theta}{g}$$
$$= \frac{21^2 \sin 2(55°)}{9.8}$$
$$\approx 42.3$$

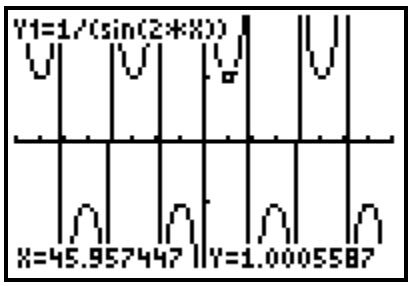
The ball will travel approximately 42.3 m.

b)
$$\frac{v_o^2 \sin 2\theta}{g} = \frac{v_o^2 2\sin\theta\cos\theta}{g}$$
$$= \frac{2v_o^2 \sin^2\theta\cos\theta}{g\sin\theta}$$
$$= \frac{2v_o^2 \sin^2\theta}{g\tan\theta}$$
$$= \frac{2v_o^2(1-\cos^2\theta)}{g\tan\theta}$$

Section 6.3 Page 314 Question 10

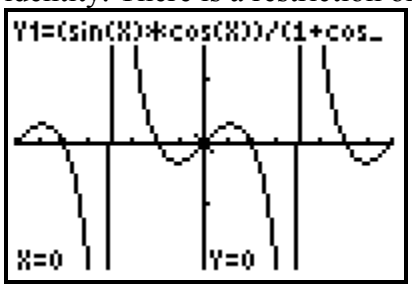
a) The graphs of each side appear to be the same, so the equation is potentially an identity.

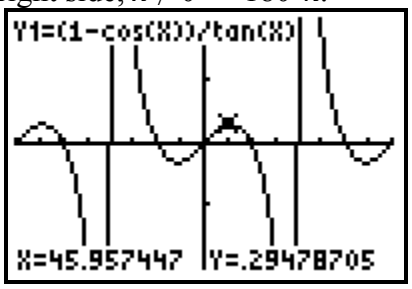




$$\text{Left Side} = \frac{\csc x}{2\cos x}$$
$$= \frac{1}{2\sin x\cos x}$$
$$= \frac{1}{\sin 2x}$$
$$= \csc 2x$$
$$= \text{Right Side}$$

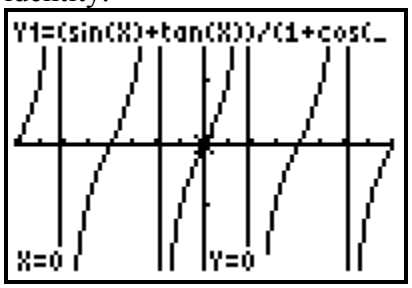
b) The graphs of each side appear to be the same, so the equation is potentially an identity. There is a restriction on the right side, $x \neq 0° + 180°n$.

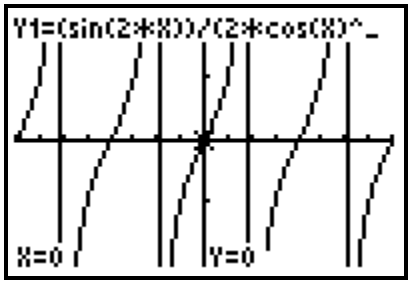




$$\begin{aligned}\text{Left Side} &= \frac{\sin x \cos x}{1+\cos x} \\ &= \frac{(\sin x \cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)} \\ &= \frac{\sin x \cos x - \sin x \cos^2 x}{\sin^2 x} \\ &= \frac{\cos x - \cos^2 x}{\sin x} \\ &= \frac{1-\cos x}{\tan x} \\ &= \text{ Right Side}\end{aligned}$$

c) The graphs of each side appear to be the same, so the equation is potentially an identity.





$$\begin{aligned}\text{Left Side} &= \frac{\sin x + \tan x}{1+\cos x} \\ &= \left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right) \div (1+\cos x) \\ &= \left(\frac{\sin x \cos x + \sin x}{\cos x}\right)\left(\frac{1}{1+\cos x}\right) \\ &= \left(\frac{\sin x(1+\cos x)}{\cos x}\right)\left(\frac{1}{1+\cos x}\right) \\ &= \frac{\sin x}{\cos x}\end{aligned}$$

Section 6.3 Page 314 Question 11

a)

$$\begin{aligned}\text{Left Side} &= \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} \\ &= \frac{2\sin x\cos x}{\cos x} + \frac{1-2\sin^2 x}{\sin x} \\ &= 2\sin x + \csc x - 2\sin x \\ &= \csc x \\ &= \text{ Right Side}\end{aligned}$$

b)

$$\begin{aligned}\text{Left Side} &= \csc^2 x + \sec^2 x \\ &= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \\ &= \frac{\sin^2 x + \cos^2 x}{\sin^2 x\cos^2 x} \\ &= \frac{1}{\sin^2 x\cos^2 x} \\ &= \csc^2 x\sec^2 x \\ &= \text{ Right Side}\end{aligned}$$

c)

$$\begin{aligned}\text{Left Side} &= \frac{\cot x - 1}{1 - \tan x} \\ &= \frac{1 - \tan x}{\tan x(1 - \tan x)} \\ &= \frac{1}{\tan x} \\ &= \frac{\csc x}{\sec x} \\ &= \text{ Right Side}\end{aligned}$$

Section 6.3 Page 315 Question 12

a) Left Side = $\sin(90° + \theta)$
$= \sin 90° \cos\theta + \cos 90° \sin\theta$
$= \cos\theta$
Right Side = $\sin(90° - \theta)$
$= \sin 90° \cos\theta - \cos 90° \sin\theta$
$= \cos\theta$
Left Side = Right Side, so $\sin(90° + \theta) = \sin(90° - \theta)$.

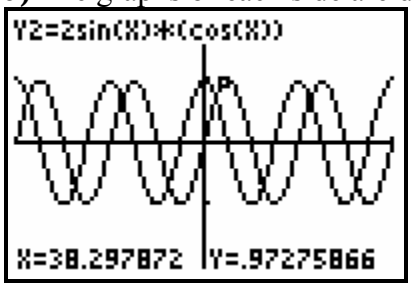
b) Left Side = $\sin(2\pi - \theta)$
$= \sin(2\pi)\cos(\theta) - \cos(2\pi)\sin(\theta)$
$= -\sin\theta$
= Right Side
So, $\sin(2\pi - \theta) = -\sin\theta$.

Section 6.3 Page 315 Question 13

Left Side = $2 \cos x \cos y$
Right Side = $\cos(x + y) + \cos(x - y)$
$= \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$
$= 2 \cos x \cos y$

Section 6.3 Page 315 Question 14

a) The graphs of each side are different so the equation is not an identity.



b) Try $x = 30°$.
Left Side = $\cos 2x$
$= \cos 2(30°)$
$= \cos 60°$
$= 0.5$

Right Side = $2 \sin x \cos x$
$= 2 \sin 30° \cos 30°$
$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{\sqrt{3}}{2}$

Left Side ≠ Right Side, so $\cos 2x = 2 \sin x \sec x$ is not an identity.

Section 6.3 Page 315 Question 15

a) For $\frac{\sin 2x}{1 - \cos 2x} = \cot x$:
The left side denominator cannot be zero.
So, $\cos 2x \neq 1$, or $2x \neq 0°$, $x \neq 0° + 180°n$.
For the right side, $\cot x = \frac{\cos x}{\sin x}$, so $\sin x \neq 0$, or $x \neq 0°$.
In general, the non-permissible values are $x \neq 180°n$, $n \in \mathrm{I}$.

b) $\text{Left Side} = \dfrac{\sin 2x}{1-\cos 2x}$

$= \dfrac{2\sin x\cos x}{1-1+2\sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$= \cot x$

$=$ Right Side

Section 6.3 Page 315 Question 16

$\text{Right Side} = \dfrac{\sin 4x-\sin 2x}{\cos 4x+\cos 2x}$

$= \dfrac{2\sin 2x\cos 2x-2\sin x\cos x}{\cos 4x+2\cos^2 x-1}$

$= \dfrac{2(2\sin x\cos x)(2\cos^2 x-1)-2\sin x\cos x}{2\cos^2 2x-1+2\cos^2 x-1}$

$= \dfrac{(2\sin x\cos x)(2(2\cos^2 x-1)-1)}{2(2\cos^2 x-1)^2+2\cos^2 x-2}$

$= \dfrac{(2\sin x\cos x)(4\cos^2 x-3)}{2(4\cos^4 x-4\cos^2 x+1)+2\cos^2 x-2}$

$= \dfrac{(2\sin x\cos x)(4\cos^2 x-3)}{8\cos^4 x-6\cos^2 x}$

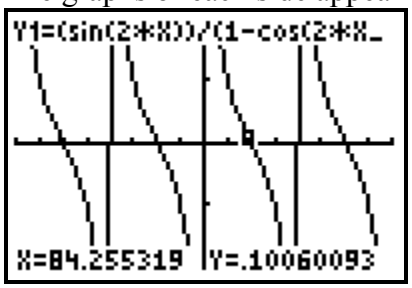
$= \dfrac{2\sin x\cos x}{2\cos^2 x}$

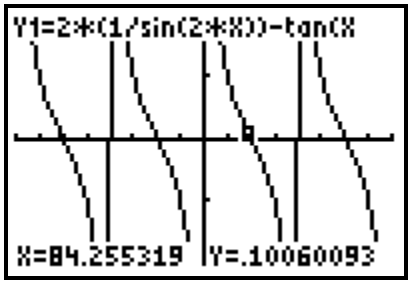
$= \tan x$

$=$ Left Side

Section 6.3 Page 315 Question 17

The graphs of each side appear to be the same.





$$\begin{aligned}\text{Left Side} &= \frac{\sin 2x}{1-\cos 2x} \\ &= \frac{\sin 2x + \sin 2x \cos 2x}{1-\cos^2 2x} \\ &= \frac{\sin 2x + \sin 2x \cos 2x}{\sin^2 2x} \\ &= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} \\ &= \frac{1}{\sin 2x} + \frac{1-2\sin^2 x}{\sin 2x} \\ &= \frac{2}{\sin 2x} - \frac{2\sin^2 x}{\sin 2x} \\ &= 2\csc 2x - \frac{2\sin^2 x}{2\sin x \cos x} \\ &= 2\csc 2x - \tan x \\ &= \text{ Right Side}\end{aligned}$$

Section 6.3 Page 315 Question 18

$$\begin{aligned}\text{Left Side} &= \frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} \\ &= \frac{\cos^2 x - 2\cos x}{\cos^2 x - \cos x - 2} \\ &= \frac{\cos x(\cos x - 2)}{(\cos x - 2)(\cos x + 1)} \\ &= \frac{\cos x}{(\cos x + 1)} \\ &= \frac{1}{1+\sec x} \\ &= \text{ Right Side}\end{aligned}$$

Section 6.3 Page 315 Question 19

a) $n_1 \sin\theta_i = n_2 \sin\theta_t$

$$\frac{n_1 \sin\theta_i}{n_2} = \sin\theta_t$$

b) Using $\sin^2\theta + \cos^2\theta = 1$, substitute $\cos\theta = \sqrt{1-\sin^2\theta}$.

$$R = \left(\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right)^2$$
$$= \left(\frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \sin^2\theta_t}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \sin^2\theta_t}} \right)^2$$

c) From part a) substitute $\sin\theta_t = \frac{n_1 \sin\theta_i}{n_2}$, or $\sin^2\theta_t = \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i$, in

$$\left(\frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \sin^2\theta_t}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \sin^2\theta_t}} \right)^2.$$

$$\left(\frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \sin^2\theta_t}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \sin^2\theta_t}} \right)^2 = \left(\frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}} \right)^2$$

Section 6.3 Page 315 Question C1

The graphs may appear to be the same but there may be some values that one function does not take; there may be discontinuities in the graph. Only pure luck would identify these values either by graphing or by checking numerically.

Section 6.3 Page 315 Question C2

$$\text{Left Side} = \cos\left(\frac{\pi}{2} - x\right)$$
$$= \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x$$
$$= \sin x$$
$$= \text{ Right Side}$$

Section 6.3 Page 315 Question C3

a) In the equation, the radical must be positive and the radicand cannot be negative. So, $\cos x \geq 0$ and $1 - \sin^2 x \geq 0$. The second condition is always true.
From the first condition, x is in quadrant I or IV. The non-permissible values are any values of x in quadrant II or III. In general, the non-permissible values are
$\frac{\pi}{2} + 2\pi n < x < \frac{3\pi}{2} + 2\pi n,\ n \in \text{I}$

b) The equation is true for $x = 1$.

Left Side = $\cos 1 \approx 0.5403$ Right Side = $\sqrt{1-\sin^2(1)} \approx 0.5403$

c) The equation is not true for $x = \pi$.

Left Side = $\cos \pi = -1$ Right Side = $\sqrt{1-\sin^2(\pi)} = 1$

d) An identity is always true for all values for which each side of the equation is defined. An equation may be true for a restricted domain.

Section 6.4 Solving Trigonometric Equations Using Identities

Section 6.4 Page 320 Question 1

In the domain $0 \le x < 2\pi$:

a) $\tan^2 x - \tan x = 0$

$\tan x (\tan x - 1) = 0$

$\tan x = 0$ or $\tan x = 1$

$x = 0, \pi$, or $x = \frac{\pi}{4}, \frac{5\pi}{4}$

b) $\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x (2 \cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = 0, \pi$, or $x = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $\sin^2 x - 4 \sin x = 5$

$\sin^2 x - 4 \sin x - 5 = 0$

$(\sin x - 5)(\sin x + 1) = 0$

$\sin x = 5$ or $\sin x = -1$

The first value for $\sin x$ is impossible.

$x = \frac{3\pi}{2}$

d) $\cos 2x = \sin x$

$1 - 2 \sin^2 x = \sin x$

$2 \sin^2 x + \sin x - 1 = 0$

$(2 \sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $x = \frac{3\pi}{2}$

Section 6.4 Page 320 Question 2

In the domain $0° \le x < 360°$:

a) $\cos x - \cos 2x = 0$

$\cos x - (2 \cos^2 x - 1) = 0$

$2 \cos^2 x - \cos x - 1 = 0$

$(2 \cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

$x = 120°, 240°$ or $x = 0°$

b) $\sin^2 x - 3 \sin x = 4$

$\sin^2 x - 3 \sin x - 4 = 0$

$(\sin x - 4)(\sin x + 1) = 0$

$\sin x = 4$ or $\sin x = -1$

The first value for $\sin x$ is impossible.

$x = 270°$

c) $\tan x \cos x \sin x - 1 = 0$

$$\left(\frac{\sin x}{\cos x}\right)\cos x \sin x - 1 = 0$$

$$\sin^2 x - 1 = 0$$

$$(\sin x - 1)(\sin x + 1) = 0$$

$\sin x = 1$ or $\sin x = -1$

$x = 90°$ or $270°$

However, the initial equation has restrictions $\cos 90° = 0$ and $\cos 270° = 0$ are not permissible.

So, there is no solution.

d) $\tan^2 x + \sqrt{3}\tan x = 0$

$$\tan x(\tan x + \sqrt{3}) = 0$$

$\tan x = 0$ or $\tan x = -\sqrt{3}$

$x = 0°, 180°$, or $x = 120°, 300°$

Section 6.4 Page 320 Question 3

In the domain $0 \le x < 2\pi$:

a) $\cos 2x - 3 \sin x = 2$

$1 - 2\sin^2 x - 3 \sin x = 2$

$2\sin^2 x + 3 \sin x + 1 = 0$

$(2 \sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = -1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ or $x = \frac{3\pi}{2}$

b) $2\cos^2 x - 3 \sin x - 3 = 0$

$2(1 - \sin^2 x) - 3 \sin x - 3 = 0$

$2\sin^2 x + 3 \sin x + 1 = 0$

$(2 \sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = -1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ or $x = \frac{3\pi}{2}$

c) $3 \csc x - \sin x = 2$

$$3\left(\frac{1}{\sin x}\right) - \sin x = 2$$

$$3 - \sin^2 x = 2 \sin x$$

$\sin^2 x + 2 \sin x - 3 = 0$

$(\sin x + 3)(\sin x - 1) = 0$

$\sin x = -3$ or $\sin x = 1$

The first value for $\sin x$ is impossible.

$x = \frac{\pi}{2}$

d) $\tan^2 x + 2 = 0$

$$\frac{\sin^2 x}{\cos^2 x} + 2 = 0$$

$$\sin^2 x + 2\cos^2 x = 0$$

$$\sin^2 x + 2(1 - \sin^2 x) = 0$$

$$2 - \sin^2 x = 0$$

$\sin^2 x = 2$ is impossible, so the equation has no solution.

Section 6.4 Page 320 Question 4

$4 \sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

In the domain $-180° \le x < 180°$: $x = -150°, -30°, 30°, 150°$

Section 6.4 Page 320 Question 5

$2 \tan^2 x + 3 \tan x - 2 = 0$

$(2 \tan x - 1)(\tan x + 2) = 0$

$\tan x = \frac{1}{2}$ or $\tan x = -2$

In the domain $0 \le x < 2\pi$: $x \approx 0.4636, 3.6052,$ or $x \approx 2.0344, 5.1760$

Section 6.4 Page 321 Question 6

Sanesh should not have divided both sides by cos x. Some solutions were lost by doing that.

$2\cos^2 x = \sqrt{3}\cos x$

$2\cos^2 x - \sqrt{3}\cos x = 0$

$\cos x(2\cos x - \sqrt{3}) = 0$

$\cos x = 0$ or $\cos x = \frac{\sqrt{3}}{2}$

$x = 90° + 360°n$ and $x = 270° + 360°n$, or $x = 30° + 360°n$ and $x = 330° + 360°n$, where $n \in$ I.

Section 6.4 Page 321 Question 7

a) $\sin 2x = 0.5$

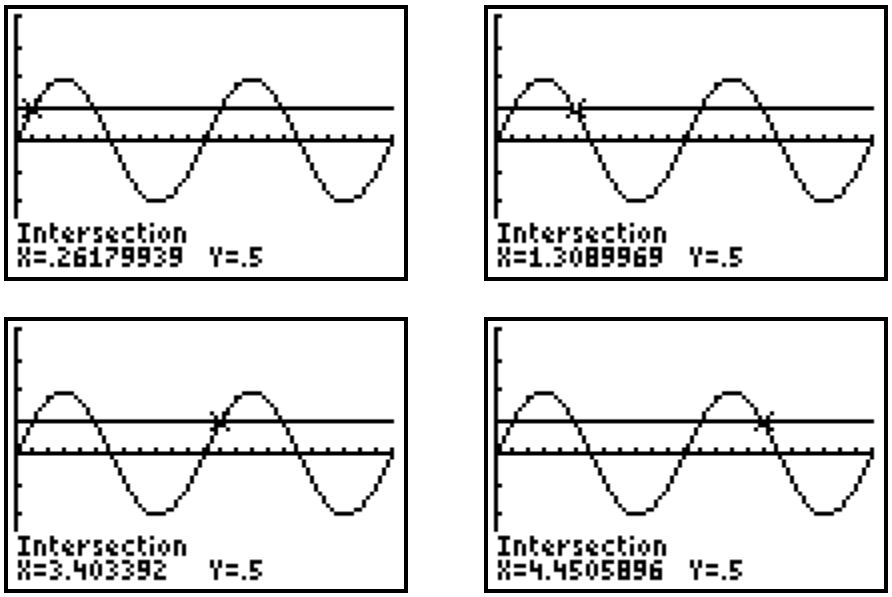
$2x = \sin^{-1}(0.5)$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$

In the domain $0 \le x < 2\pi$:

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

b) Graph $y = \sin 2x$ and $y = 0.5$ over the domain $0 \leq \theta \pi$ and find the point(s) of intersection.



Section 6.4 Page 321 Question 8

$$\sin^2 x = \cos^2 x + 1$$
$$\sin^2 x = 1 - \sin^2 x + 1$$
$$2 \sin^2 x - 2 = 0$$
$$2 (\sin^2 x - 1) = 0$$
$$\sin^2 x = 1$$
$$\sin x = \pm 1$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

In general, $x = \frac{\pi}{2} + \pi n$, $n \in \mathrm{I}$

Section 6.4 Page 321 Question 9

$$\cos x \sin 2x - 2 \sin x = -2$$
$$\cos x (2 \sin x \cos x) - 2 \sin x + 2 = 0$$
$$2 \sin x \cos^2 x - 2 \sin x + 2 = 0$$
$$\sin x(1 - \sin^2 x) - \sin x + 1 = 0$$
$$\sin x - \sin^3 x - \sin x + 1 = 0$$
$$\sin^3 x = 1$$
$$\sin x = 1$$

The general solution, in radians, is $x = \frac{\pi}{2} + 2\pi n$, $n \in \mathrm{I}$.

Section 6.4 Page 321 Question 10

The equation $(7 \sin x + 2)(3 \cos x + 3)(\tan^2 x - 2) = 0$ will have 7 solutions over the interval $0° < x \leq 360°$. The first factor yields two solutions, one each is in quadrants II and IV where x is $\sin^{-1}\left(-\frac{2}{7}\right)$. The second factor yields one solution, $\cos^{-1}(-1)$ and the third factor yields four solutions, one in each quadrant for $\tan^{-1}(\pm 2)$.

Section 6.4 Page 321 Question 11

$$\sqrt{3}\cos x \csc x = -2\cos x$$
$$2\cos x + \sqrt{3}\cos x \csc x = 0$$
$$\cos x(2 + \sqrt{3}\csc x) = 0$$
$$\cos x = 0 \text{ or } \csc x = -\frac{2}{\sqrt{3}}$$
$$\sin x = -\frac{\sqrt{3}}{2}$$

Over the domain $0 \leq x < 2\pi$:

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

Section 6.4 Page 321 Question 12

Given that $\cos x = \frac{2}{3}$ and $\cos x = -\frac{1}{3}$ are the solutions for a trigonometric equation, then the equation has the form

$(3 \cos x - 2)(3 \cos x + 1) = 0$

$9 \cos^2 x - 3 \cos x - 2 = 0$

So, in the form $9 \cos^2 x - B \cos x - C = 0$, $B = -3$ and $C = -2$.

Section 6.4 Page 321 Question 13

Example: Give a general solution, in degrees to the following equation.

$\sin 2x + \sin 2x \cos x = 0$

$\sin 2x(1 + \cos x) = 0$

$\sin 2x = 0$ or $\cos x = -1$

$2x = 0°, 180°, 360°, \ldots$ or $x = 180°, 540°, \ldots$

$x = 0°, 90°, 180°, 270°, 360°, \ldots$

In general, $x = 90°n$, $n \in$ I.

Section 6.4 Page 321 Question 14

$$\sin 2x = 2\cos x \cos 2x$$
$$2\sin x \cos x - 2\cos x \cos 2x = 0$$
$$2\sin x \cos x - 2\cos x (1 - 2\sin^2 x) = 0$$
$$2\sin x \cos x - 2\cos x + 4\cos x \sin^2 x = 0$$
$$2\cos x (2\sin^2 x + \sin x - 1) = 0$$
$$2\cos x (2\sin x - 1)(\sin x + 1) = 0$$
$$\cos x = 0 \text{ or } \sin x = \frac{1}{2} \text{ or } \sin x = -1$$

So, $x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$ or $x = \frac{3\pi}{2}, \ldots$

The general solution, in radians, is $x = \left(\frac{\pi}{2}\right)(2n+1)$ or $x = \left(\frac{\pi}{6}\right) + 2\pi n$ or $\left(\frac{5\pi}{6}\right) + 2\pi n$,

where $n \in$ I.

Section 6.4 Page 321 Question 15

Over the domain $-360° < x \leq 360°$:

$$\cos 2x \cos x - \sin 2x \sin x = 0$$
$$(1 - 2\sin^2 x)\cos x - 2\sin x \cos x \sin x = 0$$
$$\cos x - 4\sin^2 x \cos x = 0$$
$$\cos x (1 - 4\sin^2 x) = 0$$
$$\cos x = 0 \text{ or } \sin x = \pm\frac{1}{2}$$

$x = -270°, -90°, 90°, 270°$ or $x = -330°, -210°, -150°, -30°, 30°, 150°, 210°, 330°$

There are 12 solutions in the given domain.

Section 6.4 Page 321 Question 16

$$\sec x + \tan^2 x - 3\cos x = 2$$
$$\frac{1}{\cos x} + \frac{\sin^2 x}{\cos^2 x} - 3\cos x = 2$$
$$\cos x + \sin^2 x - 3\cos^3 x - 2\cos^2 x = 0$$
$$\cos x + 1 - \cos^2 x - 3\cos^3 x - 2\cos^2 x = 0$$
$$3\cos^3 x + 3\cos^2 x - \cos x - 1 = 0$$

Checking the equation, considering it as $3x^3 + 3x^2 - x - 1 = 0$, with the factor theorem reveals that $\cos x + 1$ is one factor.

$$(\cos x + 1)(3\cos^2 x - 1) = 0$$
$$\cos x = -1 \text{ or } \cos x = \pm\sqrt{\frac{1}{3}}$$

The general solution, in radians, is $x = \pi + 2\pi n$, or $x \approx \pm 0.9553 + \pi n$, where $n \in$ I.

Section 6.4 Page 321 Question 17

$$4\sin^2 x = 3\tan^2 x - 1$$
$$4(1-\cos^2 x) = 3\left(\frac{\sin^2 x}{\cos^2 x}\right) - 1$$
$$4\cos^2 x - 4\cos^4 x = 3\sin^2 x - \cos^2 x$$
$$4\cos^2 x - 4\cos^4 x = 3(1-\cos^2 x) - \cos^2 x$$
$$4\cos^2 x - 4\cos^4 x = 3 - 4\cos^2 x$$
$$4\cos^4 x - 8\cos^2 x + 3 = 0$$
$$(2\cos^2 x - 3)(2\cos^2 x - 1) = 0$$
$$\cos^2 x = \frac{3}{2} \text{ or } \cos^2 x = \frac{1}{2}$$

The first equation is impossible, so $\cos x = \pm\frac{1}{\sqrt{2}}$.

Then, the general solution, in radians, is $x = \pm\frac{\pi}{4} + \pi n$, $n \in \text{I}$.

Section 6.4 Page 321 Question 18

$$\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = -\frac{1}{3}$$
$$3 - 3\sin^2 x - 6\cos x = -\cos^2 x + \cos x + 2$$
$$3\cos^2 x + \cos^2 x - 7\cos x - 2 = 0$$
$$4\cos^2 x - 7\cos x - 2 = 0$$
$$(4\cos x + 1)(\cos x - 2) = 0$$

The second factor does not yield any possible solutions.
From the first factor, $\cos x = -0.25$.
In the domain $-\pi \le x \le \pi$, $x \approx 1.8235$ or $x = -1.8235$.

Section 6.4 Page 321 Question 19

$$4(16^{\cos^2 x}) = 2^{6\cos x}$$
$$2^2(2^{4\cos^2 x}) = 2^{6\cos x}$$
$$2^{2+4\cos^2 x} = 2^{6\cos x}$$

Then, $2 + 4\cos^2 x = 6\cos x$

$$2\cos^2 x - 3\cos x + 1 = 0$$
$$(2\cos x - 1)(\cos x - 1) = 0$$

So, $\cos x = \frac{1}{2}$ or $\cos x = 1$.

The general solution, in radians, is $x = \pm\frac{\pi}{3} + 2\pi n$ or $x = 2\pi n$, $n \in \mathrm{I}$.

Section 6.4 Page 321 Question 20

$\sin^2 \alpha + \cos^2 \beta = m^2$ ①
$\cos^2 \alpha + \sin^2 \beta = m$ ②
Add ① + ②.
$\sin^2 \alpha + \cos^2 \alpha + \cos^2 \beta + \sin^2 \beta = m^2 + m$
$2 = m^2 + m$
$0 = m^2 + m - 2$
$0 = (m + 2)(m - 1)$
$m = -2$ or $m = 1$

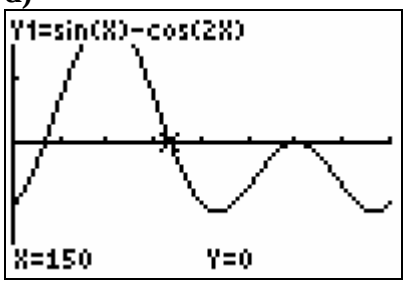
Section 6.4 Page 321 Question C1

a) To express the equation $\sin x - \cos 2x = 0$ in terms of one trigonometric function, sine, use the identity $\cos 2x = 1 - 2\sin^2 x$.

b) Substitute $\cos 2x = 1 - 2\sin^2 x$.
$\sin x - \cos 2x = 0$
$\sin x - (1 - 2\sin^2 x) = 0$
$2\sin^2 x + \sin x - 1 = 0$
$(2\sin x - 1)(\sin x + 1) = 0$

c) $2\sin x - 1 = 0$ or $\sin x + 1 = 0$
$\sin x = \frac{1}{2}$ or $\sin x = -1$
For the domain $0° \le x < 360°$,
$x = 30°, 150°, 270°$

d)



Section 6.4 Page 321 Question C2

a) It is not possible to factor $3\cos^2 x + \cos x - 1$ because there are no two integers with a sum of 1 and a product of 3.

b) $\cos x = \dfrac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)}$

$= \dfrac{-1 \pm \sqrt{13}}{6}$

≈ -0.7676 or 0.4343

c) Over the domain $0° \le x < 720°$, $x \approx 140.14°$, $219.86°$, $500.14°$, $579.86°$, or $64.26°$, $295.74°$, $424.26°$, $655.74°$.

Section 6.4 Page 321 Question C3

Example: $\sin 2x \cos x + \cos x = 0$. This is not an identity because it is not true for all value of x. For example, when $x = 0°$ the left side has value 1 and the right side has value 0.

$\sin 2x \cos x + \cos x = 0$

$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -\dfrac{1}{2}$

In general, $x = 90° + 180°n$, or $x = 135° + 180°n$, where $n \in \text{I}$.

Chapter 6 Review

Chapter 6 Review Page 322 Question 1

In each, the denominator cannot be 0.

a) For the expression $\dfrac{3\sin x}{\cos x}$, $\cos x \ne 0$. So the restriction is $x \ne \dfrac{\pi}{2} + \pi n,\ n \in \text{I}$.

b) For the expression $\dfrac{\cos x}{\tan x}$, $\tan x \ne 0$. So the restriction is $x \ne \left(\dfrac{\pi}{2}\right)n,\ n \in \text{I}$.

c) For the expression $\dfrac{\sin x}{1 - 2\cos x}$, $1 - 2\cos x \ne 0$. Then, $\cos x \ne \dfrac{1}{2}$. So, the restriction is $x \ne \pm\dfrac{\pi}{3} + 2\pi n,\ n \in \text{I}$.

d) For the expression $\frac{\cos x}{\sin^2 x - 1}$, $\sin^2 x - 1 \neq 0$. Then, $\sin x = \pm 1$. So, the restriction is $x \neq \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

Chapter 6 Review Page 322 Question 2

a)
$$\begin{aligned}\frac{\sin x}{\tan x} &= \sin x \div \left(\frac{\sin x}{\cos x}\right)\\ &= \sin x \left(\frac{\cos x}{\sin x}\right)\\ &= \cos x\end{aligned}$$

b)
$$\begin{aligned}\frac{\sec x}{\csc x} &= \frac{1}{\cos x} \div \left(\frac{1}{\sin x}\right)\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

c)
$$\begin{aligned}\frac{\sin x + \tan x}{1 + \cos x} &= \frac{\sin x + \frac{\sin x}{\cos x}}{1 + \cos x}\\ &= \frac{\sin x(\cos x + 1)}{\cos x} \div (1 + \cos x)\\ &= \tan x\end{aligned}$$

d)
$$\begin{aligned}\frac{\csc x - \sin x}{\cot x} &= \left(\frac{1}{\sin x} - \sin x\right) \div \left(\frac{\cos x}{\sin x}\right)\\ &= \left(\frac{1 - \sin^2 x}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)\\ &= \frac{\cos^2 x}{\cos x}\\ &= \cos x\end{aligned}$$

Chapter 6 Review Page 322 Question 3

a)
$$\begin{aligned}\tan x \cot x &= \left(\frac{\sin x}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)\\ &= 1\end{aligned}$$

b)
$$\begin{aligned}\frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} &= \sin^2 x + \cos^2 x\\ &= 1\end{aligned}$$

c)
$$\begin{aligned}\sec^2 x - \tan^2 x &= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1 - \sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x}\\ &= 1\end{aligned}$$

Chapter 6 Review Page 322 Question 4

When $x = 30°$:

$$\text{Left Side} = \frac{\cos x}{1-\sin x}$$
$$= \frac{\cos 30°}{1-\sin 30°}$$
$$= \frac{\sqrt{3}}{2} \div \left(1-\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2}(2)$$
$$= \sqrt{3}$$

$$\text{Right Side} = \frac{1+\sin x}{\cos x}$$
$$= \frac{1+\sin 30°}{\cos 30°}$$
$$= \left(1+\frac{1}{2}\right) \div \frac{\sqrt{3}}{2}$$
$$= \left(\frac{3}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$$
$$= \sqrt{3}$$

Left Side = Right Side

When $x = \frac{\pi}{4}$:

$$\text{Left Side} = \frac{\cos x}{1-\sin x}$$
$$= \frac{\cos \frac{\pi}{4}}{1-\sin \frac{\pi}{4}}$$
$$= \frac{1}{\sqrt{2}} \div \left(1-\frac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$
$$= \frac{1}{\sqrt{2}-1} \text{ or } \sqrt{2}+1$$

$$\text{Right Side} = \frac{1+\sin x}{\cos x}$$
$$= \frac{1+\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}}$$
$$= \left(1+\frac{1}{\sqrt{2}}\right) \div \frac{1}{\sqrt{2}}$$
$$= \left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{1}\right)$$
$$= \sqrt{2}+1$$

Left Side = Right Side

Chapter 6 Review Page 322 Question 5

a) Examples: When $x = \frac{\pi}{4}$:

$$\text{Left Side} = \sqrt{\tan^2\left(\frac{\pi}{4}\right)+1} \qquad \text{Right Side} = \frac{1}{\cos\left(\frac{\pi}{4}\right)}$$
$$= \sqrt{1+1} \qquad = \sqrt{2}$$
$$= \sqrt{2}$$

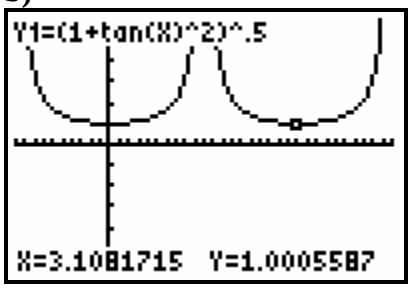
Left Side = Right Side

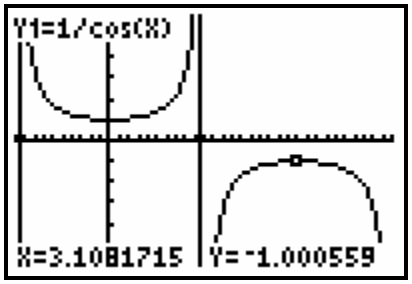
When $x = 1$:

$$\text{Left Side} = \sqrt{\tan^2(1)+1} \qquad \text{Right Side} = \frac{1}{\cos 1}$$
$$\approx 1.8508 \qquad \approx 1.8508$$

Left Side = Right Side

b)





The graphs appear to be the same for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ but differ for $\frac{\pi}{2} \le x < \frac{3\pi}{2}$.

c) The graph shows that both sides of the equation do not have the same value for all values in the given domain. There are many values of x for which the two sides of the equation have different values.

Chapter 6 Review Page 322 Question 6

a) $f(x) = \sin x + \cos x + \sin 2x + \cos 2x$
$f(0) = \sin 0 + \cos 0 + \sin 2(0) + \cos 2(0)$
$= 0 + 1 + 0 + 1$
$= 2$

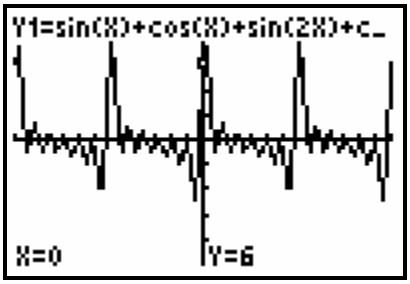
$$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) + \sin 2\left(\frac{\pi}{6}\right) + \cos 2\left(\frac{\pi}{6}\right)$$
$$= \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2}$$
$$= 1 + \sqrt{3}$$

b) $f(x) = \sin x + \cos x + \sin 2x + \cos 2x$
$= \sin x + \cos x + 2 \sin x \cos x + (1 - 2 \sin^2 x)$
$= \sin x + \cos x + 2 \sin x \cos x - 2 \sin^2 x + 1$

c) This Fourier series cannot be written using only sine or only cosine because the three terms $\sin x + \cos x + 2 \sin x \cos x$ cannot be expressed in terms of one of the ratios.

d) $f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x$
$f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x + \sin 4x + \cos 4x$
$f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x + \sin 4x + \cos 4x + \sin 5x + \cos 5x$
$f(x) = \sin x + \cos x + \sin 2x + \cos 2x + \sin 3x + \cos 3x + \sin 4x + \cos 4x + \sin 5x + \cos 5x + \sin 6x + \cos 6x$

The curved does not smooth out perfectly, but the last equation above gives a reasonable approximation, as shown.



Chapter 6 Review Page 322 Question 7

a) $\sin 25° \cos 65° + \cos 25° \sin 65°$
$= \sin (25° + 65°)$
$= \sin 90°$
$= 1$

b) $\sin 54° \cos 24° - \cos 54° \sin 24°$
$= \sin (54° - 24°)$
$= \sin 30°$
$= \frac{1}{2}$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12} + \sin\frac{\pi}{4}\sin\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$
$= \cos\left(\frac{3\pi - \pi}{12}\right)$
$= \cos\frac{\pi}{6}$
$= \frac{\sqrt{3}}{2}$

d) $\cos\frac{\pi}{6}\cos\frac{\pi}{12}-\sin\frac{\pi}{6}\sin\frac{\pi}{12}=\cos\left(\frac{\pi}{6}+\frac{\pi}{12}\right)$

$$=\cos\frac{3\pi}{12}$$

$$=\cos\frac{\pi}{4}$$

$$=\frac{1}{\sqrt{2}}$$

Chapter 6 Review Page 323 Question 8

a) $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 30° \sin 45°$

$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$=\frac{\sqrt{3}-1}{2\sqrt{2}} \text{ or } \frac{\sqrt{6}-\sqrt{2}}{4}$$

b) $\cos\left(-\frac{\pi}{12}\right)=\cos\left(\frac{3\pi}{12}-\frac{4\pi}{12}\right)$

$$=\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$$

$$=\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$=\frac{1+\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}+\sqrt{6}}{4}$$

c)

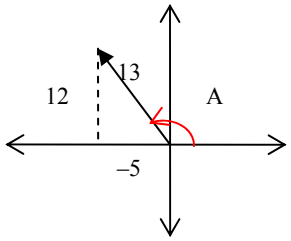
$$\begin{aligned}\tan 165° &= \tan(120° + 45°)\\ &= \frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}\\ &= \frac{-\sqrt{3}+1}{1-(-\sqrt{3})}\\ &= \frac{\left(1-\sqrt{3}\right)\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}\\ &= \frac{4-2\sqrt{3}}{-2}\\ &= \sqrt{3}-2\end{aligned}$$

d)

$$\begin{aligned}\sin\frac{5\pi}{12} &= \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\\ &= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)\\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ or } \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

Chapter 6 Review Page 323 Question 9

Given $\cos A = -\frac{5}{13}$, $\frac{\pi}{2} \le A \le \pi$, A is in quadrant II and $\sin A = \frac{12}{13}$.



a)

$$\begin{aligned}\cos\left(A-\frac{\pi}{4}\right) &= \cos A\cos\frac{\pi}{4}+\sin A\sin\frac{\pi}{4}\\ &= \left(-\frac{5}{13}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{12}{13}\right)\left(\frac{1}{\sqrt{2}}\right)\\ &= \frac{7}{13\sqrt{2}} \text{ or } \frac{7\sqrt{2}}{26}\end{aligned}$$

b)

$$\begin{aligned}\sin\left(A+\frac{\pi}{3}\right) &= \sin A\cos\left(\frac{\pi}{3}\right)+\cos A\sin\left(\frac{\pi}{3}\right)\\ &= \left(\frac{12}{13}\right)\left(\frac{1}{2}\right)+\left(\frac{-5}{13}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{12-5\sqrt{3}}{26}\end{aligned}$$

c)

$$\begin{aligned}\sin 2A &= 2\sin A\cos A\\ &= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)\\ &= -\frac{120}{169}\end{aligned}$$

Chapter 6 Review Page 323 Question 10

$$\left(\sin\frac{\pi}{8}+\cos\frac{\pi}{8}\right)^2 = \sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$$
$$= 1+\sin 2\left(\frac{\pi}{8}\right)$$
$$= 1+\sin\frac{\pi}{4}$$
$$= 1+\frac{1}{\sqrt{2}}$$

Chapter 6 Review Page 323 Question 11

$$\frac{\cos^2 x-\cos 2x}{0.5\sin 2x} = \frac{\cos^2 x-(\cos^2 x-\sin^2 x)}{0.5(2\sin x\cos x)}$$
$$= \frac{\sin^2 x}{\sin x\cos x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

Chapter 6 Review Page 323 Question 12

a) $$\frac{1-\sin^2 x}{\cos x\sin x-\cos x} = \frac{\cos^2 x}{\cos x(\sin x-1)}$$
$$= \frac{\cos x}{\sin x-1}$$

b) $\tan^2 x - \cos^2 x \tan^2 x = \tan^2 x(1 - \cos^2 x)$
$= \tan^2 x \sin^2 x$

Chapter 6 Review Page 323 Question 13

a) Left Side $= 1 + \cot^2 x$
$$= 1+\frac{\cos^2 x}{\sin^2 x}$$
$$= \frac{\sin^2 x+\cos^2 x}{\sin^2 x}$$
$$= \frac{1}{\sin^2 x}$$
$= \csc^2 x$
= Right Side

b) Right Side $= \csc 2x - \cot 2x$
$$= \frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}$$
$$= \frac{1-(2\cos^2 x-1)}{2\sin x\cos x}$$
$$= \frac{2\sin^2 x}{2\sin x\cos x}$$
$= \tan x$
= Left Side

c) Left Side $= \sec x + \tan x$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$
$$= \frac{1+\sin x}{\cos x}$$
$$= \frac{1-\sin^2 x}{(1-\sin x)\cos x}$$
$$= \frac{\cos x}{1-\sin x}$$
$$= \text{Right Side}$$

d) Left Side $= \frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

$$= \frac{1-\cos x}{1-\cos^2 x} + \frac{1+\cos x}{1-\cos^2 x}$$
$$= \frac{2}{\sin^2 x}$$
$$= 2\csc^2 x$$
$$= \text{Right Side}$$

Chapter 6 Review Page 323 Question 14

a) When $x = \frac{\pi}{4}$:

Left Side = $\sin 2x$

$$= \sin 2\left(\frac{\pi}{4}\right)$$
$$= \sin \frac{\pi}{2}$$
$$= 1$$

Right Side $= \frac{2\tan x}{1+\tan^2 x}$

$$= \frac{2\tan\left(\frac{\pi}{4}\right)}{1+\tan^2\left(\frac{\pi}{4}\right)}$$
$$= \frac{2(1)}{1+1}$$
$$= 1$$

Left Side = Right Side

The fact that an equation is true for one particular value does not prove that it is an identity. An identity is true for all permissible values.

b) The non-permissible values occur when $\tan x$ is undefined.

$x = \frac{\pi}{2} + n\pi,\ n \in \mathrm{I}$

c) Left Side $= \sin 2x$

$$= 2\sin x \cos x$$
$$= \frac{2\sin x \cos^2 x}{\cos x}$$
$$= \frac{2\tan x}{\sec^2 x}$$
$$= \frac{2\tan x}{1+\tan^2 x}$$
$$= \text{Right Side}$$

Chapter 6 Review Page 323 Question 15

a) $\text{Left Side} = \dfrac{\cos x + \cot x}{\sec x + \tan x}$

$$= \frac{\cos x + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$$

$$= \frac{\dfrac{\sin x \cos^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}}{1 + \sin x}$$

$$= \frac{\dfrac{(\sin x + 1)\cos^2 x}{\sin x}}{1 + \sin x}$$

$$= \frac{\cos x \cos x}{\sin x}$$

$$= \cos x \cot x$$

$$= \text{Right Side}$$

b) $\text{Left Side} = \sec x + \tan x$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{(1 - \sin x)\cos x}$$

$$= \frac{\cos x}{1 - \sin x}$$

$$= \text{Right Side}$$

Chapter 6 Review Page 323 Question 16

a) To prove that $\cos 2x = 2 \sin x \sec x$ is an identity, you might use algebraic reasoning or compare the graphs of each side. It is definitely not an identity if you find a value for which the left side is not equal to the right side.

b) When $x = 0$, the left side has value 1 and the right side has value 0. So, the equation is not an identity.

Chapter 6 Review Page 323 Question 17

Use the domain $0 \le x < 2\pi$.

a) $\sin 2x + \sin x = 0$

$2 \sin x \cos x + \sin x = 0$

$\sin x (2 \cos x + 1) = 0$

$\sin x = 0$ or $\cos x = -\dfrac{1}{2}$

$x = 0, \pi,$ or $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$x = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

b) $\cot x + \sqrt{3} = 0$

$$\cot x = -\sqrt{3}$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

c) $2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 2$ (which is not possible)

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

d) $\sin^2 x = \cos x - \cos 2x$

$1 - \cos^2 x = \cos x - (2\cos^2 x - 1)$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or 0

Chapter 6 Review Page 323 Question 18

a) $2\sin 2x = 1$

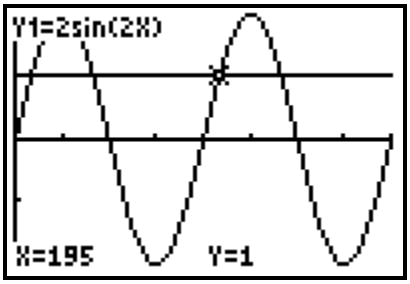
$\sin 2x = \frac{1}{2}$

$2x = 30°$ or 150°, or 390°, 510°, …

So, in the domain $0° \le x < 360°$,

$x = 15°$, 75°, or 195°, 255°.

Graph $y = 2\sin 2x$ and $y = 1$ to verify the four values.



b) $\sin^2 x = 1 + \cos^2 x$

$\sin^2 x - \cos^2 x = 1$

$-\cos 2x = 1$

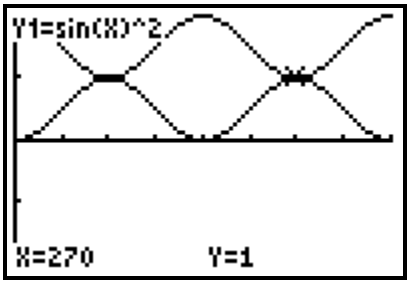
$\cos 2x = -1$

$2x = 180°, 540°, …$

So, in the domain $0° \le x < 360°$,

$x = 90°, 270°$.

Graph $y = \sin^2 x$ and $y = 1 + \cos^2 x$ to verify the two values.



c) $2\cos^2 x = \sin x + 1$

$2(1 - \sin^2 x) = \sin x + 1$

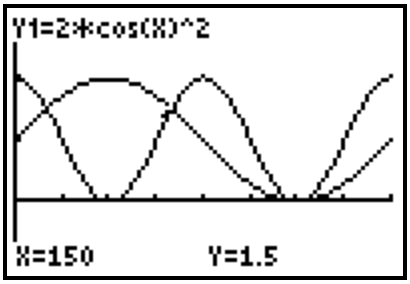
$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = 30°$, 150°, or $x = 270°$

Graph $y = 2\cos^2 x$ and $y = \sin x + 1$ to verify the three values.



d) $\cos x \tan x - \sin^2 x = 0$

$$\cos x\left(\frac{\sin x}{\cos x}\right) - \sin^2 x = 0$$

$$\sin x - \sin^2 x = 0$$

$$\sin x(1 - \sin x) = 0$$

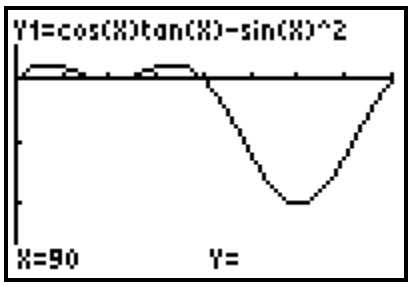
$\sin x = 0$ or $1 - \sin x = 0$

$\sin x = 1$

$x = 0°, 180°$ $x = 90°$

Graph $y = \cos x \tan x - \sin^2 x$ to verify the roots. The other possible root, $x = 90°$, does not check.

The solution is $x = 0°, 180°$.



Chapter 6 Review Page 323 Question 19

$$4\cos^2 x - 1 = 0$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm\frac{1}{2}$$

The general solution, in radians, is $x = \pm\frac{\pi}{3} + n\pi, n \in \mathrm{I}$

Chapter 6 Review Page 323 Question 20

$$2\cos^2 x + \sin^2 x = \frac{41}{25}$$

$$2\cos^2 x + 1 - \cos^2 x = \frac{41}{25}$$

$$\cos^2 x = \frac{41}{25} - 1$$

$$\cos^2 x = \frac{16}{25}$$

$$\cos x = \pm\frac{4}{5}$$

Chapter 6 Review Page 323 Question 21

$$2 \sin x \cos x = 3 \sin x$$

$$2 \sin x \cos x - 3 \sin x = 0$$

$$\sin x (2 \cos x - 3) = 0$$

$\sin x = 0$ or $\cos x = \frac{3}{2}$

The second equation is impossible.
So, in the domain $-2\pi \le x \le 2\pi$, $x = -2\pi, -\pi, 0, \pi, 2\pi$.

Chapter 6 Practice Test Page 324 Question 1

$$\begin{aligned}\frac{\cos 2x - 1}{\sin 2x} &= \frac{-2\sin^2 x}{2\sin x \cos x} \\ &= -\frac{\sin x}{\cos x} \\ &= -\tan x\end{aligned}$$

The correct answer is **A**.

Chapter 6 Practice Test Page 324 Question 2

$$\begin{aligned}\cot\theta + \tan\theta &= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \\ &= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \\ &= \frac{1}{\sin\theta\cos\theta}\end{aligned}$$

The correct answer is **A**.

Chapter 6 Practice Test Page 324 Question 3

$$\begin{aligned}&\tan^2\theta\csc\theta + \frac{1}{\sin\theta} \\ &= \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\sin\theta}\right) + \frac{1}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin\theta} \\ &= \frac{1}{\cos^2\theta\sin\theta} \\ &= \sec^2\theta\csc\theta\end{aligned}$$

The correct answer is **D**.

Chapter 6 Practice Test Page 324 Question 4

$$\cos\frac{\pi}{5}\cos\frac{\pi}{6}-\sin\frac{\pi}{5}\sin\frac{\pi}{6}$$
$$=\cos\left(\frac{\pi}{5}+\frac{\pi}{6}\right)$$
$$=\cos\frac{11\pi}{30}$$

The correct answer is **D**.

Chapter 6 Practice Test Page 324 Question 5

$$4\cos^2 x - 2 = 2(2\cos^2 x - 1)$$
$$= 2\cos 2x$$

The correct answer is **A**.

Chapter 6 Practice Test Page 324 Question 6

If $\sin\theta = c$ and $0 \le \theta < \frac{\pi}{2}$,

$\cos(\pi + \theta)$ must be in quadrant III, with $y = -\sqrt{1-c^2}$.

So $\cos(\pi + \theta) = -\sqrt{1-c^2}$.

The correct answer is **D**.

Chapter 6 Practice Test Page 324 Question 7

a) $\cos 105° = \cos(60° + 45°)$
$= \cos 60° \cos 45° - \sin 60° \sin 45°$
$$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$=\frac{1-\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}-\sqrt{6}}{4}$$

b) $\sin\frac{5\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$

Chapter 6 Practice Test Page 324 Question 8

Left Side = $\cot\theta - \tan\theta$

$= \frac{1}{\tan\theta} - \tan\theta$

$= \frac{1-\tan^2\theta}{\tan\theta}$

$= 2\cot 2\theta$

= Right Side

$\theta = \left(\frac{\pi}{2}\right)n,\ n \in \mathrm{I}$

Chapter 6 Practice Test Page 324 Question 9

Theo's Formula = $I_0\cos^2\theta$

$= I_0 - I_0\sin^2\theta$

$= I_0 - \frac{I_0}{\csc^2\theta}$

= Sany's Formula

Chapter 6 Practice Test Page 324 Question 10

a) $\sec A + 2 = 0$

$\frac{1}{\cos A} = -2$

$\cos A = -\frac{1}{2}$

The reference angle for A = $\frac{\pi}{3}$ and A is in quadrant II or III.

So, $A = \frac{2\pi}{3} + 2\pi n$, or $\frac{4\pi}{3} + 2\pi n$, where $n \in I$.

b) $2 \sin B = 3 \tan^2 B$

$$2\sin B = 3\left(\frac{\sin^2 B}{\cos^2 B}\right)$$

$$2\sin B\cos^2 B - 3\sin^2 B = 0$$
$$2\sin B(1-\sin^2 B) - 3\sin^2 B = 0$$
$$2\sin B - 2\sin^3 B - 3\sin^2 B = 0$$
$$\sin B(2 - 3\sin B - 2\sin^2 B) = 0$$
$$\sin B(2+\sin B)(1-2\sin B) = 0$$
$$\sin B = 0 \text{ or } \sin B = \frac{1}{2}$$

So, $B = \pi n$, $n \in I$ or $B = \frac{\pi}{6} + 2\pi n$, or $\frac{5\pi}{6} + 2\pi n$, $n \in I$.

c) $\sin 2\theta \sin \theta + \cos^2 \theta = 1$

$$2 \sin \theta \cos \theta \sin \theta + \cos^2 \theta - 1 = 0$$
$$2 \sin^2 \theta \cos \theta - \sin^2 \theta = 0$$
$$\sin^2 \theta(2 \cos \theta - 1) = 0$$
$$\sin \theta = 0 \text{ or } \cos \theta = \frac{1}{2}$$

So, $\theta = \pi n$, $n \in I$ or $\theta = \pm\frac{\pi}{3} + 2\pi n$, $n \in I$

Chapter 6 Practice Test Page 324 Question 11

$$\sin 2x + 2 \cos x = 0$$
$$2 \sin x \cos x + 2 \cos x = 0$$
$$2 \cos x (\sin x + 1) = 0$$
$$\cos x = 0 \text{ or } \sin x = -1$$

So, $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

The general solution, in radians, is $x = \frac{\pi}{2} + \pi n$, $n \in I$.

Chapter 6 Practice Test Page 324 Question 12

Given $\sin\theta = -\frac{4}{5}$ and θ is in quadrant III, then $\cos\theta = -\frac{3}{5}$.

$$\cos\left(\theta-\frac{\pi}{6}\right)=\cos\theta\cos\frac{\pi}{6}+\sin\theta\sin\frac{\pi}{6}$$
$$=\left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(-\frac{4}{5}\right)\left(\frac{1}{2}\right)$$
$$=\frac{-3\sqrt{3}-4}{10}$$

Chapter 6 Practice Test Page 324 Question 13

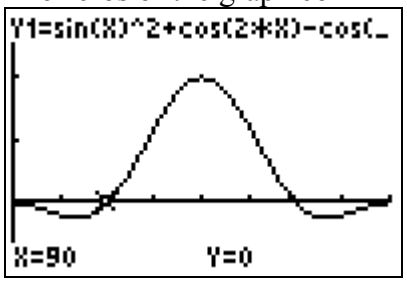
$2\tan x\cos^2 x=1$
$$2\left(\frac{\sin x}{\cos x}\right)\cos^2 x=1$$
$$2\sin x\cos x=1$$
$$\sin 2x=1$$
$$2x=\frac{\pi}{2}$$
$$x=\frac{\pi}{4}$$

So, in the domain $0\le x<2\pi$, $x=\frac{\pi}{4}$ or $\frac{5\pi}{4}$.

Chapter 6 Practice Test Page 324 Question 14

$\sin^2 x+\cos 2x-\cos x=0$
$\sin^2 x+\cos^2 x-\sin^2 x-\cos x=0$
$\cos^2 x-\cos x=0$
$\cos x(\cos x-1)=0$
$\cos x=0$ or $\cos x=1$

In the domain $0°\le x<360°$, $x=0°, 90°, 270°$.
The zeros of the graph confirm these three values.



Chapter 6 Practice Test Page 324 Question 15

a) $\text{Left Side} = \dfrac{\cot x}{\csc x - 1}$

$= \dfrac{\cot x(\csc x + 1)}{\csc^2 x - 1}$

$= \dfrac{\cot x(\csc x + 1)}{1 + \cot^2 x - 1}$

$= \dfrac{(\csc x + 1)}{\cot x}$

$= \text{Right Side}$

b) $\text{Left Side} = \sin(x + y)\sin(x - y)$

$= (\sin x \cos y + \sin y \cos x)(\sin x \cos y - \sin y \cos x)$

$= \sin^2 x \cos^2 y - \sin^2 y \cos^2 x$

$= \sin^2 x(1 - \sin^2 y) - \sin^2 y(1 - \sin^2 x)$

$= \sin^2 x - \sin^2 y$

$= \text{Right Side}$

Chapter 6 Practice Test Page 324 Question 16

$2\cos^2 x + 3\sin x - 3 = 0$

$2(1 - \sin^2 x) + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \dfrac{1}{2}$ or $\sin x = 1$

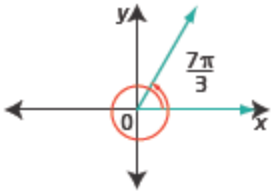
The general solution, in radians, is $x = \dfrac{\pi}{6} + 2\pi n,\ \dfrac{5\pi}{6} + 2\pi n,\ n \in \text{I}$,

or $x = \dfrac{\pi}{2} + 2\pi n,\ n \in \text{I}$.

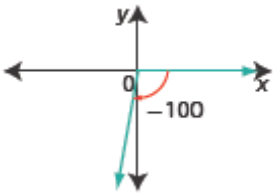
Cumulative Review, Chapters 4-6

Cumulative Review, Chapters 4-6 Page 326 Question 1

a) $\dfrac{7\pi}{3} \pm 2\pi n, n \in \text{N}$



b) $-100° \pm (360°)\, n,\ n \in \mathrm{N}$



Cumulative Review, Chapters 4-6 Page 326 Question 2

a) $\pi = 180°$

So, $4 = 4\left(\dfrac{180°}{\pi}\right)$

$\approx 229°$

b) $\dfrac{-5\pi}{3} = \dfrac{-5(180°)}{3}$

$= -300°$

Cumulative Review, Chapters 4-6 Page 326 Question 3

a) $210° = \dfrac{210\pi}{180}$

$= \dfrac{7\pi}{6}$

b) $-500° = \dfrac{-500\pi}{180}$

$= -\dfrac{25\pi}{9}$

Cumulative Review, Chapters 4-6 Page 326 Question 4

a) $\text{arc length} = \dfrac{\text{circumference}}{42}$

$= \dfrac{\pi(175)}{42}$

≈ 13.1

The arc length between each gondola is 13.1 ft, to the nearest tenth of a foot.

b) $\dfrac{\text{arc length}}{\text{circumference}} = \dfrac{70}{360}$

$\text{arc length} = \pi(175)\left(\dfrac{70}{360}\right)$

≈ 106.9

In rotating through 70°, the gondola travels 106.9 ft, to the nearest tenth of a foot.

Cumulative Review, Chapters 4-6 Page 326 Question 5

a) Substitute $r = 5$ in $x^2 + y^2 = r^2$.

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

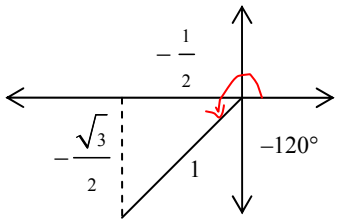
b) Substitute $x = 3$ and $y = \sqrt{7}$ in $x^2 + y^2 = r^2$.

$$3^2 + \left(\sqrt{7}\right)^2 = r^2$$
$$9 + 7 = r^2$$
$$r^2 = 16$$

The equation of the circle through P(3, $\sqrt{7}$) is $x^2 + y^2 = 16$.

Cumulative Review, Chapters 4-6 Page 326 Question 6

a) $P(\theta) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, then θ terminates in quadrant III.



b) The reference angle of θ is $\frac{\pi}{3}$. In the interval $-2\pi \le \theta \le 2\pi$, $\theta = -\frac{2\pi}{3}, \frac{4\pi}{3}$.

c) The coordinates of $P\left(\theta + \frac{\pi}{2}\right)$ are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. When the given quadrant III angle is rotated through $\frac{\pi}{2}$, its terminal arm is in quadrant IV and its coordinates are switched and the signs adjusted.

d) The coordinates of $P(\theta - \pi)$ are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. When the given quadrant III angle is rotated through $-\pi$, its terminal arm is in quadrant I and its coordinates are the same but the signs adjusted.

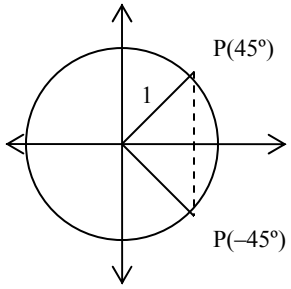
Cumulative Review, Chapters 4-6 Page 326 Question 7

a) 45° is in an isosceles right triangle with sides $\frac{1}{\sqrt{2}} : \frac{1}{\sqrt{2}} : 1$.

$P(-45°) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$P(45°) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

The points have the same *x*-coordinates but opposite *y*-coordinates.



b) 675° = 360° + 315°, so P(675°) is coterminal with P(315°).
This angle has a reference angle of 45° and terminates in quadrant IV.

$$P(675°) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right).$$

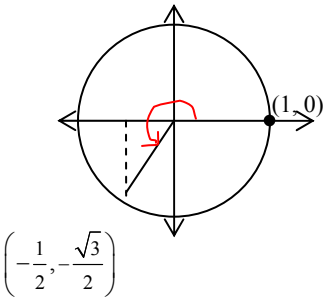
765° = 720° + 45°, so P(765°) is coterminal with P(45°).
This angle has a reference angle of 45° and terminates in quadrant I.

$$P(765°) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$

The points have the same *x*-coordinates but opposite *y*-coordinates.

Cumulative Review, Chapters 4-6 Page 326 Question 8

a) A rotation of $\frac{4\pi}{3}$ takes the terminal arm of the angle into quadrant III, with reference angle $\frac{\pi}{3}$ as shown.



$$\sin\frac{4\pi}{3} = \frac{y}{r}$$
$$= -\frac{\sqrt{3}}{2}$$

b) A rotation of 300° takes the terminal arm of the angle into quadrant IV, with reference angle 60°. In quadrant IV, cosine is positive.

$$\cos 300° = \frac{1}{2}$$

c) A rotation of –570° is –(360° + 210°) and takes the terminal arm of the angle into quadrant II, with reference angle 30°. In quadrant II, tangent is negative.

$$\tan(-570°) = -\frac{1}{\sqrt{3}}$$

d) A rotation of 135° takes the terminal arm of the angle into quadrant II, with reference angle 45°. In quadrant II, sine and cosecant are positive.

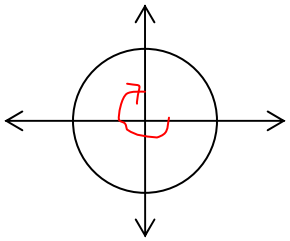
$$\csc 135° = \frac{1}{\sin 45°}$$
$$= \sqrt{2}$$

e) A rotation of $-\frac{3\pi}{2}$ is on the *y*-axis above the origin.

$$\sec\left(-\frac{3\pi}{2}\right) = \frac{1}{\cos\left(-\frac{3\pi}{2}\right)}$$

$$= \frac{1}{0}$$

This is undefined.



f) A rotation of $\frac{23\pi}{6} = 2\pi + \frac{11\pi}{6}$ and takes the terminal arm of the angle into quadrant IV, with reference angle $\frac{\pi}{6}$. In quadrant IV, tangent and cotangent are negative.

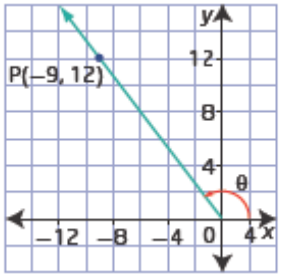
$$\cot\frac{23\pi}{6} = \frac{1}{\tan\frac{23\pi}{6}}$$

$$= \frac{1}{-\frac{1}{\sqrt{3}}}$$

$$= -\sqrt{3}$$

Cumulative Review, Chapters 4-6 Page 326 Question 9

a)



b) First determine r. Substitute $x = -9$ and $y = 12$ into $x^2 + y^2 = r^2$.

$$x^2 + y^2 = r^2$$

$$(-9)^2 + 12^2 = r^2$$

$$r^2 = 225$$

$$r = 15$$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$= \frac{12}{15} \qquad = \frac{-9}{15} \qquad = \frac{12}{-9}$$

$$= \frac{4}{5} \qquad = -\frac{3}{5} \qquad = -\frac{4}{3}$$

Then, $\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

c) Since θ is in quadrant II, and the reference angle is $\sin^{-1}(0.8)$,
$\theta = 126.87° + 360°n$, where $n \in$ I.

Cumulative Review, Chapters 4-6 Page 326 Question 10

a) For $\sin\theta = -\frac{1}{2}$ the reference angle is $\frac{\pi}{6}$ and θ is in quadrant III or IV.

So, in the domain $-2\pi \le \theta \le 2\pi$, $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$.

b) For $\sec\theta = \frac{2\sqrt{3}}{3}$, or $\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$, the reference angle is 30° and θ is in quadrant I or IV.
So, in the domain $-180° \le \theta \le 180°$, θ = –30° or 30°.

c) For $\tan\theta = -1$ the reference angle is $\frac{\pi}{4}$ and θ is in quadrant II or IV.

So, in the domain $0 \le \theta \le 2\pi$, $\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$.

Cumulative Review, Chapters 4-6 Page 326 Question 11

a) For $\cos\theta = -\frac{\sqrt{2}}{2}$ the reference angle is $\frac{\pi}{4}$ and θ is in quadrant II or III.

Then, the general solution is $\theta = \frac{3\pi}{4} + 2\pi n$ or $\frac{5\pi}{4} + 2\pi n$, $n \in$ I.

b) For csc θ = 1, or sin θ = 1, θ is $\frac{\pi}{2}$. Then, the general solution is $\theta = \frac{\pi}{2} + 2\pi n$, $n \in$ I.

c) For cot θ = 0, or tan θ is undefined, θ is $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Then, the general solution is

$\theta = \frac{\pi}{2} + \pi n$, $n \in$ I.

Cumulative Review, Chapters 4-6 Page 326 Question 12

a) $\sin\theta = \sin\theta \tan\theta$
$\sin\theta - \sin\theta \tan\theta = 0$
$\sin\theta (1 - \tan\theta) = 0$

$\sin\theta = 0$ or $\tan\theta = 1$

In the domain $0 \le \theta \le 2\pi$, $\theta = 0, \pi, 2\pi$ or $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

b) $2\cos^2\theta + 5\cos\theta + 2 = 0$

$(2\cos\theta + 1)(\cos\theta + 2) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -2$ (which is impossible)

In the domain $0 \le \theta \le 2\pi$, $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$.

Cumulative Review, Chapters 4-6 Page 326 Question 13

a) $4\tan^2\theta - 1 = 0$

$(2\tan\theta - 1)(2\tan\theta + 1) = 0$

$\tan\theta = \frac{1}{2}$ or $\tan\theta = -\frac{1}{2}$

In the domain $0° \le \theta \le 360°$, $\theta \approx 27°, 153°, 207°, 333°$.

b) $3\sin^2\theta - 2\sin\theta = 1$

$3\sin^2\theta - 2\sin\theta - 1 = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{3}$ or $\sin\theta = 1$

In the domain $0° \le \theta \le 360°$, $\theta \approx 199°, 341°$ or $\theta = 90°$.

Cumulative Review, Chapters 4-6 Page 326 Question 14

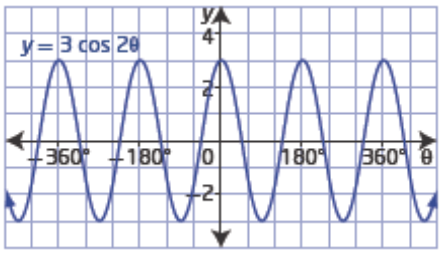
For the sine function in the form $y = a\sin b(x - c) + d$, the amplitude $a = 3$, the period $b = \frac{1}{2}$, and the horizontal shift is $c = -\frac{\pi}{4}$.

The equation is $y = 3\sin\frac{1}{2}\left(x + \frac{\pi}{4}\right)$.

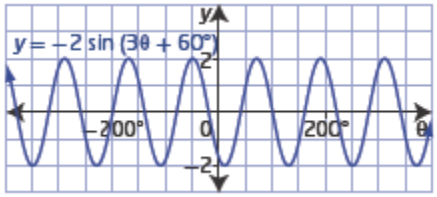
Cumulative Review, Chapters 4-6 Page 327 Question 15

a) $y = 3\cos 2\theta$

amplitude is 3, period is $\frac{360°}{2}$ or $180°$, there is no phase shift or vertical displacement

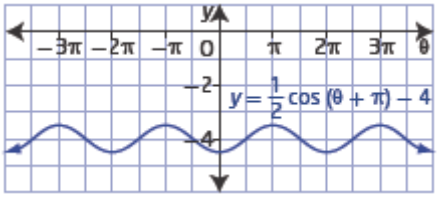


b) $y = -2 \sin (3\theta + 60°)$
$y = -2 \sin 3(\theta + 20°)$
amplitude is 2, period is 120°, the phase shift is 20° to the left, there is no vertical displacement



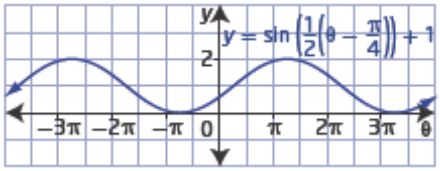
c) $y = \frac{1}{2}\cos(\theta + \pi) - 4$

amplitude is $\frac{1}{2}$, period is 2π, phase shift is π units to the left, and the vertical displacement is 4 units down.



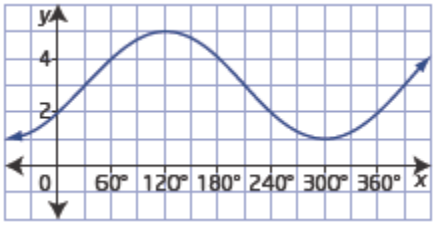
d) $y = \sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{4}\right)\right) + 1$

amplitude is 1, period is 4π, phase shift is $\frac{\pi}{4}$ units to the right and the vertical displacement is 1 unit up.



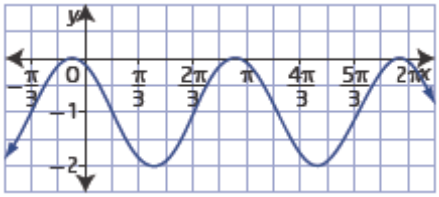
Cumulative Review, Chapters 4-6 **Page 327 Question 16**

a)



From the graph, the amplitude is 2,the period is 360°, the phase shift is 30° to the right and the vertical displacement is 3 units up.So, in the equation $a = 2$, $b = 1$, $c = 30°$, and $d = 3$.
The equation is $y = 2 \sin (x - 30°) + 3$ or $y = 2 \cos (x - 120°) + 3$.

b)



From the graph, the amplitude is 1,the period is π, the phase shift is $\frac{\pi}{3}$ to the left and the vertical displacement is 1 unit down. So, in the equation $a = 1$, $b = 2$, $c = -\frac{\pi}{3}$, and $d = 1$.
The equation is $y = \sin 2\left(x + \frac{\pi}{3}\right) - 1$ or $y = \cos 2\left(x + \frac{\pi}{12}\right) - 1$.

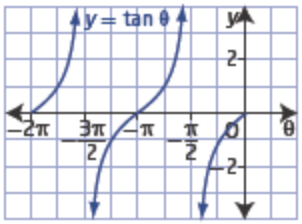
Cumulative Review, Chapters 4-6 Page 327 Question 17

$a = 4$, $b = \dfrac{360°}{300°} = 1.2$, $c = –30°$, $d = –3$

The equation of this cosine function is $y = 4 \cos 1.2 (x + 30°) – 3$.

Cumulative Review, Chapters 4-6 Page 327 Question 18

a)



b) The equations of the asymptotes in the domain $–2\pi \le \theta \le 0$ are

$x = -\dfrac{3\pi}{2}$ and $x = -\dfrac{\pi}{2}$.

Cumulative Review, Chapters 4-6 Page 327 Question 19

a) Assume that the horizontal axis passes through the centre of the wheel. The height of the wheel varies 25 m above and below the centre, so the amplitude is 25. The centre of the wheel is 26 m above the ground so the vertical displacement is 1. The wheel rotates twice in 22 min, so the period is $\dfrac{4\pi}{22}$ or $\dfrac{2\pi}{11}$. For a passenger starting at the lowest point on the Ferris wheel, an equation representing their motion is

$h(x) = –25 \cos \dfrac{2\pi}{11} x + 26$.

b) Determine x when $h(x) = 30$.

$$30 = –25 \cos \frac{2\pi}{11} x + 26$$

$$25 \cos \frac{2\pi}{11} x = 26 – 30$$

$$\cos \frac{2\pi}{11} x = -\frac{4}{25}$$

$$\frac{2\pi}{11} x = \cos^{-1}\left(-\frac{4}{25}\right)$$

$$x = \frac{11}{2\pi} \cos^{-1}\left(-\frac{4}{25}\right)$$

$$x \approx 3.03$$

The passenger is 30 m above the ground after 3.0 min, to the nearest tenth of a minute.

Cumulative Review, Chapters 4-6 Page 327 Question 20

a) For $\frac{1-\cos^2\theta}{\cos^2\theta}$, $\cos^2\theta \neq 0$. So, the non-permissible values are $\theta \neq \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$.

$$\begin{aligned}\frac{1-\cos^2\theta}{\cos^2\theta} &= \frac{\sin^2\theta}{\cos^2\theta} \\ &= \tan^2\theta\end{aligned}$$

b) For sec *x* csc *x* tan *x*, sin $x \neq 0$ and cos $x \neq 0$. So, the non-permissible values are $x \neq \left(\frac{\pi}{2}\right)n,\ n \in \mathrm{I}$.

$$\begin{aligned}\sec x \csc x \tan x &= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right) \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x\end{aligned}$$

Cumulative Review, Chapters 4-6 Page 327 Question 21

a) $\sin 195° = \sin(45° + 150°)$

$$\begin{aligned}&= \sin 45° \cos 150° + \cos 45° \sin 150° \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{1-\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

b)

$$\begin{aligned}\cos\left(-\frac{5\pi}{12}\right) &= \cos\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right) \\ &= \cos\left(\frac{\pi}{4} - \frac{2\pi}{3}\right) \\ &= \cos\frac{\pi}{4}\cos\frac{2\pi}{3} + \sin\frac{\pi}{4}\sin\frac{2\pi}{3} \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ or } \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

Cumulative Review, Chapters 4-6 Page 327 Question 22

a) $2\cos^2\frac{3\pi}{8}-1=\cos 2\left(\frac{3\pi}{8}\right)$

$=\cos\frac{3\pi}{4}$

$=-\frac{1}{\sqrt{2}}$

b) $\sin 10° \cos 80° + \cos 10° \sin 80° = \sin (10° + 80°)$

$= \sin 90°$

$= 1$

c) $\frac{\tan\frac{5\pi}{12}+\tan\frac{23\pi}{12}}{1-\tan\frac{5\pi}{12}\tan\frac{23\pi}{12}}=\tan\left(\frac{5\pi}{12}+\frac{23\pi}{12}\right)$

$=\tan\frac{28\pi}{12}$

$=\tan\frac{7\pi}{3}$

$=\sqrt{3}$

Cumulative Review, Chapters 4-6 Page 327 Question 23

a) Substitute A = 30°:

Left Side $= \sin^2 A + \cos^2 A + \tan^2 A$

$= \sin^2 30° + \cos^2 30° + \tan^2 30°$

$=\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2$

$=\frac{1}{4}+\frac{3}{4}+\frac{1}{3}$

$=\frac{4}{3}$

Right Side $= \sec^2 A$

$= \sec^2 30°$

$=\frac{1}{\cos^2 30°}$

$=\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$=\frac{4}{3}$

Left Side = Right Side

b) Left Side $= \sin^2 A + \cos^2 A + \tan^2 A$

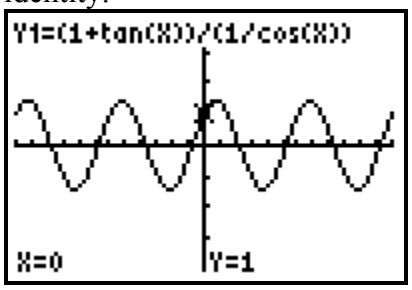
$= 1 + \tan^2 A$

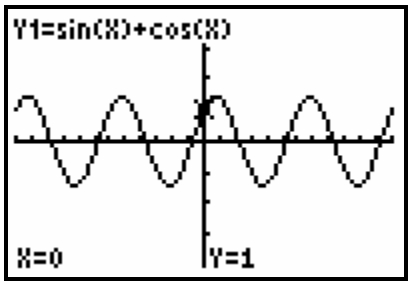
$=\sec^2 A$

= Right Side

Cumulative Review, Chapters 4-6 Page 327 Question 24

a) The graphs of each side of the equation look the same, so the equation may be an identity.





b) $\text{Left Side} = \dfrac{1+\tan x}{\sec x}$

$= \dfrac{1}{\sec x} + \dfrac{\tan x}{\sec x}$

$= \cos x + \dfrac{\sin x}{\cos x} \div \dfrac{1}{\cos x}$

$= \cos x + \sin x$

$= \text{Right Side}$

Cumulative Review, Chapters 4-6 Page 327 Question 25

$\text{Left Side} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$ $\qquad$ $\text{Right Side} = \dfrac{\cos 2\theta}{1+\sin 2\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta}$

$= \dfrac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)(\cos\theta + \sin\theta)}$

$= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

Left Side = Right Side

Cumulative Review, Chapters 4-6 Page 327 Question 26

a) $\sec^2 x = 4 \tan^2 x$

$1 + \tan^2 x = 4 \tan^2 x$

$1 = 3 \tan^2 x$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6} + \pi n, \ \frac{5\pi}{6} + \pi n, \ n \in \mathrm{I}$

b) $\sin 2x + \cos x = 0$

$2 \sin x \cos x + \cos x = 0$

$\cos x(2 \sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2} + \pi n, \ n \in \mathrm{I}$ or

$x = \frac{7\pi}{6} + 2\pi n, \ \frac{11\pi}{6} + 2\pi n, \ n \in \mathrm{I}$

Cumulative Review, Chapters 4-6 Page 327 Question 27

a) $(\sin \theta + \cos \theta)^2 - \sin 2\theta = 1$

$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = 1$

$\sin^2 \theta + \cos^2 \theta = 1$

This is an identity, so the solution is all values of θ.

b) Yes, the equation is an identity because the left side simplifies to 1.

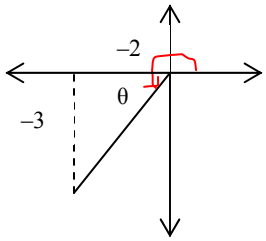
Unit 2 Test

Unit 2 Test Page 328 Question 1

If $\tan \theta = \frac{3}{2}$ and $\cos \theta < 0$, then θ is in quadrant III.

$r^2 = (-2)^2 + (-3)^2$

$r = \sqrt{13}$



Then, $\cos 2\theta = 2 \cos^2 \theta - 1$

$= 2\left(\frac{-2}{\sqrt{13}}\right)^2 - 1$

$= \frac{8}{13} - 1$

$= -\frac{5}{13}$

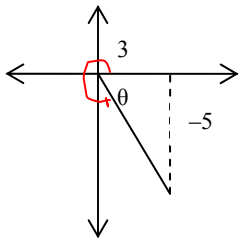
The best answer is **B**.

Unit 2 Test **Page 328** **Question 2**

The point (3, –5) is is quadrant IV.

$r^2 = (3)^2 + (-5)^2$

$r = \sqrt{34}$



$\sin(\pi - \theta) = \sin\pi\cos\theta - \sin\theta\cos\pi$

$= 0\left(\frac{3}{\sqrt{34}}\right) - \left(\frac{-5}{\sqrt{34}}\right)(-1)$

$= -\frac{5}{\sqrt{34}}$

The best answer is **D**.

Unit 2 Test **Page 328** **Question 3**

If the range is $-2 \le y \le 6$, the amplitude is 4 and the vertical displacement is 2 units up.
So, $a = 4$ and $d = 2$.
The best answer is **C**.

Unit 2 Test **Page 328** **Question 4**

For the function $f(x) = 3\cos\left(4x + \frac{\pi}{2}\right)$ or $f(x) = 3\cos 4\left(x + \frac{\pi}{8}\right)$,

the period is $\frac{2\pi}{4}$, or $\frac{\pi}{2}$, and the phase shift is $\frac{\pi}{8}$ units to the left.

The best answer is **C**.

Unit 2 Test **Page 328** **Question 5**

When the graph of $y = \cos x$ is translated to the right $\frac{\pi}{2}$ units it is the same as the graph of $y = \sin x$. So, $y = 3\cos\left(x - \frac{\pi}{2}\right)$ has a graph that is equivalent to $y = 3\sin x$.

The best answer is **B**.

Unit 2 Test **Page 328** **Question 6**

$y = \tan x$ is not defined when $\cos x = 0$. This occurs when $x = 90° + 180°n$, $n \in \mathrm{I}$.
The best answer is **D**.

Unit 2 Test Page 328 Question 7

$$\frac{\sin\theta+\tan\theta}{1+\cos\theta}=\frac{\left(\sin\theta+\frac{\sin\theta}{\cos\theta}\right)}{1+\cos\theta}$$
$$=\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta(1+\cos\theta)}$$
$$=\frac{\sin\theta(1+\cos\theta)}{\cos\theta(1+\cos\theta)}$$
$$=\tan\theta$$

The best answer is **C**.

Unit 2 Test Page 328 Question 8

By inspection, the equation $\frac{\sec\theta\csc\theta}{\cot\theta}=\sec\theta$ cannot possibly be true.

Consider B: Left Side $=\tan^2\theta-\sin^2\theta$
$$=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta$$
$$=\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}$$
$$=\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$
$$=\tan^2\theta\sin^2\theta$$
$$=\text{Right Side}$$

Consider C: Left Side $=\frac{1-\cos2\theta}{2}$
$$=\frac{1-(1-2\sin^2\theta)}{2}$$
$$=\sin^2\theta$$
$$=\text{Right Side}$$

Consider D: Left Side $=\frac{\tan^2\theta}{1+\tan^2\theta}$
$$=\frac{\tan^2\theta}{\sec^2\theta}$$
$$=\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\div\left(\frac{1}{\cos^2\theta}\right)$$
$$=\sin^2\theta$$
$$=\text{Right Side}$$

Equations B, C and D are identities.
The best answer is **A**.

Unit 2 Test **Page 328** **Question 9**

$$\frac{17\pi}{3} = \frac{12\pi + 5\pi}{3}$$
$$= 4\pi + \frac{5\pi}{3}$$

So $\frac{17\pi}{3}$ is coterminal with $\frac{5\pi}{3}$ and is in quadrant IV with reference angle $\frac{\pi}{3}$.

$\sin\frac{17\pi}{3} = \frac{-\sqrt{3}}{2}$.

The exact value of $\sin\frac{17\pi}{3}$ is $\frac{-\sqrt{3}}{2}$.

Unit 2 Test **Page 328** **Question 10**

If $P\left(x, \frac{\sqrt{5}}{3}\right)$ is on the unit circle then

$$x^2 + \left(\frac{\sqrt{5}}{3}\right)^2 = 1$$
$$x^2 + \frac{5}{9} = 1$$
$$x^2 = \frac{4}{9}$$
$$x = \pm\frac{2}{3}$$

The possible values of x are $\frac{2}{3}$ and $-\frac{2}{3}$.

Unit 2 Test **Page 328** **Question 11**

If $\cos\theta = \frac{-5}{13}$ and $\frac{\pi}{2} \le \theta \le \pi$, then θ is in quadrant II and $y = 12$.

Then, $\sin\left(\theta+\frac{\pi}{4}\right)=\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}$

$$=\left(\frac{12}{13}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{-5}{13}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$=\frac{7}{13\sqrt{2}} \text{ or } \frac{7\sqrt{2}}{26}$$

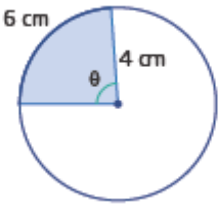
The exact value of $\sin\left(\theta+\frac{\pi}{4}\right)$ is $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$.

Unit 2 Test **Page 328** **Question 12**

$$\frac{\theta}{2\pi}=\frac{6}{2\pi(4)} \qquad \text{or} \qquad \frac{\theta}{360°}=\frac{6}{2\pi(4)}$$

$$\theta=\frac{3}{2} \qquad \theta=\frac{3(360°)}{4\pi}$$

$$\approx 85.9°$$



The measures of θ in radians and degrees, to the nearest tenth of a unit, are 1.5 and 85.9°.

Unit 2 Test **Page 329** **Question 13**

If $\sqrt{3}\sec\theta-2=0$

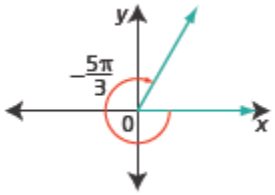
$$\sec\theta=\frac{2}{\sqrt{3}}$$

So θ is in quadrant I or IV with reference angle $\frac{\pi}{6}$.

For $-2\pi\le\theta\le 2\pi$, $\theta=-\frac{11\pi}{6},-\frac{\pi}{6},\frac{\pi}{6},$ and $\frac{11\pi}{6}$.

Unit 2 Test **Page 329** **Question 14**

a)



b) $-\frac{5\pi}{3}=-\frac{5\pi(180°)}{3\pi}$

$$=-300°$$

c) All coterminal angles are given by $-\frac{5\pi}{3}\pm 2\pi n,\ n\in\mathrm{N}$.

d) No. $\frac{10\pi}{3} = 2\pi + \frac{4\pi}{3}$; its terminal arm is in quadrant III, and so this angle is never coterminal with $-\frac{5\pi}{3}$.

Unit 2 Test Page 329 Question 15

$$5\sin^2\theta + 3\sin\theta - 2 = 0$$
$$(5\sin\theta - 2)(\sin\theta + 1) = 0$$
$$5\sin\theta - 2 = 0 \text{ or } \sin\theta + 1 = 0$$
$$\sin\theta = 0.4 \text{ or } \sin\theta = -1$$

For $0 \le \theta \le 2\pi$, $\theta \approx 0.412, 2.730$, or 4.712.

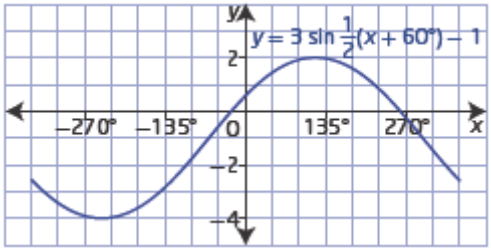
Unit 2 Test Page 329 Question 16

Sam is correct. In the first step, Pat has lost two solutions by forgetting the negative square root.
The correct solution is:

$$4\sin^2 x = 3$$
$$2\sin x = \pm\sqrt{3}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Unit 2 Test Page 329 Question 17

a)



b) The range is $-4 \le y \le 2$.

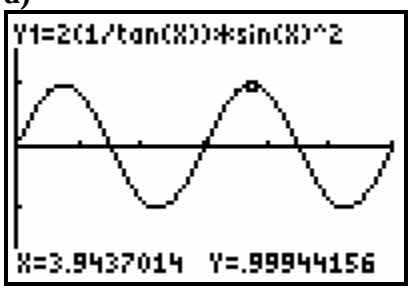
c) The amplitude is 3, the period is 720°, the phase shift is 60° to the left, and the vertical displacement is 1 unit down.

d) $3\sin\frac{1}{2}(x + 60°) - 1 = 0$

$$\sin\frac{1}{2}(x + 60°) = \frac{1}{3}$$
$$\frac{1}{2}(x + 60°) \approx 19.5° \text{ or } 160.5°$$
$$x \approx -21° \text{ or } 261°$$

Unit 2 Test **Page 329** **Question 18**

a)



b) $g(\theta) = \sin 2\theta$

c) $f(\theta) = 2\cot\theta\sin^2\theta$

$$= 2\left(\frac{\cos\theta}{\sin\theta}\right)\sin^2\theta$$

$$= 2\cos\theta\sin\theta$$

$$= \sin 2\theta$$

Unit 2 Test **Page 329** **Question 19**

a) For $x = \frac{2\pi}{3}$:

$$\text{Left Side} = \tan\frac{2\pi}{3} + \frac{1}{\tan\frac{2\pi}{3}}$$

$$= -\sqrt{3} + \frac{1}{-\sqrt{3}}$$

$$= \frac{-3-1}{\sqrt{3}}$$

$$= -\frac{4\sqrt{3}}{3}$$

$$\text{Right Side} = \frac{\sec\frac{2\pi}{3}}{\sin\frac{2\pi}{3}}$$

$$= -2\left(\frac{2}{\sqrt{3}}\right)$$

$$= -\frac{4\sqrt{3}}{3}$$

Left Side = Right Side

b) Non-permissible values occur when $\sin x = 0$ and $\cos x = 0$. In general, the non-permissible values are $x \neq \frac{\pi n}{2}, n \in \mathrm{I}$.

c) $\text{Left Side} = \tan x + \frac{1}{\tan x}$

$$= \frac{\tan^2 x + 1}{\tan x}$$

$$= \frac{\sec^2 x}{\tan x}$$

$$= \sec x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{\sec x}{\sin x}$$

$$= \text{ Right Side}$$

Unit 2 Test Page 329 Question 20

a) For the function $h(t) = 2.962 \sin (0.508t - 0.107) + 3.876$,
the amplitude is 2.962 and the vertical displacement is 3.876 and the maximum height of the tide presumably occurs at the maximum of the curve which is 2.962 + 3.876. The maximum height of the tide is predicted to be 6.838 m.

b) The period is $\frac{2\pi}{0.508}$, or 12.368. The period of the function is approximately 12.37 h.

c) At noon $t = 12$.
$h(12) = 2.962 \sin (0.508(12) - 0.107) + 3.876$
≈ 3.017
The height of the tide at 12 noon was about 3.017 m.

Chapter 7 Exponential Functions

Section 7.1 Characteristics of Exponential Functions

Section 7.1 Page 342 Question 1

a) The function $y = x^3$ is a polynomial function, not an exponential function.

b) The function $y = 6^x$ is an exponential function. The base is greater than 0 and the independent variable is the exponent.

c) The function $y = x^{\frac{1}{2}}$ is a square root function, not an exponential function.

d) The function $y = 0.75^x$ is an exponential function. The base is greater than 0 and the independent variable is the exponent.

Section 7.1 Page 342 Question 2

a) For $x = 5$,

$f(x) = 4^x$ $\quad g(x) = \left(\frac{1}{4}\right)^x$ $\quad h(x) = 2^x$

$f(5) = 4^5$ $\quad g(5) = \left(\frac{1}{4}\right)^5$ $\quad h(5) = 2^5$

$f(5) = 1024$ $\quad g(5) = \frac{1}{1024}$ $\quad h(5) = 32$

The function $f(x)$ has the greatest value when $x = 5$.

b) For $x = -5$,

$f(x) = 4^x$ $\quad g(x) = \left(\frac{1}{4}\right)^x$ $\quad h(x) = 2^x$

$f(-5) = 4^{-5}$ $\quad g(-5) = \left(\frac{1}{4}\right)^{-5}$ $\quad h(-5) = 2^{-5}$

$f(-5) = \frac{1}{1024}$ $\quad g(-5) = 1024$ $\quad h(-5) = \frac{1}{32}$

The function $g(x)$ has the greatest value when $x = -5$.

c) Any base raised to the exponent 0 is 1.

$f(x) = 4^x$ $\quad g(x) = \left(\frac{1}{4}\right)^x$ $\quad h(x) = 2^x$

$f(0) = 4^0$ $\quad g(0) = \left(\frac{1}{4}\right)^0$ $\quad h(0) = 2^0$

$f(0) = 1$ $g(0) = 1$ $h(0) = 1$

Section 7.1 Page 342 Question 3

a) For $y = 5^x$, $c > 1$ so the graph is increasing. The graph will pass through the point (1, 5). Graph **B**.

b) For $y = \left(\frac{1}{4}\right)^x$, $c < 1$ so the graph is decreasing. The graph will pass through the point (1, 0.25). Graph **C**.

c) For $y = \left(\frac{2}{3}\right)^x$, $c < 1$ so the graph is decreasing. The graph will pass through the approximate point (1, 0.67). Graph **A**.

Section 7.1 Page 343 Question 4

a) There is a pattern in the ordered pairs.

x	*y*
0	1
1	3
2	9

As the value of x increases by 1 unit, the value of y increases by a factor of 3. Therefore, for this function, $c = 3$.
Use the point (1, 3) to check the function $y = 3^x$:

Left Side	Right Side
y	3^x
$= 3$	$= 3^1$
	$= 3$

The function equation for the graph is $y = 3^x$.

b) There is a pattern in the ordered pairs.

x	*y*
–2	25
–1	5
0	1

As the value of x increases by 1 unit, the value of y decreases by a factor of $\frac{1}{5}$. Therefore, for this function, $c = \frac{1}{5}$.

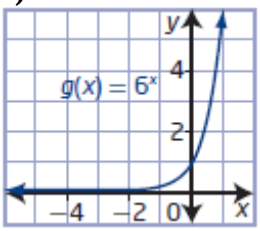
Use the point (–1, 5) to check the function $y = \left(\frac{1}{5}\right)^x$:

Left Side	Right Side
y	$\left(\frac{1}{5}\right)^x$
$= 5$	$= \left(\frac{1}{5}\right)^{-1}$
	$= 5$

The function equation for the graph is $y = \left(\frac{1}{5}\right)^x$.

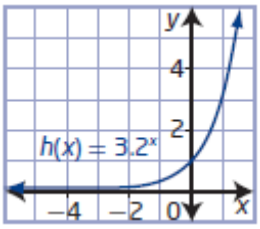
Section 7.1 Page 343 Question 5

a)



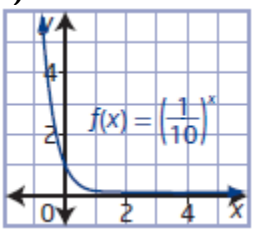
The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is increasing.
The equation of the horizontal asymptote is $y = 0$.

b)



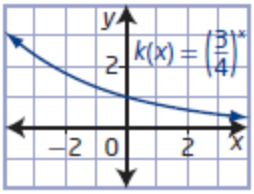
The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is increasing.
The equation of the horizontal asymptote is $y = 0$.

c)



The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is decreasing.
The equation of the horizontal asymptote is $y = 0$.

d)



The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is decreasing.
The equation of the horizontal asymptote is $y = 0$.

Section 7.1 Page 343 Question 6

a) The bacteria in a Petri dish doubling their number every hour represents growth. So, $c > 1$.

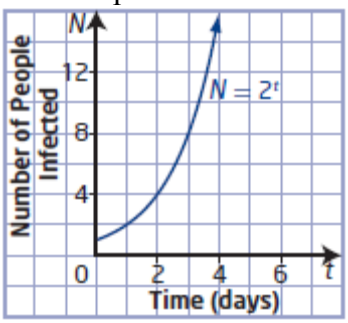
b) The half-life of the radioactive isotope actinium-225 represents decay. So, $c < 1$.

c) The amount of light passing through water decreases with depth represents decay. So, $c < 1$.

d) The population of an insect colony tripling every hour represents growth. So, $c > 1$.

Section 7.1 Page 343 Question 7

a) The function $N = 2^t$ is exponential since the base is greater than zero and the variable t is an exponent.



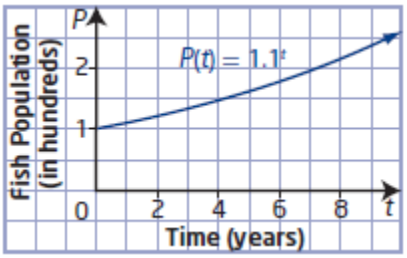
b)

i) For $t = 0$, $N = 2^0$ $= 1$ At the start, 1 person has the virus.	**ii)** For $t = 1$, $N = 2^1$ $= 2$ After 1 day, 2 people have the virus.	**iii)** For $t = 4$, $N = 2^4$ $= 16$ After 4 days, 16 people have the virus.	**iv)** For $t = 10$, $N = 2^{10}$ $= 1024$ After 10 days, 1024 people have the virus.

Section 7.1 Page 343 Question 8

a) If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110%, or 1.1 written as a decimal.

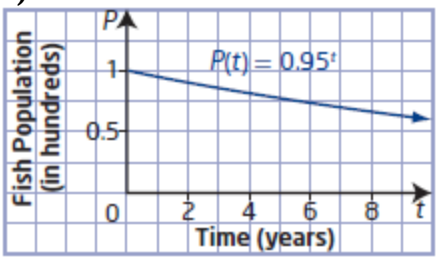
b)



The domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$, and the range is $\{P \mid P \geq 100, P \in \mathrm{R}\}$.

c) If the population decreases by 5% each year, the population becomes 95% of the previous year's population. So, the growth rate is 95%, or 0.95 written as a decimal.

d)

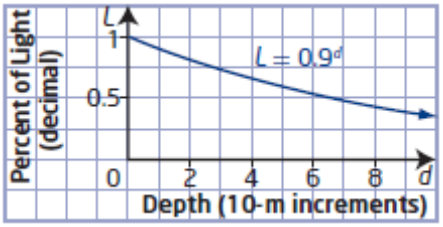


The domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$, and the range is $\{P \mid 0 < P \leq 100, P \in \mathrm{R}\}$.

Section 7.1 Page 344 Question 9

a) The exponential function that relates the amount, L, as a percent expressed as a decimal, of light available to the depth, d, in 10-m increments, is $L = 0.9^d$.

b)



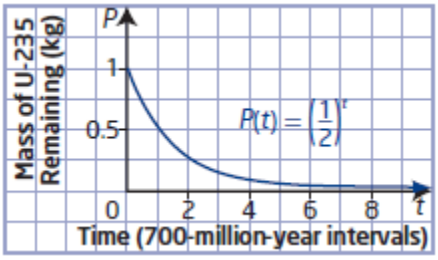
c) The domain is $\{d \mid d \geq 0, d \in \mathrm{R}\}$, and the range is $\{L \mid 0 < L \leq 1, L \in \mathrm{R}\}$.

d) 25 m is the same as 2.5 10-m increments.
For 2.5,
$L = 0.9^d$
$= 0.9^{2.5}$
$= 0.7684\ldots$
The percent of light that will reach Petra if she dives to a depth of 25 m is approximately 76.8%.

Section 7.1 Page 344 Question 10

a) Let P represent the percent, as a decimal, of U-235 remaining. Let t represent time, in 700-million-year intervals. Then, the exponential function that represents the radioactive decay of 1 kg of U-235 is $P(t) = \left(\frac{1}{2}\right)^t$.

b)

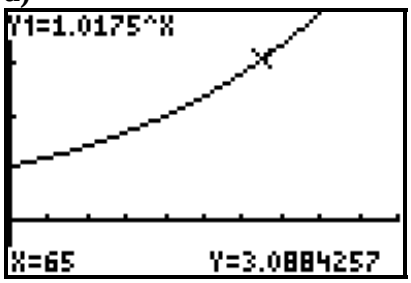


c) From the graph, it will take 3 700-million-year intervals, or 2 100 000 000 years, for 1 kg of U-235 to decay to 0.125 kg.

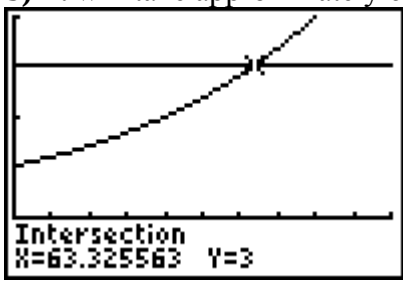
d) The sample in part c) will never decay to 0 kg, since $P = 0$ is the horizontal asymptote.

Section 7.1 Page 344 Question 11

a)

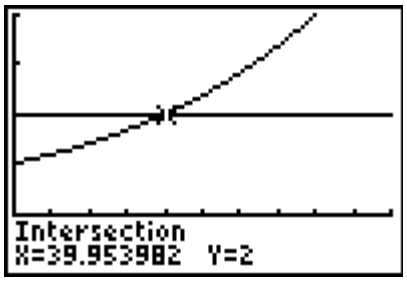


b) It will take approximately 64 years for the deposit to triple in value.



c) The amount of time it takes for a deposit to triple does not depend on the value of the initial deposit. Since each $1 amount invested triples, it does not matter what the initial investment was.

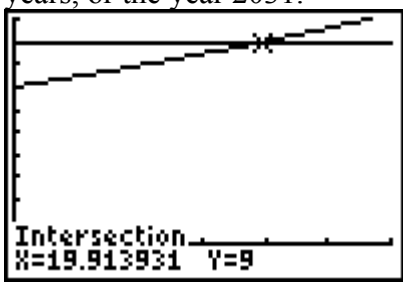
d) From the graph, the approximate doubling time for this investment is 40 years.



From the rule of 72, the approximate doubling time for this investment is $\frac{72}{1.75}$, or 42 years.

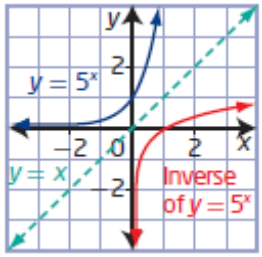
Section 7.1 Page 344 Question 12

Let P represent the world population, in billions. Let t represent time, in years, since 2011. Then, the exponential function that represents world population over time is $P(t) = 7(1.0127)^t$. The population of the world will reach 9 billion in approximately 20 years, or the year 2031.



Section 7.1 Page 344 Question 13

a)



b) The points (x, y) on the graph $y = 5^x$ become the points (y, x) on the graph of the inverse of the function. Thus, the domains and ranges are interchanged. Also, the horizontal asymptote of the graph $y = 5^x$ becomes a vertical asymptote of the graph of the inverse of the function.

c) The equation of the inverse of the function is $x = 5^y$.

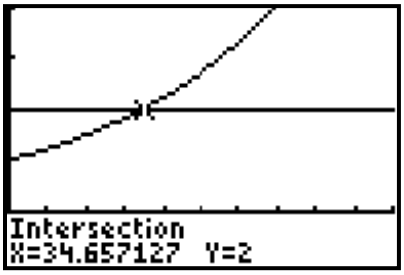
Section 7.1 Page 345 Question 14

a) The function $D = 2^{-\varphi}$ can be written as $D = \left(\frac{1}{2}\right)^{\varphi}$. The coarser the material, the greater the diameter. Therefore, a negative value of φ represents a greater value of D.

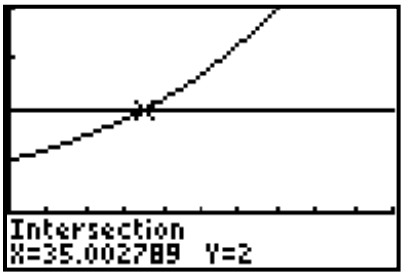
b) The diameter of fine sand is $\left(\frac{1}{2}\right)^{3}$, or 0.125 mm. The diameter of coarse gravel is $\left(\frac{1}{2}\right)^{-5}$, or 32 mm. Thus, fine sand is $\frac{1}{256}$ the diameter of coarse gravel.

Section 7.1 Page 345 Question 15

a) Graph $A(t) = (2.7183)^{0.02t}$ and $A(t) = 2$ and determine the point of intersection. The approximate doubling period is 34.7 years.



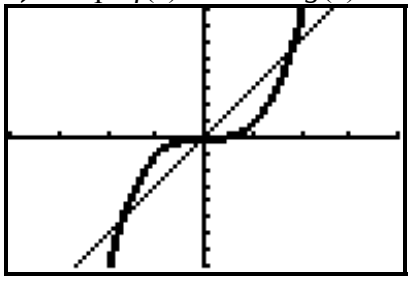
b) Graph $A = (1.02)^{t}$ and $A = 2$ and determine the point of intersection. The approximate doubling period is 35 years.

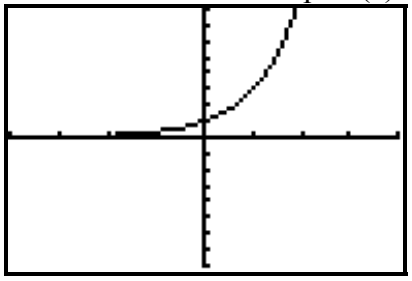


c) The results are similar, but the continuous compounding function gives a shorter doubling period by approximately 0.3 years.

Section 7.1 Page 345 Question C1

a) Graph $f(x) = 3x$ and $g(x) = x^3$ on the same set of axes. Graph $h(x) = 3^x$ separately.





b)

Feature	$f(x) = 3x$	$g(x) = x^3$	$h(x) = 3^x$
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid y \in R\}$	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
intercepts	x-intercept 0, y-intercept 0	x-intercept 0, y-intercept 0	no x-intercept, y-intercept 1
equations of asymptotes	none	none	$y = 0$

c) Example: All three functions have the same domain, and each of their graphs has a y-intercept. The functions $f(x)$ and $g(x)$ have all key features in common.

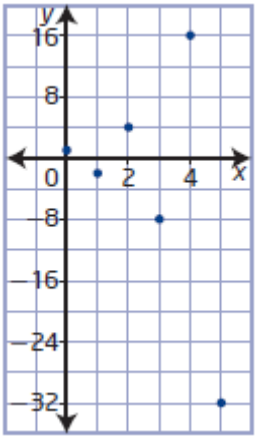
d) Example: The function $h(x)$ is the only function with an asymptote, which restricts its range and results in no x-intercept.

Section 7.1 Page 345 Question C2

a)

x	$f(x)$
0	1
1	−2
2	4
3	−8
4	16
5	−32

b)



c) No, the points do not form a smooth curve. The locations of the points alternate between above the x-axis and below the x-axis.

d) Using technology to evaluate $f\left(\frac{1}{2}\right)$ and $f\left(\frac{5}{2}\right)$ results in an error: non-real answer.

For $x = \frac{1}{2}$,

$f(x) = (-2)^x$

$f\left(\frac{1}{2}\right) = (-2)^{\frac{1}{2}}$

$f\left(\frac{1}{2}\right) = \sqrt{-2}$

For $x = \frac{5}{2}$,

$f(x) = (-2)^x$

$f\left(\frac{5}{2}\right) = (-2)^{\frac{5}{2}}$

$f\left(\frac{5}{2}\right) = \sqrt{(-2)^5}$

Both values are undefined.

e) Example: Exponential functions are defined to only include positive bases, because only positive bases result in smooth curves.

Section 7.2 Transformations of Exponential Functions

Section 7.2 Page 354 Question 1

Compare each function to the form $y = a(c)^{b(x-h)} + k$.

a) For $y = 2(3)^x$, $a = 2$. This is a vertical stretch by a factor of 2: choice **C**.

b) For $y = 3^{x-2}$, $h = 2$. This is a horizontal translation of 2 units to the right: choice **D**.

c) For $y = 3^x + 4$, $k = 4$. This is a vertical translation of 4 units up: choice **A**.

d) For $y = 3^{\frac{x}{5}}$, $b = \frac{1}{5}$. This is a horizontal stretch by a factor of 5: choice **B**.

Section 7.2 Page 354 Question 2

Compare each function to the form $y = a(c)^{b(x-h)} + k$.

a) For $y = \left(\frac{3}{5}\right)^{x+1}$, $h = -1$. This is a horizontal translation of 1 unit to the left: choice **D**.

b) For $y = -\left(\frac{3}{5}\right)^x$, $a = -1$. This is a reflection in the x-axis: choice **A**.

c) For $y = \left(\frac{3}{5}\right)^{-x}$, $b = -1$. This is a reflection in the y-axis: choice **B**.

d) For $y = \left(\frac{3}{5}\right)^x - 2$, $k = -2$. This is a vertical translation of 2 units down: choice **C**.

Section 7.2 Page 354 Question 3

Compare each function to the form $y = a(c)^{b(x-h)} + k$.

a) For $f(x) = 2(3)^x - 4$,
$a = 2$, vertical stretch by a factor of 2
$b = 1$, no horizontal stretch
$h = 0$, no horizontal translation
$k = -4$, vertical translation of 4 units down

b) For $g(x) = 6^{x-2} + 3$,
$a = 1$, no vertical stretch
$b = 1$, no horizontal stretch
$h = 2$, horizontal translation of 2 units to the right
$k = 3$, vertical translation of 3 units up

c) For $m(x) = -4(3)^{x+5}$,
$a = -4$, vertical stretch by a factor of 4 and a reflection in the x-axis
$b = 1$, no horizontal stretch
$h = -5$, horizontal translation of 5 units to the left
$k = 0$, no vertical translation

d) For $y = \left(\frac{1}{2}\right)^{3(x-1)}$,
$a = 1$, no vertical stretch
$b = 3$, horizontal stretch by a factor of $\frac{1}{3}$
$h = 1$, horizontal translation of 1 unit to the right
$k = 0$, no vertical translation

e) For $n(x) = -\frac{1}{2}(5)^{2(x-4)} + 3$,
$a = -\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis
$b = 2$, horizontal stretch by a factor of $\frac{1}{2}$
$h = 4$, horizontal translation of 4 units to the right
$k = 3$, vertical translation of 3 units up

f) For $y = -\left(\frac{2}{3}\right)^{2x-2}$, or $y = -\left(\frac{2}{3}\right)^{2(x-1)}$
$a = -1$, reflection in the x-axis
$b = 2$, horizontal stretch by a factor of $\frac{1}{2}$
$h = 1$, horizontal translation of 1 unit to the right
$k = 0$, no vertical translation

g) For $y = 1.5(0.75)^{\frac{x-4}{2}} - \frac{5}{2}$,
$a = 1.5$, vertical stretch by a factor of 1.5
$b = \frac{1}{2}$, horizontal stretch by a factor of 2
$h = 4$, horizontal translation of 4 units to the right

$k = -\frac{5}{2}$, vertical translation of $\frac{5}{2}$ units down

Section 7.2 Page 355 Question 4

a) Since the graph has been reflected in the x-axis, $a < 0$ and $0 < c < 1$. The graph has also been translated 2 units up, so $k = 2$. Choice **C**.

b) The graph has also been translated 1 unit to the right and 2 units down, so $h = 1$ and $k = -2$. Choice **A**.

c) Since the graph has been reflected in the x-axis, $a < 0$ and $c > 1$. The graph has also been translated 2 units up, so $k = 2$. Choice **D**.

d) The graph has also been translated 2 units to the right and 1 unit up, so $h = 2$ and $k = 1$. Choice **B**.

Section 7.2 Page 355 Question 5

a) For $y = \frac{1}{2}(4)^{-(x-3)} + 2$,

$a = \frac{1}{2}$, vertical stretch by a factor of $\frac{1}{2}$

$b = -1$, reflection in the y-axis

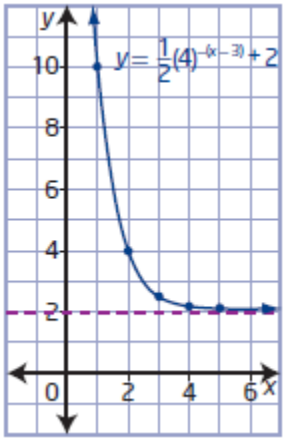
$h = 3$, horizontal translation of 3 units to the right

$k = 2$, vertical translation of 2 units up

b)

$y = 4^x$	$y = 4^{-x}$	$y = \frac{1}{2}(4)^{-x}$	$y = \frac{1}{2}(4)^{-(x-3)} + 2$
$\left(-2, \frac{1}{16}\right)$	$\left(2, \frac{1}{16}\right)$	$\left(2, \frac{1}{32}\right)$	$\left(5, \frac{65}{32}\right)$
$\left(-1, \frac{1}{4}\right)$	$\left(1, \frac{1}{4}\right)$	$\left(1, \frac{1}{8}\right)$	$\left(4, \frac{17}{8}\right)$
(0, 1)	(0, 1)	$\left(0, \frac{1}{2}\right)$	$\left(3, \frac{5}{2}\right)$
(1, 4)	(−1, 4)	(−1,2)	(2, 4)
(2, 16)	(−2, 16)	(−2, 8)	(1, 10)

c)

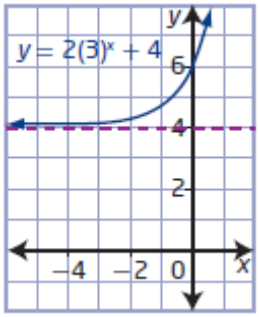


d) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 2, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 2$.
The y-intercept is 34.

Section 7.2 Page 355 Question 6

a) i), ii) For $y = 2(3)^x + 4$,
$a = 2$, vertical stretch by a factor of 2
$b = 1$, no horizontal stretch
$h = 0$, no horizontal translation
$k = 4$, vertical translation of 4 units up

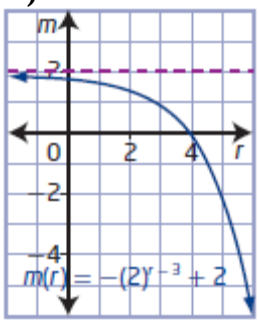
iii)



iv) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 4, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 4$.
The y-intercept is 6.

b) i), ii) For $m(r) = -(2)^{r-3} + 2$,
$a = -1$, reflection in the x-axis
$b = 1$, no horizontal stretch
$h = 3$, horizontal translation of 3 units to the right
$k = 2$, vertical translation of 2 units up

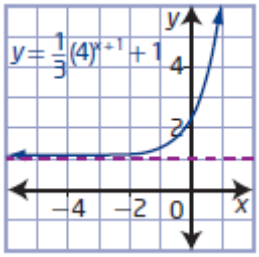
iii)



iv) The domain is $\{r \mid r \in \mathrm{R}\}$, and the range is $\{m \mid m < 2, m \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $m = 2$.
The m-intercept is $\frac{15}{8}$, and the r-intercept is 4.

c) i), ii) For $y = \frac{1}{3}(4)^{x+1} + 1$,
$a = \frac{1}{3}$, vertical stretch by a factor of $\frac{1}{3}$
$b = 1$, no horizontal stretch
$h = -1$, horizontal translation of 1 unit to the left
$k = 1$, vertical translation of 1 unit up

iii)



iv) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 1, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 1$.
The y-intercept is $\frac{7}{3}$.

d) i), ii) For $n(s) = -\frac{1}{2}\left(\frac{1}{3}\right)^{\frac{1}{4}x} - 3$,

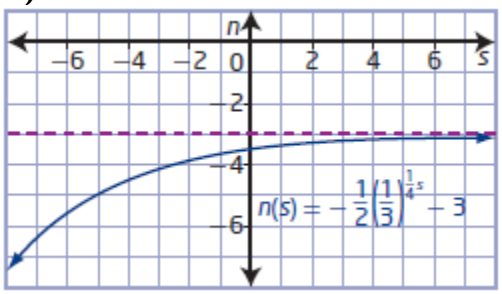
$a = -\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis

$b = \frac{1}{4}$, horizontal stretch by a factor of 4

$h = 0$, no horizontal translation

$k = -3$, vertical translation of 3 units down

iii)



iv) The domain is $\{s \mid s \in \mathrm{R}\}$, and the range is $\{n \mid n < -3, n \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $n = -3$.
The n-intercept is $\frac{15}{8}$.

Section 7.2 Page 355 Question 7

a) To obtain the graph of $y = f(x - 2) + 1$, the graph of $f(x)$ must be translated 2 units to the right and 1 unit up: $y = \left(\frac{1}{2}\right)^{x-2} + 1$.

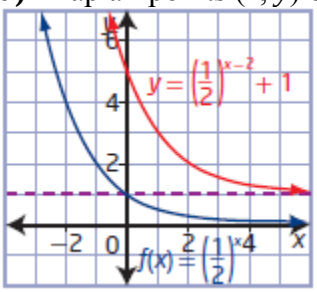
b) To obtain the graph of $y = -0.5f(x - 3)$, the graph of $f(x)$ must be vertically stretched by a factor of 0.5, reflected in the x-axis, and translated 3 units to the right:
$y = -0.5(5)^{x-3}$.

c) To obtain the graph of $y = -f(3x) + 1$, the graph of $f(x)$ must be reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{3}$, and translated 1 unit up: $y = \left(\frac{1}{4}\right)^{3x} + 1$.

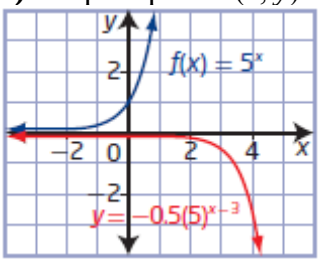
d) To obtain the graph of $y = 2f\left(-\frac{1}{3}(x-1)\right) - 5$, the graph of $f(x)$ must be vertically stretched by a factor of 2, horizontally stretched by a factor of 3, reflected in the y-axis, and translated 1 unit to the right and 5 units down: $y = 2(4)^{-\frac{1}{3}(x-1)} - 5$.

Section 7.2 Page 356 Question 8

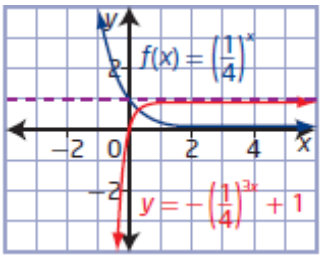
a) Map all points (x, y) on the graph of $f(x)$ to $(x + 2, y + 1)$.



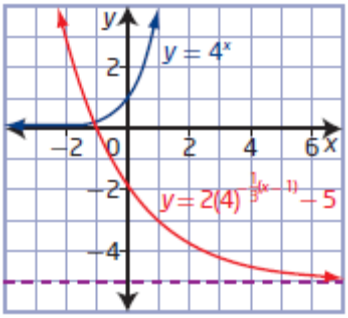
b) Map all points (x, y) on the graph of $f(x)$ to $(x + 3, –0.5y)$.



c) Map all points (x, y) on the graph of $f(x)$ to $\left(\frac{1}{3}x, -y + 1\right)$.



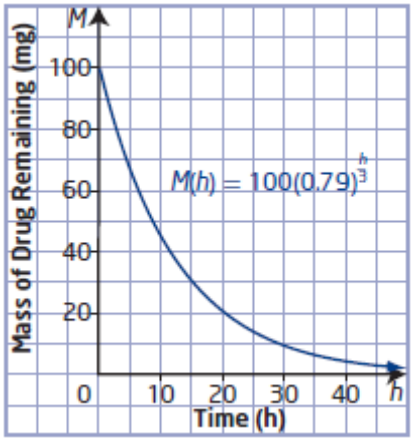
d) Map all points (x, y) on the graph of $f(x)$ to $(–3x + 1, 2y – 5)$.



Section 7.2 Page 356 Question 9

a) The number 0.79 represents the 79% of the drug remaining in the body of a dose taken. The number $\frac{1}{3}$ represents the decay rate of the dose taken. The dose decreases by 79% every $\frac{1}{3}$ h.

b)



c) The M-intercept represents the dose of 100 mg.

d) For this situation, the domain is $\{h \mid h \geq 0, h \in \mathrm{R}\}$ and the range is $\{M \mid 0 < M \leq 100, M \in \mathrm{R}\}$.

Section 7.2 Page 356 Question 10

a) Substitute $T_i = 95$ and $T_f = 20$ into $T(t) = (T_i - T_f)(0.9)^{\frac{t}{5}} + T_f$:

$T(t) = 75(0.9)^{\frac{t}{5}} + 20$.

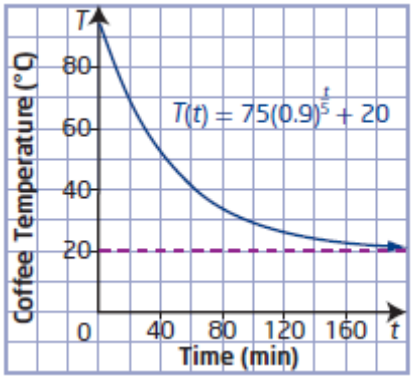
$a = 75$, vertical stretch by a factor of 75

$b = \frac{1}{5}$, horizontal stretch by a factor of 5

$h = 0$, no horizontal translation

$k = 20$, vertical translation of 20 units up

b)



c) Substitute $t = 100$.

$$T(t) = 75(0.9)^{\frac{t}{5}} + 20$$

$$T(100) = 75(0.9)^{\frac{100}{5}} + 20$$

$$T(100) = 29.1182...$$

The temperature of the coffee after 100 min is approximately 29.1 °C.

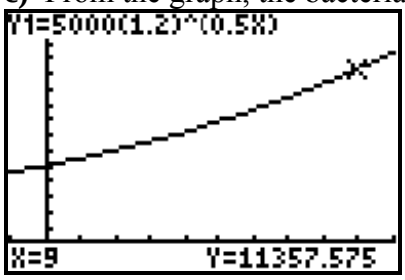
d) The equation of the horizontal asymptote is $T = 20$. This represents 20 °C, the final temperature of the coffee.

Section 7.2 Page 356 Question 11

a) For 5000 bacteria, $a = 5000$. For an increase of 20%, $c = 1.2$. For an increase that happens every 2 days, $b = \frac{1}{2}$. Then, the transformed exponential function for this situation is $P = 5000(1.2)^{\frac{x}{2}}$.

b) $a = 5000$, vertical stretch by a factor of 5000
$b = \frac{1}{2}$, horizontal stretch by a factor of 2
$h = 0$, no horizontal translation
$k = 0$, no vertical translation

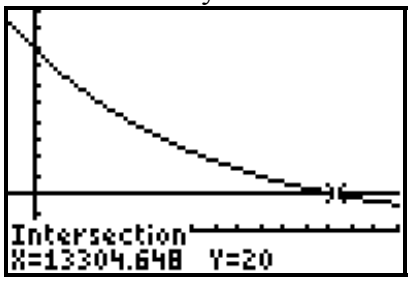
c) From the graph, the bacteria population after 9 days is approximately 11 357.



Section 7.2 Page 356 Question 12

a) The initial percent of C-14 in an organism is 100%, so $a = 100$. For half-life, $c = \frac{1}{2}$. Since the half-life of C-14 is about 5730 years, $b = \frac{1}{5730}$. Then, the transformed exponential function that represents the percent, P, of C-14 remaining after t years is $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

b) From the graph, the approximate age of a dead organism that has 20% of original C-14 is 13 305 years old.



Section 7.2 Page 357 Question 13

a) Let A represent the area covered by the bacteria. Let t represent time, in hours. The doubling time for the area is 10 h, so $c = 2$ and $b = \frac{1}{10}$. Since the initial area was 100 cm^2, $a = 100$. Then, $A = 100(2)^{\frac{t}{10}}$.

Substitute $t = 24$,

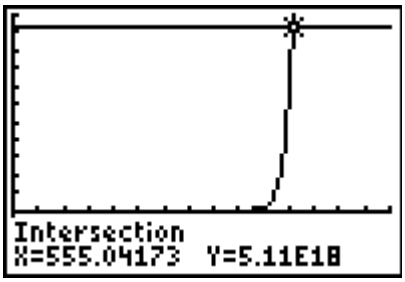
$$\begin{aligned} A &= 100(2)^{\frac{t}{10}} \\ &= 100(2)^{\frac{24}{10}} \\ &= 527.803... \end{aligned}$$

By Tuesday morning (24 h later), the bacteria covers an area of approximately 527.8 cm^2.

b) Determine the surface area of Earth: 6378 km = 637 800 000 cm

$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi(637\ 800\ 000)^2 \\ &\approx 5.11 \times 10^{18} \end{aligned}$$

Graph $A = 100(2)^{\frac{t}{10}}$ and $A = 5.11 \times 10^{18}$. It would take these bacteria about 555 h to cover the surface of Earth.



Section 7.2 Page 357 Question 14

a) Let P represent the fox population. Let t represent time, in years.

The initial fox population was 325 15 years ago, so $a = 325$ and $h = -15$. The population doubled in 15 years, so $b = \frac{1}{15}$ and $c = 2$. Then, $P = 325(2)^{\frac{1}{15}(t+15)}$.

Substitute $t = 20$,

$$\begin{aligned} P &= 325(2)^{\frac{1}{15}(t+15)} \\ &= 325(2)^{\frac{1}{15}(20+15)} \\ &= 1637.897... \end{aligned}$$

The fox population in 20 years will be about 1637.

b) Example: Disease or lack of food can change the rate of growth of the foxes. Exponential growth suggests that the population will grow without bound, and therefore the fox population will grow beyond the possible food sources, which is not good if not controlled.

Section 7.2 Page 357 Question C1

Example: The graph of an exponential function of the form $y = c^x$ has a horizontal asymptote at $y = 0$. Since $y \neq 0$, the graph cannot have an x-intercept.

Section 7.2 Page 357 Question C2

a) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a and k can affect the x-intercept. If $a > 0$ and $k < 0$ or $a < 0$ and $k > 0$, then the graph of the exponential function will have an x-intercept.

b) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a, h, and k can affect the y-intercept. The point $(0, y)$ on the graph of $y = c^x$ gets mapped to $(h, ay + k)$.

Section 7.3 Solving Exponential Equations

Section 7.3 Page 364 Question 1

a) $4^6 = (2^2)^6$
$= 2^{12}$

b) $8^3 = (2^3)^3$
$= 2^9$

c) $\left(\frac{1}{8}\right)^2 = \left(\left(\frac{1}{2}\right)^3\right)^2$
$= \left(2^{-3}\right)^2$
$= 2^{-6}$

d) $16 = 2^4$

Section 7.3 Page 364 Question 2

a) 2^3 and $4^2 = 2^3$

b) $9^x = (3^2)^x$ and $27 = 3^3$
$= 3^{2x}$

c) $\left(\frac{1}{2}\right)^{2x}$ and $\left(\frac{1}{4}\right)^{x-1} = \left(\left(\frac{1}{2}\right)^2\right)^{x-1}$
$= \left(\frac{1}{2}\right)^{2x-2}$

d) $\left(\frac{1}{8}\right)^{x-2} = \left(2^{-3}\right)^{x-2}$ and $16^x = (2^4)^x$
$= 2^{-3x+6}$ $= 2^{4x}$

Section 7.3 Page 364 Question 3

a) $\left(\sqrt{16}\right)^2 = (4)^2$

$= 4^2$

b) $\sqrt[3]{16} = \sqrt[3]{4^2}$

$= 4^{\frac{2}{3}}$

c) $\sqrt{16}\left(\sqrt[3]{64}\right)^2 = 4(4)^2$

$= 4(4^2)$

$= 4^3$

d) $\left(\sqrt{2}\right)^8\left(\sqrt[4]{4}\right)^4 = 2^4(4)$

$= 4^2(4)$

$= 4^3$

Section 7.3 Page 364 Question 4

a) $2^{4x} = 4^{x+3}$

$2^{4x} = (2^2)^{x+3}$

$2^{4x} = 2^{2x+6}$

Equate the exponents.

$4x = 2x + 6$

$2x = 6$

$x = 3$

Check:

Left Side	Right Side
2^{4x}	4^{x+3}
$= 2^{4(3)}$	$= 4^{3+3}$
$= 2^{12}$	$= 4^6$
$= 4096$	$= 4096$

Left Side = Right Side

The solution is $x = 3$.

b) $25^{x-1} = 5^{3x}$

$(5^2)^{x-1} = 5^{3x}$

$5^{2x-2} = 5^{3x}$

Equate the exponents.

$2x - 2 = 3x$

$x = -2$

Check:

Left Side	Right Side
25^{x-1}	5^{3x}
$= 25^{-2-1}$	$= 5^{3(-2)}$
$= 25^{-3}$	$= 5^{-6}$
$= \frac{1}{15\,625}$	$= \frac{1}{15\,625}$

Left Side = Right Side

The solution is $x = -2$.

c) $3^{w+1} = 9^{w-1}$

$3^{w+1} = (3^2)^{w-1}$

$3^{w+1} = 3^{2w-2}$

Equate the exponents.

$w + 1 = 2w - 2$

$w = 3$

Check:

Left Side	Right Side
3^{w+1}	9^{w-1}
$= 3^{3+1}$	$= 9^{3-1}$
$= 3^4$	$= 9^2$
$= 81$	$= 81$

Left Side = Right Side

The solution is $w = 3$.

d) $36^{3m-1} = 6^{2m+5}$

$(6^2)^{3m-1} = 6^{2m+5}$

$6^{6m-2} = 6^{2m+5}$

Equate the exponents.

$6m - 2 = 2m + 5$

$4m = 7$

$m = \frac{7}{4}$

Check:

Left Side	Right Side
36^{3m-1}	6^{2m+5}
$= 36^{3\left(\frac{7}{4}\right)-1}$	$= 6^{2\left(\frac{7}{4}\right)+5}$
$= 36^{\frac{21}{4}-1}$	$= 6^{\frac{7}{2}+5}$
$= 36^{\frac{17}{4}}$	$= 6^{\frac{17}{2}}$
$= 4\,114\,202.164...$	$= 4\,114\,202.164...$

Left Side = Right Side

The solution is $m = \frac{7}{4}$.

Section 7.3 Page 364 Question 5

a) $4^{3x} = 8^{x-3}$

$(2^2)^{3x} = (2^3)^{x-3}$

$2^{6x} = 2^{3x-9}$

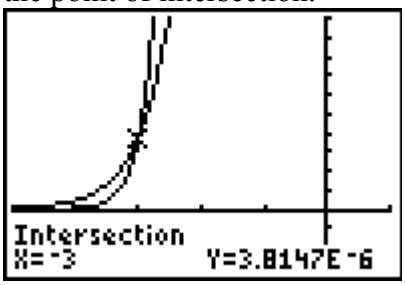
Equate the exponents.

$6x = 3x - 9$

$3x = -9$

$x = -3$

Check: Graph $y = 4^{3x}$ and $y = 8^{x-3}$ and find the point of intersection.



The solution is $x = -3$.

b) $27^x = 9^{x-2}$

$(3^3)^x = (3^2)^{x-2}$

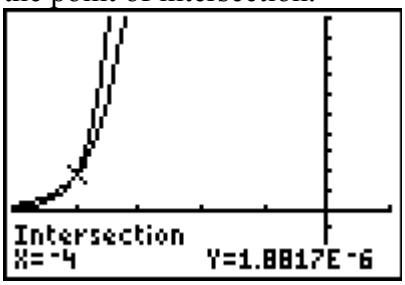
$3^{3x} = 3^{2x-4}$

Equate the exponents.

$3x = 2x - 4$

$x = -4$

Check: Graph $y = 27^x$ and $y = 9^{x-2}$ and find the point of intersection.



The solution is $x = -4$.

c) $125^{2y-1} = 25^{y+4}$

$(5^3)^{2y-1} = (5^2)^{y+4}$

$5^{6y-3} = 5^{2y+8}$

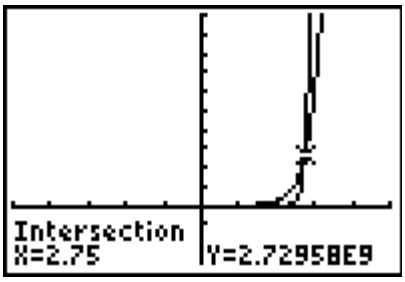
Equate the exponents.

$6y - 3 = 2y + 8$

$4y = 11$

Check: Graph $y = 125^{2x-1}$ and $y = 25^{x+4}$ and find the point of intersection.

$y = \frac{11}{4}$



The solution is $y = \frac{11}{4}$.

d) $16^{2k-3} = 32^{k+3}$

$(2^4)^{2k-3} = (2^5)^{k+3}$

$2^{8k-12} = 2^{5k+15}$

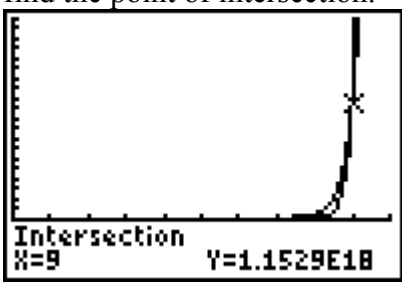
Equate the exponents.

$8k - 12 = 5k + 15$

$3k = 27$

$k = 9$

Check: Graph $y = 16^{2k-3}$ and $y = 32^{k+3}$ and find the point of intersection.



The solution is $k = 9$.

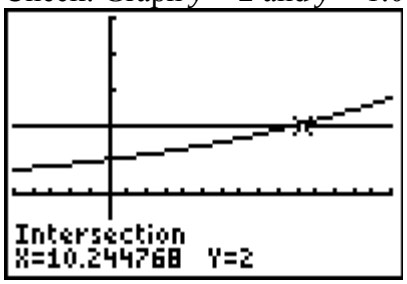
Section 7.3 Page 364 Question 6

a) Use systematic trial to solve $2 = 1.07^x$.

x	1.07^x	Mathematical Reasoning
12	2.252…	Try $x = 12$. This gives a value greater than 2. Try a lesser value.
10	1.967…	Too low. The correct value is between 10 and 12, but much closer to 10. Try 10.3.
10.3	2.007…	Close, but too high.
10.2	1.993…	This is very close and a reasonable approximation.

The solution is $x \approx 10.2$.

Check: Graph $y = 2$ and $y = 1.07^x$ and find the point of intersection.



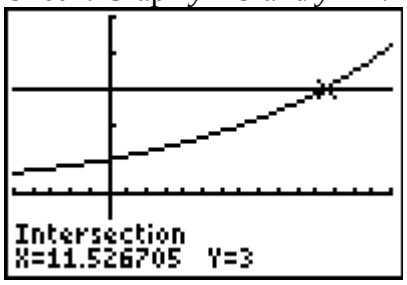
The solution is $x \approx 10.2$.

b) Use systematic trial to solve $3 = 1.1^x$.

x	1.1^x	Mathematical Reasoning
12	3.138…	Try $x = 12$. This gives a value greater than 3. Try a lesser value.
11	2.853…	Too low. The correct value is between 11 and 12. Try 11.6.
11.6	3.021…	Close, but too high.
11.5	2.992…	This is very close and a reasonable approximation.

The solution is $x \approx 11.5$.

Check: Graph $y = 3$ and $y = 1.1^x$ and find the point of intersection.



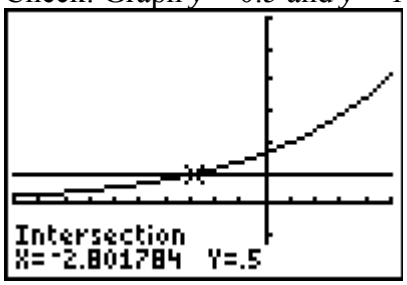
The solution is $x \approx 11.5$.

c) Use systematic trial to solve $0.5 = 1.2^{x-1}$.

x	1.2^{x-1}	Mathematical Reasoning
–4	0.4018…	Try $x = -4$. This gives a value lesser than 0.5. Try a greater value.
–2	0.5787…	Too high. The correct value is between –4 and –2. Try –3.
–3	0.4822…	Too low.
–2.9	0.4911…	Close, but too low.
–2.8	0.5001…	This is very close and a reasonable approximation.

The solution is $x \approx -2.8$.

Check: Graph $y = 0.5$ and $y = 1.2^{x-1}$ and find the point of intersection.



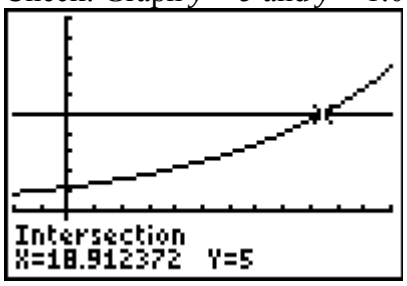
The solution is $x \approx -2.8$.

d) Use systematic trial to solve $5 = 1.08^{x+2}$.

x	1.08^{x+2}	Mathematical Reasoning
20	5.436…	Try $x = 20$. This gives a value greater than 5. Try a lesser value.
18	4.660…	Too low. The correct value is between 18 and 20. Try 19.
19	5.033…	Close, but too high.
18.9	4.995…	This is very close and a reasonable approximation.

The solution is $x \approx 18.9$.

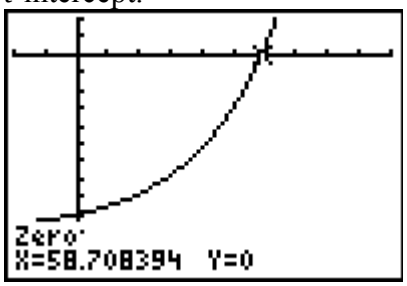
Check: Graph $y = 5$ and $y = 1.08^{x+2}$ and find the point of intersection.



The solution is $x \approx 18.9$.

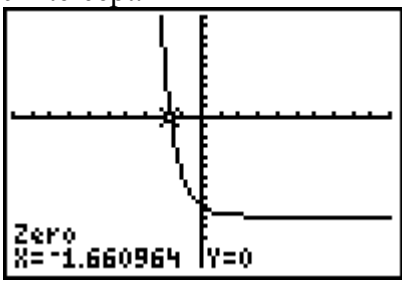
Section 7.3 Page 364 Question 7

a) Graph $y = 10(1.04)^t - 100$ and find the t-intercept.



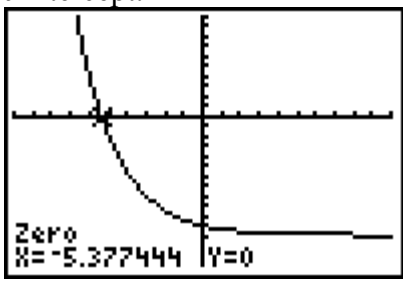
The solution is $t \approx 58.71$.

b) Graph $y = \left(\frac{1}{2}\right)^{2t} - 10$ and find the t-intercept.



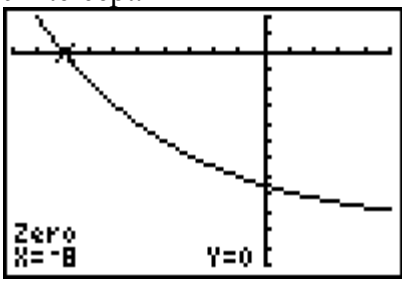
The solution is $t \approx -1.66$.

c) Graph $y = \left(\frac{1}{4}\right)^{\frac{t}{3}} - 12$ and find the t-intercept.



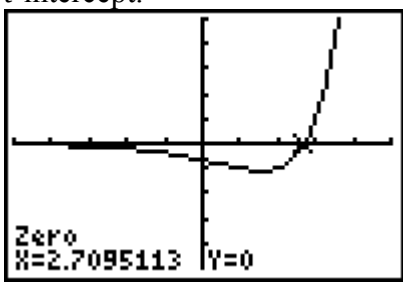
The solution is $t \approx -5.38$.

d) Graph $y = 25\left(\frac{1}{2}\right)^{\frac{t}{4}} - 100$ and find the t-intercept.



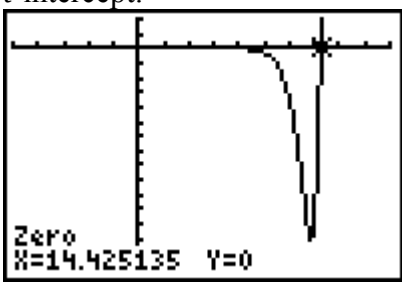
The solution is $t = -8$.

e) Graph $y = 3^{t-1} - 2^t$ and find the t-intercept.



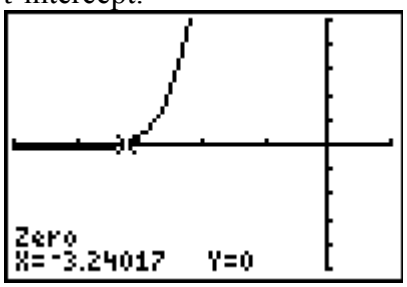
The solution is $t \approx 2.71$.

f) Graph $y = 5^{t-2} - 4^t$ and find the t-intercept.



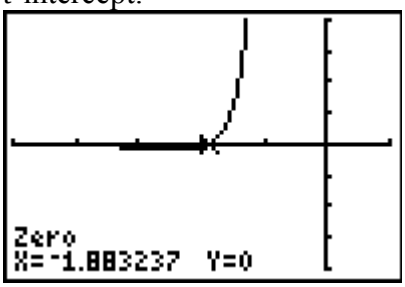
The solution is $t \approx 14.43$.

g) Graph $y = 8^{t+1} - 3^{t-1}$ and find the t-intercept.



The solution is $t \approx -3.24$.

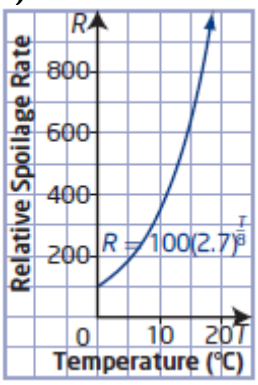
h) Graph $y = 7^{2t+1} - 4^{t-2}$ and find the t-intercept.



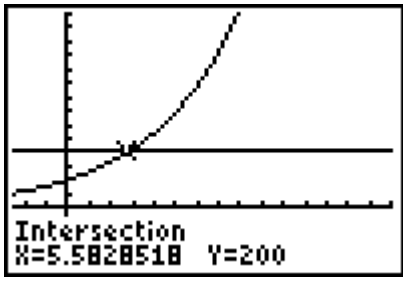
The solution is $t \approx -1.88$.

Section 7.3 Page 364 Question 8

a)



b) Graph $R = 100(2.7)^{\frac{T}{8}}$ and $R = 200$ and find the point of intersection.



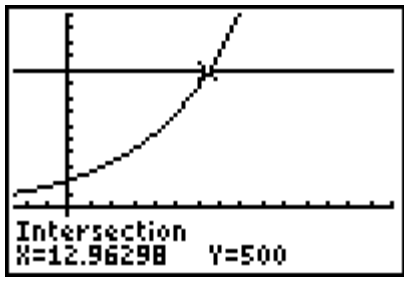
The temperature at which the relative spoilage rate doubles to 200 is approximately 5.6 °C.

c) Substitute $T = 15$.

$$R = 100(2.7)^{\frac{T}{8}}$$
$$= 100(2.7)^{\frac{15}{8}}$$
$$= 643.883...$$

The relative spoilage rate at 15 °C is approximately 643.

d) Graph $R = 100(2.7)^{\frac{T}{8}}$ and $R = 500$ and find the point of intersection.



The maximum storage temperature is approximately 13.0 °C.

Section 7.3 Page 364 Question 9

Let N represent the number of bacteria. Let t represent time, in hours.
The initial bacteria count is 2000, so $a = 2000$. The bacteria double every 0.75 h, so $b = \frac{4}{3}$ and $c = 2$. Then, $N = 2000(2)^{\frac{4}{3}t}$.
Substitute $N = 32\,000$,

$$N = 2000(2)^{\frac{4}{3}t}$$
$$32\,000 = 2000(2)^{\frac{4}{3}t}$$
$$16 = (2)^{\frac{4}{3}t}$$
$$2^4 = (2)^{\frac{4}{3}t}$$

Equate the exponents.

$$\frac{4}{3}t = 4$$
$$t = 3$$

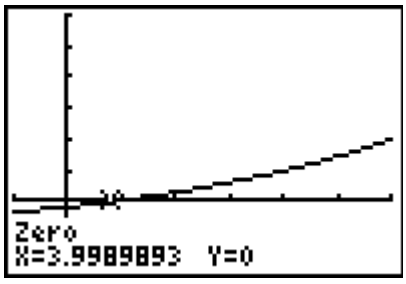
After 3 h the bacteria will be 32 000.

Section 7.3 Page 364 Question 10

Use the formula $A = P(1 + i)^n$, where $A = 7000$, $P = 6000$, and $i = 0.0393$.

$$A = P(1 + i)^n$$
$$7000 = 6000(1 + 0.0393)^n$$
$$\frac{7}{6} = 1.0393^n$$

Graph $y = 1.0393^x - \frac{7}{6}$ and find the x-intercept.



Simionie would have to invest his money in a GIC for 4 years.

Section 7.3 Page 365 Question 11

a) Use the formula $A = P(1 + i)^n$, where $P = 1000$ and $i = 0.02$: $A = 1000(1.02)^n$.

b) Substitute $n = 16$, the number of compounding periods in 4 years.

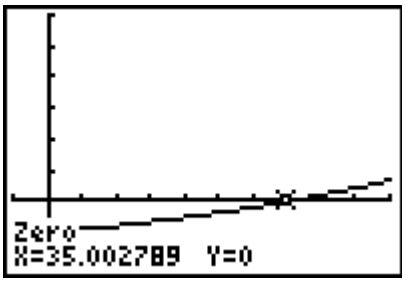
$A = 1000(1.02)^n$
$= 1000(1.02)^{16}$
$= 1372.785\ldots$

The value of the investment after 4 years is \$1372.79.

c) Substitute $A = 2000$.

$A = 1000(1.02)^n$
$2000 = 1000(1.02)^n$
$2 = 1.02^n$

Graph $y = 1.02^x - 2$ and find the x-intercept.



From the graph, it appears that it will take 35 compounding periods, or 8.75 years, for the investment to double in value. However, substituting $n = 35$ into the original function $A = 1000(1.02)^n$ results in a value of \$1999.89. So, it will take 36 compounding periods, or 9 years, for the investment to actually double in value.

Section 7.3 Page 365 Question 12

a) Let the initial sample of Co-60 be 1, so $a = 1$. For half-life, $c = \frac{1}{2}$. Since the half-life of Co-60 is about 5.3 years, $b = \frac{1}{5.3}$. Then, the exponential function that represents this situation is $m = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$, where m is the amount of Co-60 remaining after t years.

b) Substitute $t = 26.5$.

$$m = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$
$$= \left(\frac{1}{2}\right)^{\frac{26.5}{5.3}}$$
$$= \left(\frac{1}{2}\right)^{5}$$
$$= \frac{1}{32}$$

The fraction of a sample of Co-60 that will remain after 26.5 years is $\frac{1}{32}$.

c) Substitute $m = \frac{1}{512}$.

$$m = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$\left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

Equate the exponents.

$$\frac{t}{5.3} = 9$$

$$t = 47.7$$

It will take a sample of Co-60 47.7 years to decay to $\frac{1}{512}$ of its original mass.

Section 7.3 Page 365 Question 13

a) Use the formula $A = P(1 + i)^n$, where $P = 500$ and $i = 0.033$: $A = 500(1.033)^n$.

b) Substitute $n = 10$, the number of compounding periods in 5 years.

$A = 500(1.033)^n$

$= 500(1.033)^{10}$

$= 691.788\ldots$

The value of the investment after 5 years is \$691.79.

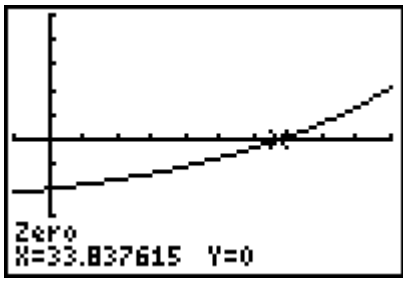
c) Substitute $A = 1500$.

$A = 500(1.033)^n$

$1500 = 500(1.033)^n$

$3 = 1.033^n$

Graph $y = 1.033^x - 3$ and find the x-intercept.



It will take 34 compounding periods, or 17 years, for the investment to triple in value.

Section 7.3 Page 365 Question 14

Use the formula $A = P(1 + i)^n$, where $A = 20\,000$, $i = 0.035$, and $n = 36$.

$$A = P(1 + i)^n$$
$$20\,000 = P(1 + 0.035)^{36}$$
$$P = \frac{20\,000}{1.035^{36}}$$
$$P = 5796.654\ldots$$

Glenn and Arlene will need to invest \$5796.65 today.

Section 7.3 Page 365 Question 15

a) i) $2^{3x} > 4^{x+1}$

$$2^{3x} > (2^2)^{x+1}$$
$$2^{3x} > 2^{2x+2}$$

Equate the exponents.

$3x > 2x + 2$

$x > 2$

ii) $81^x < 27^{2x+1}$

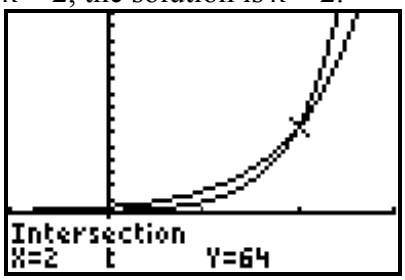
$$(3^4)^x < (3^3)^{2x+1}$$
$$3^{4x} < 3^{6x+3}$$

Equate the exponents.

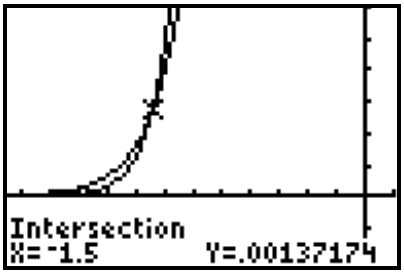
$4x < 6x + 3$

$x > -\frac{3}{2}$

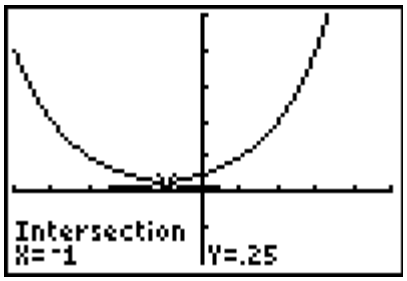
b) i) Since the graph of $y = 2^{3x}$ is greater than (above) the graph of $y = 4^{x+1}$ when $x > 2$, the solution is $x > 2$.



ii) Since the graph of $y = 81^x$ is less than (below) the graph of $y = 27^{2x+1}$ when $x > -\frac{3}{2}$, the solution is $x > -\frac{3}{2}$.



c) Example: Solve the inequality $\left(\frac{1}{2}\right)^{x+3} > 2^{x-1}$.



Since the graph of $y = \left(\frac{1}{2}\right)^{x+3}$ is greater than (above) the graph of $y = 2^{x-1}$ when $x < -1$, the solution is $x < -1$.

Section 7.3 Page 365 Question 16

$$4^{2x} + 2(4^x) - 3 = 0$$
$$(4^x)^2 + 2(4^x) - 3 = 0$$
$$(4^x + 3)(4^x - 1) = 0$$
$$4^x + 3 = 0 \text{ or } 4^x - 1 = 0$$
$$4^x = -3 \qquad 4^x = 1$$
$$x = 0$$

Since the value 4^x is always greater than zero, there is no real value of x for which $4^x = -3$. The real solution is $x = 0$.

Section 7.3 Page 365 Question 17

$$4^x - 4^{x-1} = 24$$
$$4^x - 4^x(4)^{-1} = 24$$
$$4^x(1 - 4^{-1}) = 24$$
$$4^x(0.75) = 24$$
$$4^x = 32$$
$$2^{2x} = 2^5$$

Equate the exponents.

$$2x = 5$$
$$x = \frac{5}{2}$$

Substitute $x = \frac{5}{2}$ into $(2^x)^x$.

$$(2^x)^x = \left(2^{\frac{5}{2}}\right)^{\frac{5}{2}}$$
$$= 2^{\frac{25}{4}}$$
$$= 76.109...$$

The value of $(2^x)^x$ is approximately 76.1.

Section 7.3 Page 365 Question 18

Use the formula $PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$ with $PMT = 831.90$, $PV = 150\ 000$, and $i = 0.0025$.

$$PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$$

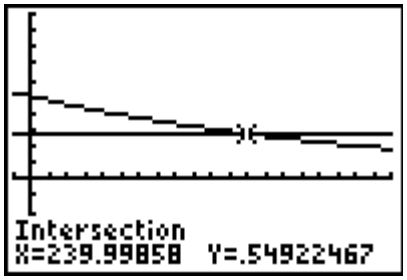
$$831.90 = 150\,000\left[\frac{0.0025}{1-(1+0.0025)^{-n}}\right]$$

$$0.005\,546 = \left[\frac{0.0025}{1-(1.0025)^{-n}}\right]$$

$$1-(1.0025)^{-n} = \frac{0.0025}{0.005\,546}$$

$$(1.0025)^{-n} = 1-\frac{0.0025}{0.005\,546}$$

Graph $y = 1.0025^{-n}$ and $y = 1-\frac{0.0025}{0.005\,546}$ and find the point of intersection.



It will take Tyseer 240 payment periods, or 20 years, to pay off the mortgage.

Section 7.3 Page 365 Question C1

a) You can express 16^2 with a base of 4 by writing 16 as 4^2 and simplifying.

$16^2 = (4^2)^2$
$= 4^4$

b) Example: You can express 16^2 with a base of 2 by writing 16 as 2^4 and simplifying.

$16^2 = (2^4)^2$
$= 2^8$

You can express 16^2 with a base of $\frac{1}{2}$ by writing 16 as $\left(\frac{1}{2}\right)^{-4}$ and simplifying.

$$16^2 = \left(\left(\frac{1}{2}\right)^{-4}\right)^2$$

$$= \left(\frac{1}{2}\right)^{-8}$$

Section 7.3 Page 365 Question C2

a) $16^{2x} = 8^{x-3}$

$(2^4)^{2x} = (2^3)^{x-3}$ Express 16 and 8 as powers of 2.

$2^{8x} = 2^{3x-9}$ Apply the power of a power law.

$8x = 3x - 9$ Equate the exponents.

$5x = -9$ Isolate the x-term.

$x = -\frac{9}{5}$ Solve of x.

b) Given equation.

Chapter 7 Review

Chapter 7 Review Page 366 Question 1

a) For the population of a country, in millions, that grows at a rate of 1.5% per year, the graph would show the function $y = 1.015x$: graph **B**.

b) The graph of $y = 10^x$ contains the point (1, 10): graph **D**.

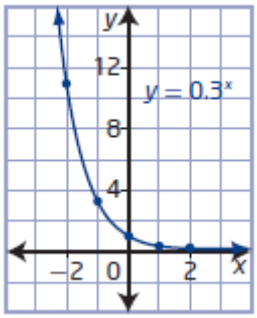
c) For Tungsten-187, a radioactive isotope that has a half-life of 1 day, the graph would show the function $y = \left(\frac{1}{2}\right)^x$: graph **A**.

d) The graph of $y = 0.2^x$ contains the point (–1, 5): graph **C**.

Chapter 7 Review Page 366 Question 2

a)

x	y
–2	$11.\overline{1}$
–1	$3.\overline{3}$
0	1
1	0.3
2	0.09



b) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
There is no x-intercept. The y-intercept is 1.
The function is decreasing for all values of x.
The equation of the horizontal asymptote is $y = 0$.

Chapter 7 Review Page 366 Question 3

There is a pattern in the ordered pairs.

x	y
0	1
–1	4
–2	16

As the value of x increases by 1 unit, the value of y decreases by a factor of $\frac{1}{4}$.

Therefore, for this function, $c = \frac{1}{4}$.

Use the point (–1, 4) to check the function $y = \left(\frac{1}{4}\right)^x$:

Left Side	Right Side
y	$\left(\frac{1}{4}\right)^x$
$= 4$	$= \left(\frac{1}{4}\right)^{-1}$
	$= 4$

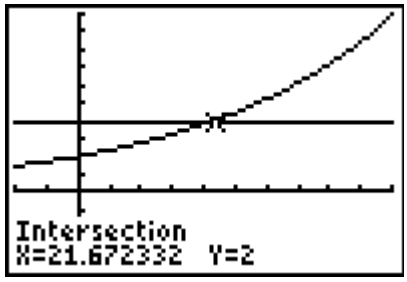
The function equation for the graph is $y = \left(\frac{1}{4}\right)^x$.

Chapter 7 Review Page 366 Question 4

a) Since the interest rate is 3.25% per year, each year the investment grows by a factor of 103.25%, or 1.0325, as a decimal.

b) Substitute $t = 10$.
$v = 1.0325^t$
$= 1.0325^{10}$
$= 1.376\ldots$
The value of \$1 if it is invested for 10 years will be \$1.38.

c) Graph $v = 1.0325^t$ and $v = 2$ and find the point of intersection.



It will take approximately 21.7 years for the value of the dollar invested to reach \$2.

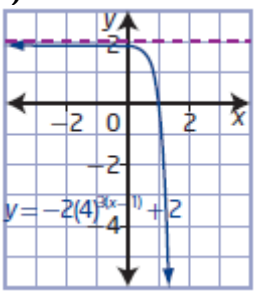
Chapter 7 Review Page 366 Question 5

a) For $y = -2(4)^{3(x-1)} + 2$,
$a = -2$, vertical stretch by a factor of 2 and reflected in the x-axis
$b = 3$, horizontal stretch by a factor of $\frac{1}{3}$
$h = 1$, horizontal translation of 1 unit to the right
$k = 2$, vertical translation of 2 units up

b)

Transformation	Parameter Value	Function Equation
horizontal stretch	$b = 3$	$y = 4^{3x}$
vertical stretch	$a = -2$	$y = -2(4)^x$
translation left/right	$h = 1$	$y = (4)^{x-1}$
translation up/down	$k = 2$	$y = 4^x + 2$

c)



d) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y < 2, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 2$.
The x-intercept is 1. The y-intercept is $\frac{63}{32}$.

Chapter 7 Review Page 367 Question 6

a) Look for a pattern in the points.

$y = 3^x$	**Transformed Function**
$\left(-1, \frac{1}{3}\right)$	$\left(2, \frac{1}{3}\right)$
(0, 1)	(3, 1)
(1, 3)	(4, 3)

The transformation can be described by the mapping $(x, y) \rightarrow (x + 3, y)$.
This represents a horizontal translation of 3 units to the right.

b) Look for a pattern in the points.

$y = 3^x$	**Transformed Function**
$\left(-1, \frac{1}{3}\right)$	$\left(-1, \frac{11}{3}\right)$
(0, 1)	(0, –3)
(1, 3)	(1, –1)

The transformation can be described by the mapping $(x, y) \rightarrow (x, y - 4)$.
This represents a vertical translation of 4 units down.

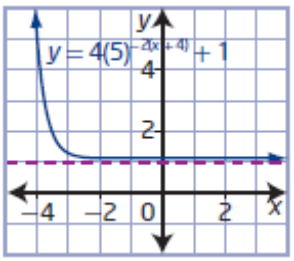
c) Look for a pattern in the points.

$y = 3^x$	**Transformed Function**
$\left(-1, \frac{1}{3}\right)$	$\left(-2, \frac{5}{3}\right)$
(0, 1)	(–1, 1)
(1, 3)	(0, –1)

The transformation can be described by the mapping $(x, y) \rightarrow (x - 1, -y + 2)$.
This represents a reflection in the x-axis and a translation of 1 unit to the right and 2 units up.

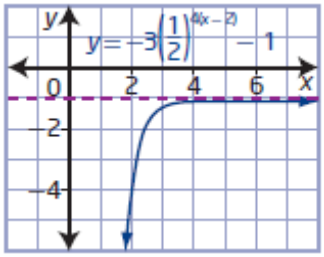
Chapter 7 Review Page 367 Question 7

a) For $f(x) = 5^x$,



• stretched vertically by a factor of 4, $a = 4$
• stretched horizontally by a factor of $\frac{1}{2}$ and reflected in the y-axis, $b = -2$
• translated 1 unit up and 4 units to the left, $k = 1$ and $h = -4$
The equation of the transformed function is $y = 4(5)^{-2(x+4)} + 1$.

b) For $g(x) = \left(\frac{1}{2}\right)^x$,



• stretched horizontally by a factor of $\frac{1}{4}$, $b = 4$
• stretched vertically by a factor of 3 and reflected in the x-axis, $a = -3$
• translated 2 units to the right and 1 unit down, $h = 2$ and $k = -1$
The equation of the transformed function is $y = -3\left(\frac{1}{2}\right)^{4(x-2)} - 1$.

Chapter 7 Review Page 367 Question 8

a) For $T = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$,

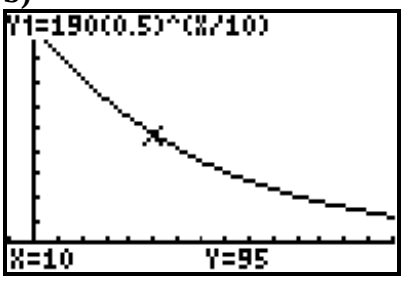
$a = 190$, vertical stretch by a factor of 190

$b = \frac{1}{10}$, horizontal stretch by a factor of 10

$h = 0$, no horizontal translation

$k = 0$, no vertical translation

b)



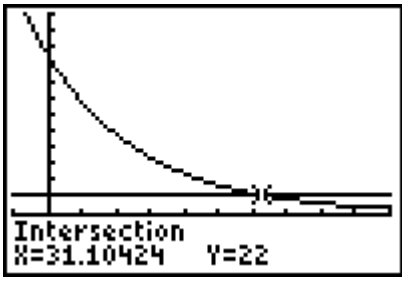
c) For this situation, the domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and the range is $\{T \mid 0 < T \leq 190, T \in \mathrm{R}\}$.

d) Substitute $T = 22$.

$$T = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$$

$$22 = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$$

Graph $T = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$ and $T = 22$ and find the point of intersection.



The milk will keep fresh at 22 °C for approximately 31.1 h.

Chapter 7 Review Page 367 Question 9

a) $36 = 6^2$ **b)** $\frac{1}{36} = 6^{-2}$ **c)** $\left(\sqrt[3]{216}\right)^5 = 6^5$

Chapter 7 Review Page 367 Question 10

a) $3^{5x} = 27^{x-1}$

$3^{5x} = (3^3)^{x-1}$

$3^{5x} = 3^{3x-3}$

Equate the exponents.

$5x = 3x - 3$

$2x = -3$

$x = -\frac{3}{2}$

b) $\left(\frac{1}{8}\right)^{2x+1} = 32^{x-3}$

$(2^{-3})^{2x+1} = (2^5)^{x-3}$

$2^{-6x-3} = 2^{5x-15}$

Equate the exponents.

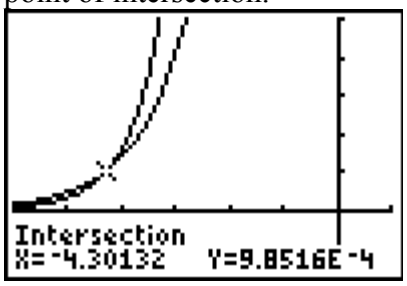
$-6x - 3 = 5x - 15$

$-11x = -12$

$x = \frac{12}{11}$

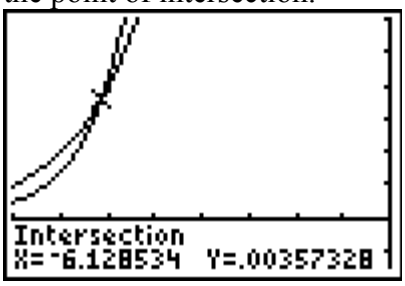
Chapter 7 Review Page 367 Question 11

a) Graph $y = 3^{x-2}$ and $y = 5^x$ and find the point of intersection.



The solution is $x \approx -4.30$.

b) Graph $y = 2^{x-2}$ and $y = 3^{x+1}$ and find the point of intersection.



The solution is $x \approx -6.13$.

Chapter 7 Review Page 367 Question 12

a) Let the initial sample of Ni-65 be 1, so $a = 1$. For half-life, $c = \frac{1}{2}$. Since the half-life of Ni-65 is 2.5 h, $b = \frac{1}{2.5}$. Then, the exponential function that represents this situation is $m = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$, where m is the amount of Ni-65 remaining after t hours.

b) Substitute $t = 10$.

$$m = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$$
$$= \left(\frac{1}{2}\right)^{\frac{10}{2.5}}$$
$$= \left(\frac{1}{2}\right)^{4}$$
$$= \frac{1}{16}$$

The fraction of a sample of Ni-65 that will remain after 10 h is $\frac{1}{16}$.

c) Substitute $m = \frac{1}{1024}$.

$$m = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$$
$$\frac{1}{1024} = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$$
$$\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$$

Equate the exponents.

$$\frac{t}{2.5} = 10$$
$$t = 25$$

It will take a sample of Ni-65 25 h to decay to $\frac{1}{1024}$ of its original mass.

Chapter 7 Practice Test

Chapter 7 Practice Test Page 368 Question 1

The functions $y = 2^x$, $y = \left(\frac{2}{3}\right)^x$, and $y = 7^x$ will all have the same y-value of 1 when $x = 0$:

Choice **B**.

Chapter 7 Practice Test Page 368 Question 2

To obtain the graph of $y = 3^{\frac{1}{4}(x-5)} - 2$, transform the graph of $y = 3^x$ by a horizontal stretch by a factor of 4 and a translation of 5 units to the right and 2 units down: Choice **C**.

Chapter 7 Practice Test Page 368 Question 3

Let V represent the value of the car. Let t represent time, in years.
The current value is 100 000, so $a = 100\,000$. The value doubles every 10 years, so $b = \frac{1}{10}$ and $c = 2$. Then, $V = 100\,000(2)^{\frac{t}{10}}$.
Substitute $t = -20$,

$$\begin{aligned} V &= 100\,000(2)^{\frac{t}{10}} \\ &= 100\,000(2)^{\frac{-20}{10}} \\ &= 25\,000 \end{aligned}$$

The value of the car 20 years ago was \$25 000: Choice **B**.

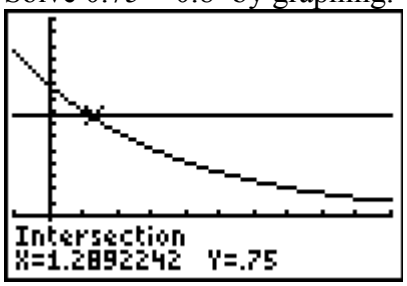
Chapter 7 Practice Test Page 368 Question 4

$$\begin{aligned} \frac{2^9}{(4^3)^2} &= \frac{2^9}{((2^2)^3)^2} \\ &= \frac{2^9}{2^{12}} \\ &= 2^{-3} \end{aligned}$$

Choice **A**.

Chapter 7 Practice Test Page 368 Question 5

Solve $0.75 = 0.8^x$ by graphing.



The glass should be approximately 1.3 mm thick: Choice **D**.

Chapter 7 Practice Test Page 368 Question 6

a) Look for a pattern in the points.

$y = 5^x$	**Transformed Function**
(0, 1)	(–3, 3)
(1, 5)	(–2, 7)
(2, 25)	(–1, 27)

The transformation can be described by the mapping $(x, y) \rightarrow (x - 3, y + 2)$.
This represents a horizontal translation of 3 units to the left and 2 units up: $h = -3$ and $k = 2$. So, the equation of the transformed function is $y = 5^{x+3} + 2$.

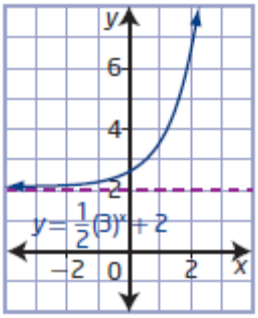
b) Look for a pattern in the points.

$y = 2^x$	**Transformed Function**
(0, 1)	$\left(1, -\frac{9}{2}\right)$
(1, 2)	(2, –5)
(2, 4)	(3, –6)

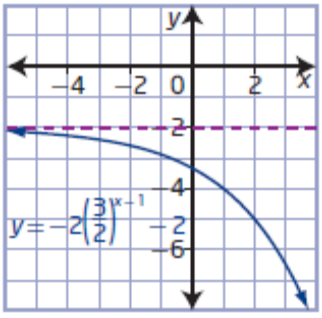
The transformation can be described by the mapping $(x, y) \rightarrow (x + 1, -0.5y - 4)$.
This represents a vertical stretch by a factor of 0.5, a reflection in the x-axis, and a translation of 1 unit to the left and 4 units down: $a = -0.5$, $h = 1$, and $k = -4$.
So, the equation of the transformed function is $y = -0.5(2)^{x-1} - 4$.

Chapter 7 Practice Test Page 369 Question 7

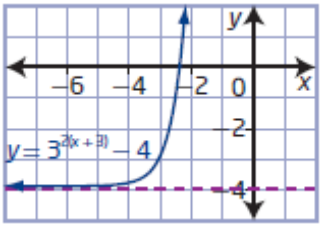
a)



b)



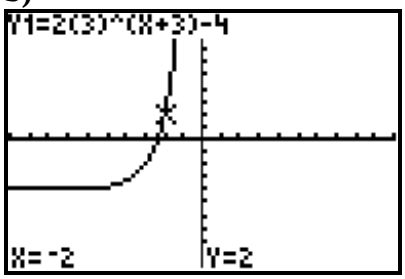
c)



Chapter 7 Practice Test Page 369 Question 8

a) The base function for $g(x) = 2(3)^{x+3} - 4$ is $f(x) = 3^x$.
For $g(x) = 2(3)^{x+3} - 4$,
$a = 2$, vertical stretch by a factor of 2
$b = 1$, no horizontal stretch
$h = -3$, horizontal translation of 3 units to the left
$k = -4$, vertical translation of 4 units down

b)



c) The domain is $\{x \mid x \in \mathrm{R}\}$, the range is $\{y \mid y > -4, y \in \mathrm{R}\}$, and the equation of the horizontal asymptote is $y = -4$.

Chapter 7 Practice Test Page 369 Question 9

a) $3^{2x} = 9^{\frac{1}{2}(x-4)}$

$3^{2x} = (3^2)^{\frac{1}{2}(x-4)}$

$3^{2x} = 3^{x-4}$

Equate the exponents.
$2x = x - 4$
$x = -4$

b) $27^{x-4} = 9^{x+3}$

$(3^3)^{x-4} = (3^2)^{x+3}$

$3^{3x-12} = 3^{2x+6}$

Equate the exponents.
$3x - 12 = 2x + 6$
$x = 18$

c) $1024^{2x-1} = 16^{x+4}$

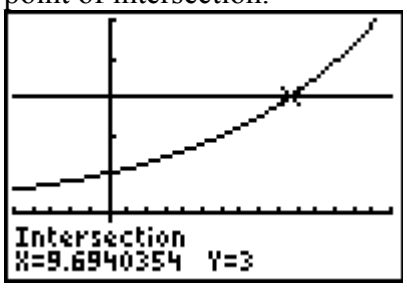
$(2^{10})^{2x-1} = (2^4)^{x+4}$

$2^{20x-10} = 2^{4x+16}$

Equate the exponents.
$20x - 10 = 4x + 16$
$16x = 26$
$x = \frac{13}{8}$

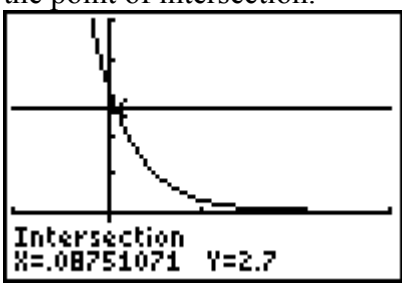
Chapter 7 Practice Test Page 369 Question 10

a) Graph $y = 3$ and $y = 1.12^x$ and find the point of intersection.



The solution is $x \approx 9.7$.

b) Graph $y = 2.7$ and $y = 0.3^{2x-1}$ and find the point of intersection.



The solution is $x \approx 0.1$.

Chapter 7 Practice Test Page 369 Question 11

a) If the growth rate remains constant at 2.77%, then the population would have been multiplied by a factor of 1.0277 after 1 year.

b) Let the initial percent of the population for Saskatoon in 2010 be 100, so $a = 100$. For the growth rate from part a), $c = 1.0277$. Then, the exponential function that represents this situation is $P = 100(1.0277)^t$, where P is percent of the population t years after 2010.

c) For this situation, the domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and the range is $\{P \mid P \geq 100, P \in \mathrm{R}\}$.

d) For Saskatoon's population to grow by 25%, substitute $P = 125$.

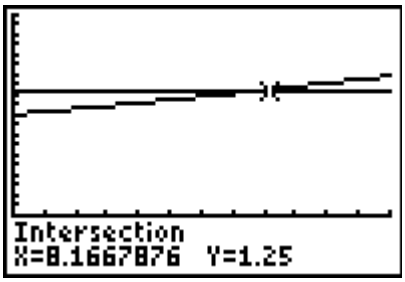
$P = 100(1.0277)^t$

$125 = 100(1.0277)^t$

$1.25 = 1.0277^t$

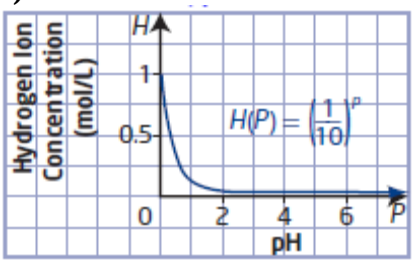
Graph $y = 1.25$ and $y = 1.0277^x$ and find the point of intersection.

It will take approximately 8.2 years for Saskatoon's population to grow by 25%.



Chapter 7 Practice Test Page 369 Question 12

a)



b) Substitute $P = 7$.

$$H(P) = \left(\frac{1}{10}\right)^{P}$$

$$H(7) = \left(\frac{1}{10}\right)^{7}$$

$$H(7) = 1 \times 10^{-7}$$

The hydrogen ion concentration for a pH of 7.0 is 1.0×10^{-7} [H+].

c) Determine the hydrogen ion concentration for a pH of 7.6. Substitute $P = 7.6$.

$$H(P) = \left(\frac{1}{10}\right)^{P}$$

$$H(7.6) = \left(\frac{1}{10}\right)^{7.6}$$

$$H(7.6) \approx 2.5 \times 10^{-8}$$

The equivalent range of hydrogen ion concentration for a pH between 7.0 and 7.6 is from 1.0×10^{-7} [H+] to 2.5×10^{-8} [H+].

Chapter 7 Practice Test Page 369 Question 13

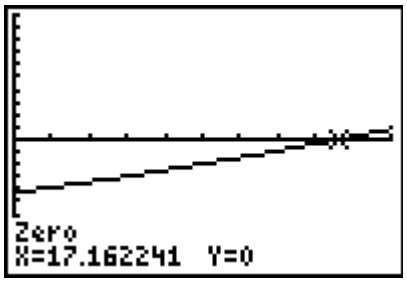
Use the formula $A = P(1 + i)^n$, where $A = 5000$, $P = 3500$ and $i = 0.021$.
Solve $5000 = 3500(1.021)^n$.

$$5000 = 3500(1.021)^n$$

$$\frac{5000}{3500} = 1.021^n$$

$$\frac{10}{7} = 1.021^n$$

Graph $y = 1.021^x - \frac{10}{7}$ and find the x-intercept.



It will take approximately 18 compounding periods, or 4.5 years, before Lucas has enough money to take the vacation he wants.

Chapter 7 Practice Test Page 369 Question 14

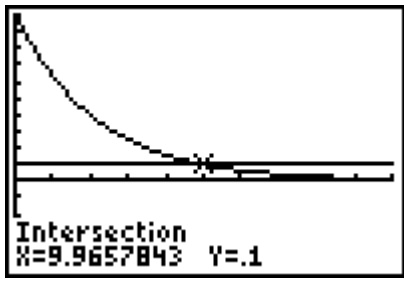
Determine when a computer purchased for \$3000 is worth 10% of its values, or \$300.
Substitute $V = 300$.

$$V(t) = 3000\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$300 = 3000\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$0.1 = \left(\frac{1}{2}\right)^{\frac{t}{3}}$$

Graph $y = 0.1$ and $y = \left(\frac{1}{2}\right)^{\frac{x}{3}}$ and find the point of intersection.



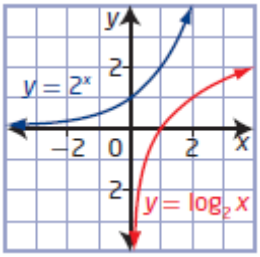
It will take about 9.97 years, for the computer to be worth 10% of its purchase price.

Chapter 8 Logarithmic Functions

Section 8.1 Understanding Logarithms

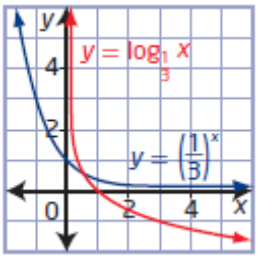
Section 8.1 Page 380 Question 1

a) i) **ii)** $y = \log_2 x$



iii) For $y = \log_2 x$,
• domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
• x-intercept: 1
• no y-intercept
• equation of the asymptote: $x = 0$

b) i) **ii)** $y = \log_{\frac{1}{3}} x$



iii) For $y = \log_{\frac{1}{3}} x$,
• domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
• x-intercept: 1
• no y-intercept
• equation of the asymptote: $x = 0$

Section 8.1 Page 380 Question 2

a) In logarithmic form, $12^2 = 144$ is $\log_{12} 144 = 2$.

b) In logarithmic form, $8^{\frac{1}{3}} = 2$ is $\log_8 2 = \frac{1}{3}$.

c) In logarithmic form, $10^{-5} = 0.000\,01$ is $\log_{10} 0.000\,01 = -5$.

d) In logarithmic form, $7^{2x} = y + 3$ is $\log_7 (y + 3) = 2x$.

Section 8.1 Page 380 Question 3

a) In exponential form, $\log_5 25 = 2$ is $5^2 = 25$.

b) In exponential form, $\log_8 4 = \frac{2}{3}$ is $8^{\frac{2}{3}} = 4$.

c) In exponential form, $\log 1\,000\,000 = 6$ is $10^6 = 1\,000\,000$.

d) In exponential form, $\log_{11} (x + 3) = y$ is $11^y = x + 3$.

Section 8.1 Page 380 Question 4

a) Since $5^3 = 125$, the value of the logarithm is 3. Therefore, $\log_5 125 = 3$.

b) Since $10^0 = 1$, the value of the logarithm is 0. Therefore, $\log 1 = 0$.

c) Let $\log_4 \sqrt[3]{4} = x$. Express in exponential form.

$$4^x = \sqrt[3]{4}$$

$$4^x = 4^{\frac{1}{3}}$$

$$x = \frac{1}{3}$$

Therefore, $\log_4 \sqrt[3]{4} = \frac{1}{3}$.

d) Let $\log_{\frac{1}{3}} 27 = x$. Express in exponential form.

$$\left(\frac{1}{3}\right)^x = 27$$

$$\left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-3}$$

$$x = -3$$

Therefore, $\log_{\frac{1}{3}} 27 = -3$.

Section 8.1 Page 380 Question 5

Write $a < \log_2 28 < b$ in exponential form: $2^a < 28 < 2^b$.
Since $2^4 = 16$ and $2^5 = 32$, then $a = 4$ and $b = 5$.

Section 8.1 Page 380 Question 6

a) For $\log_3 x$ to be a positive number, $x > 1$.

b) For $\log_3 x$ to be a negative number, $0 < x < 1$.

c) For $\log_3 x$ to be zero, $x = 1$.

d) Example: For $\log_3 x$ to be a rational number, $x = \sqrt{3}$.

Section 8.1 Page 380 Question 7

a) The base of a logarithm cannot be 0 because $0^y = 0$, $y \neq 0$.

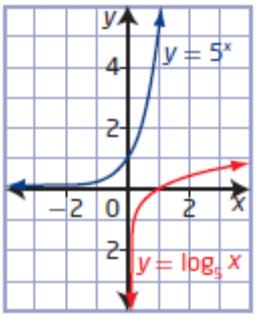
b) The base of a logarithm cannot be 1 because $1^y = 1$.

c) The base of a logarithm cannot be negative because exponential functions are only defined for $c > 0$.

Section 8.1 Page 380 Question 8

a) If $f(x) = 5^x$, then $f^{-1}(x) = \log_5 x$.

b)

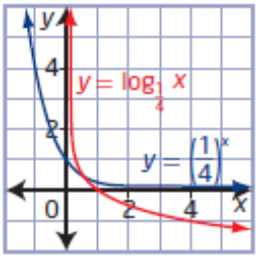


For $y = \log_5 x$,
- domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
- x-intercept: 1
- no y-intercept
- equation of the asymptote: $x = 0$

Section 8.1 Page 380 Question 9

a) If $g(x) = \log_{\frac{1}{4}} x$, then $g^{-1}(x) = \left(\frac{1}{4}\right)^x$.

b)



For $y = \left(\frac{1}{4}\right)^x$,
- domain: $\{x \mid x \in \mathrm{R}\}$ and range: $\{y \mid y > 0, y \in \mathrm{R}\}$
- no x-intercept
- y-intercept: 1
- equation of the asymptote: $y = 0$

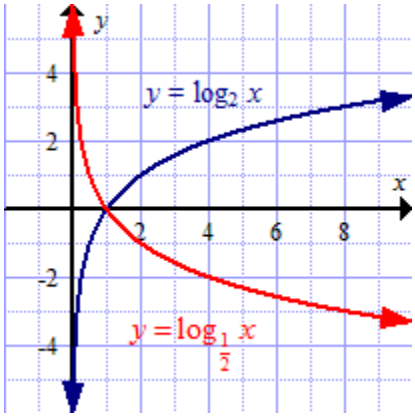
Section 8.1 Page 381 Question 10

The relationship between the characteristics of the functions $y = 7^x$ and $y = \log_7 x$ is that the graphs are reflections of each other in the line $y = x$. This means that the domain, range, y-intercept, and horizontal asymptote of the exponential function become the range, domain, x-intercept, and vertical asymptote of the logarithmic function.

Section 8.1 Page 381 Question 11

a) The graphs have the same domain, range, x-intercept, and vertical asymptote.

b) The graphs differ in that one is increasing and the other is decreasing.



Section 8.1 Page 381 Question 12

a) $\log_6 x = 3$

$6^3 = x$

$x = 216$

b) $\log_x 9 = \frac{1}{2}$

$x^{\frac{1}{2}} = 9$

$x = 9^2$

$x = 81$

c) $\log_{\frac{1}{4}} x = -3$

$\left(\frac{1}{4}\right)^{-3} = x$

$x = 4^3$

$x = 64$

d) $\log_x 16 = \frac{4}{3}$

$x^{\frac{4}{3}} = 16$

$x = 16^{\frac{3}{4}}$

$x = 8$

Section 8.1 Page 381 Question 13

a) Use the inverse property $c^{\log_c x} = x$. For $m = \log_5 7$,

$5^m = 5^{\log_5 7}$

$= 7$

b) Use the inverse property $c^{\log_c x} = x$. For $n = \log_8 6$,

$8^n = 8^{\log_8 6}$

$= 6$

Section 8.1 Page 381 Question 14

a) $\log_2 (\log_3 (\log_4 64)) = \log_2 (\log_3 3)$

$= \log_2 1$

$= 0$

b) $\log_4 (\log_2 (\log 10^{16})) = \log_4 (\log_2 16)$

$= \log_4 4$

$= 1$

Section 8.1 Page 381 Question 15

Substitute $y = 0$.

$y = \log_7 (x + 2)$

$0 = \log_7 (x + 2)$

$x + 2 = 7^0$

$x = -1$

The x-intercept of $y = \log_7 (x + 2)$ is -1.

Section 8.1 Page 381 Question 16

Use the given point $\left(\frac{1}{8}, -3\right)$ on the graph of $f(x) = \log_c x$ to determine the value of c.

$f(x) = \log_c x$

$-3 = \log_c \frac{1}{8}$

$c^{-3} = \frac{1}{8}$

$c^{-3} = 2^{-3}$

$c = 2$

So, the inverse of $f(x) = \log_2 x$ is $f^{-1}(x) = 2^x$.

For the point $(4, k)$ on the graph of the inverse, substitute $x = 4$.

$f^{-1}(x) = 2^x$

$f^{-1}(4) = 2^4$

$f^{-1}(4) = 16$

Therefore, the value of k is 16.

Section 8.1 Page 381 Question 17

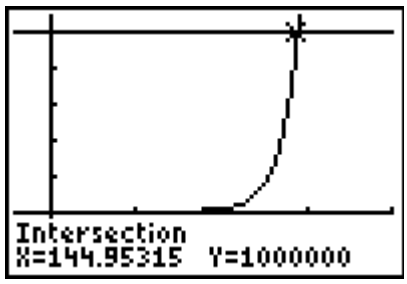
a) Given the exponential function $N(t) = 1.1^t$, the equation of the inverse is $t = \log_{1.1} N$.

b) Substitute $N = 1\ 000\ 000$.

$N(t) = 1.1^t$

$1\ 000\ 000 = 1.1^t$

Use graphing technology to graph each side of the equation and determine the point of intersection.



It will take approximately 145 days for the number of users to exceed 1 000 000.

Section 8.1 Page 381 Question 18

Determine the relative risk for each asteroid from the Palermo scale.

Substitute $P = -1.66$.

$P = \log R$

$-1.66 = \log R$

$R = 10^{-1.66}$

Substitute $P = -4.83$.

$P = \log R$

$-4.83 = \log R$

$R = 10^{-4.83}$

Compare the relative risks.

$$\frac{10^{-1.66}}{10^{-4.83}} = 10^{3.17}$$

$$= 1479.108\ldots$$

The larger asteroid had a relative risk that is about 1479 times as dangerous as the smaller asteroid.

Section 8.1 Page 381 Question 19

Determine the amplitude of each earthquake.

Nahanni earthquake:

$$M = \log \frac{A}{A_0}$$

$$6.9 = \log \frac{A}{A_0}$$

$$10^{6.9} = \frac{A}{A_0}$$

$$A = 10^{6.9} A_0$$

Saskatchewan earthquake:

$$M = \log \frac{A}{A_0}$$

$$3.9 = \log \frac{A}{A_0}$$

$$10^{3.9} = \frac{A}{A_0}$$

$$A = 10^{3.9} A_0$$

Compare the amplitudes.

$$\frac{10^{6.9}}{10^{3.9}} = 10^3$$

The seismic shaking of the Nahanni earthquake was 1000 times that of the Saskatchewan earthquake.

Section 8.1 Page 381 Question 20

If $\log_5 x = 2$, then $x = 5^2$, or 25.

$$\begin{aligned}\log_5 125x &= \log_5 125(25)\\ &= \log_5 5^3(5^2)\\ &= \log_5 5^5\\ &= 5\end{aligned}$$

Section 8.1 Page 381 Question 21

$\log_3 (m - n) = 0$

$3^0 = m - n$

$1 = m - n$ ①

$\log_3 (m + n) = 3$

$3^3 = m + n$

$27 = m + n$ ②

Solve the system of equations.

$$\begin{aligned}1 &= m - n\\ 27 &= m + n\\ \hline 28 &= 2m \quad \text{① + ②}\\ m &= 14\end{aligned}$$

Substitute $m = 14$ into ①.

$$\begin{aligned}1 &= m - n\\ 1 &= 14 - n\\ n &= 13\end{aligned}$$

Section 8.1 Page 381 Question 22

If $\log_3 m = n$, then $3^n = m$.

$$\begin{aligned}\log_3 m^4 &= \log_3 (3^n)^4\\ &= \log_3 3^{4n}\\ &= 4n\end{aligned}$$

Section 8.1 Page 381 Question 23

If $y = \log_2 (\log_3 x)$, then the inverse is

$x = \log_2 (\log_3 y)$

$2^x = \log_3 y$

$y = 3^{2^x}$

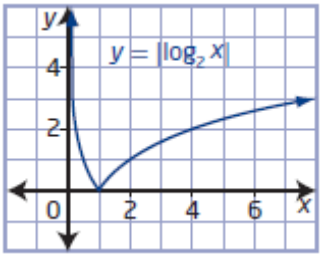
Section 8.1 Page 381 Question 24

If $m = \log_2 n$, then $2^m = n$.

$$\begin{aligned}2m + 1 &= \log_2 16n\\ 2m + 1 &= \log_2 16(2^m)\\ 2m + 1 &= \log_2 2^4(2^m)\\ 2m + 1 &= \log_2 2^{4+m}\\ 2m + 1 &= 4 + m\\ m &= 3\end{aligned}$$

Substitute $m = 3$ into $2^m = n$.
$2^m = n$
$2^3 = n$
$8 = n$

Section 8.1 Page 381 Question C1



The graph of $y = |\log_2 x|$ is the same as the graph of $y = \log_2 x$ for $x \geq 1$. The graph of $y = |\log_2 x|$ is the reflection in the x-axis of the graph of $y = \log_2 x$ for $0 < x < 1$.

Section 8.1 Page 381 Question C2

Answers may vary. Mind maps should include a graph showing how exponential functions and logarithmic functions are related, domain, range, intercept, and equation of the asymptote.

Section 8.1 Page 382 Question C3

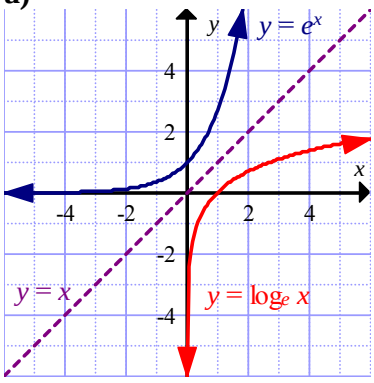
Step 1
a) $e = 2.718\ 281\ 828$

b) The minimum value of x needed to approximate e correctly to nine decimal places is 10^{10}.

x	$\left(1+\frac{1}{x}\right)^x$
10^1	2.593 742 460
10^2	2.704 813 829
10^3	2.716 923 932
10^4	2.718 145 927
10^5	2.718 268 237
10^6	2.718 280 469
10^7	2.718 281 693
10^8	2.718 281 815
10^9	2.718 281 827
10^{10}	2.718 281 828

Step 2

a)



For $y = \log_e x$,
- domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
- x-intercept: 1
- no y-intercept
- equation of the asymptote: $x = 0$

b) The inverse of $y = e^x$ is $y = \ln x$.

Step 3

a) Substitute $\theta = 2\pi$.

$$r = e^{0.14\theta}$$
$$= e^{0.14(2\pi)}$$
$$= 2.410\ldots$$

The distance, r, from point P to the origin after the point has rotated 2π is approximately 2.41.

b) i) $r = e^{0.14\theta}$

$$0.14\theta = \ln r$$
$$\theta = \frac{\ln r}{0.14}$$

The logarithmic form of $r = e^{0.14\theta}$ is $\theta = \frac{\ln r}{0.14}$.

ii) Substitute $r = 12$.

$$\theta = \frac{\ln r}{0.14}$$
$$= \frac{\ln 12}{0.14}$$
$$= 17.749\ldots$$

The angle, θ, of rotation that corresponds to a value for r of 12 is approximately 17.75.

Section 8.2 Transformations of Logarithmic Functions

Section 8.2 Page 389 Question 1

Compare each function to the form $y = a \log_5 (b(x - h)) + k$.

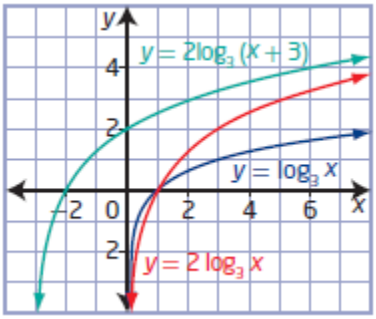
a) For $y = \log_5 (x - 1) + 6$, $h = 1$ and $k = 6$. This is a translation of 1 unit to the right and 6 units up of the graph of $y = \log_5 x$.

b) For $y = -4 \log_5 3x$, $a = -4$ and $b = 3$. This is a vertical stretch about the x-axis by a factor of 4, a reflection in the x-axis, and a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $y = \log_5 x$.

c) For $y = \frac{1}{2} \log_5 (-x) + 7$, $a = \frac{1}{2}$, $b = -1$, and $k = 7$. This is a vertical stretch about the x-axis by a factor of $\frac{1}{2}$, a reflection in the y-axis, and a translation of 7 units up of the graph of $y = \log_5 x$.

Section 8.2 Page 389 Question 2

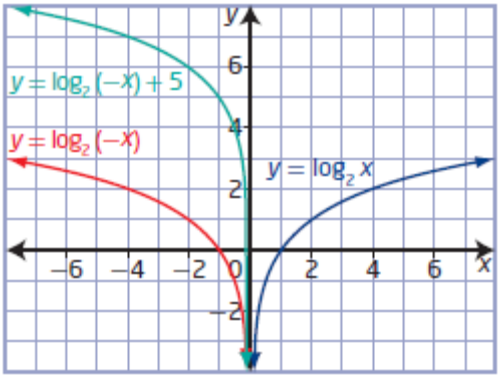
a) Given: $y = \log_3 x$
• Stretch vertically about the x-axis by a factor of 2: $a = 2$, $y = 2 \log_3 x$
• Translate 3 units to the left: $h = -3$, $y = 2 \log_3 (x + 3)$



b) $y = 2 \log_3 (x + 3)$

Section 8.2 Page 390 Question 3

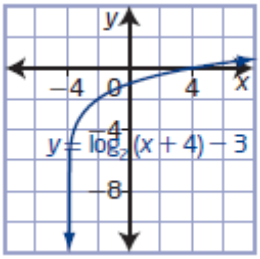
a) Given: $y = \log_2 x$
• Reflect in the y-axis: $b = -1$, $y = \log_2 (-x)$
• Translate vertically 5 units up: $k = 5$, $y = \log_2 (-x) + 5$



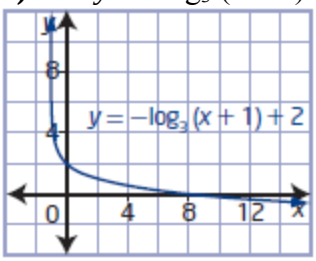
b) $y = \log_2 (-x) + 5$

Section 8.2 Page 390 Question 4

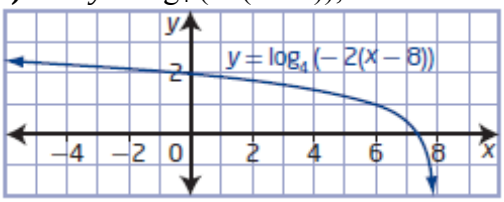
a) For $y = \log_2 (x + 4) - 3$, $h = -4$ and $k = -3$.



b) For $y = -\log_3 (x + 1) + 2$, $a = -1$, $h = -1$, and $k = 2$.



c) For $y = \log_4 (-2(x - 8))$, $b = -2$ and $h = 8$.



Section 8.2 Page 390 Question 5

a) $y = -5 \log_3 (x + 3)$

i) The equation of the vertical asymptote occurs when $x + 3 = 0$. Therefore, the equation of the vertical asymptote is $x = -3$.

ii) The domain is $\{x \mid x > -3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

iii) Substitute $x = 0$. Then, solve for y.

$y = -5 \log_3 (x + 3)$
$= -5 \log_3 (0 + 3)$
$= -5 \log_3 3$
$= -5$

The y-intercept is -5.

iv) Substitute $y = 0$. Then, solve for x.

$y = -5 \log_3 (x + 3)$
$0 = -5 \log_3 (x + 3)$
$0 = \log_3 (x + 3)$

$3^0 = x + 3$
$1 = x + 3$
$x = -2$
The x-intercept is -2.

b) $y = \log_6 (4(x + 9))$
i) The equation of the vertical asymptote occurs when $4(x + 9) = 0$. Therefore, the equation of the vertical asymptote is $x = -9$.

ii) The domain is $\{x \mid x > -9, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

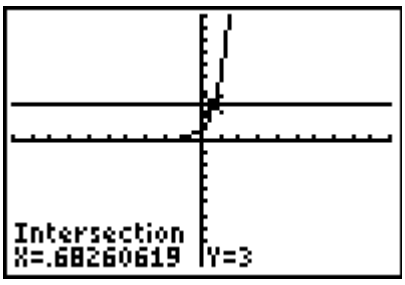
iii) Substitute $x = 0$. Then, solve for y.
$y = \log_6 (4(x + 9))$
$= \log_6 (4(0 + 9))$
$= \log_6 36$
$= 2$
The y-intercept is 2.

iv) Substitute $y = 0$. Then, solve for x.
$y = \log_6 (4(x + 9))$
$0 = \log_6 (4(x + 9))$
$6^0 = 4(x + 9)$
$1 = 4x + 36$
$4x = -35$
$x = -\frac{35}{4}$
The x-intercept is $-\frac{35}{4}$, or -8.75.

c) $y = \log_5 (x + 3) - 2$
i) The equation of the vertical asymptote occurs when $x + 3 = 0$. Therefore, the equation of the vertical asymptote is $x = -3$.

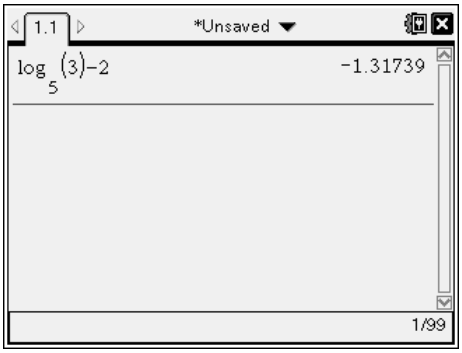
ii) The domain is $\{x \mid x > -3, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

iii) Substitute $x = 0$. Then, solve for y.
$y = \log_5 (x + 3) - 2$
$= \log_5 (0 + 3) - 2$
$= \log_5 3 - 2$
To obtain an approximate value for $\log_5 3$, graph $y = 5^x$ and $y = 3$ and find the point of intersection.
$\log_5 3 \approx 0.68$



$y \approx 0.68 - 2$
≈ -1.3
The y-intercept is about -1.3.

A possible alternative is to use a calculator that can evaluate a logarithm of any base.



$y = \log_5 3 - 2$
≈ -1.3
The y-intercept is about -1.3.

iv) Substitute $y = 0$. Then, solve for x.
$y = \log_5 (x + 3) - 2$
$0 = \log_5 (x + 3) - 2$
$2 = \log_5 (x + 3)$
$5^2 = x + 3$
$25 = x + 3$
$x = 22$
The x-intercept is 22.

d) $y = -3 \log_2 (x + 1) - 6$
i) The equation of the vertical asymptote occurs when $x + 1 = 0$. Therefore, the equation of the vertical asymptote is $x = -1$.

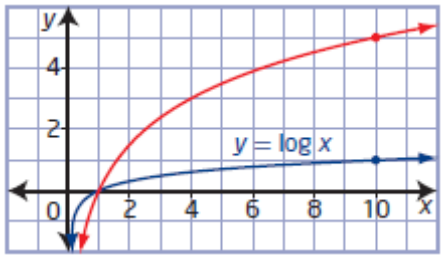
ii) The domain is $\{x \mid x > -1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

iii) Substitute $x = 0$. Then, solve for y.
$y = -3 \log_2 (x + 1) - 6$
$= -3 \log_2 (0 + 1) - 6$
$= -3 \log_2 1 - 6$
$= -3(0) - 6$
$= -6$
The y-intercept is -6.

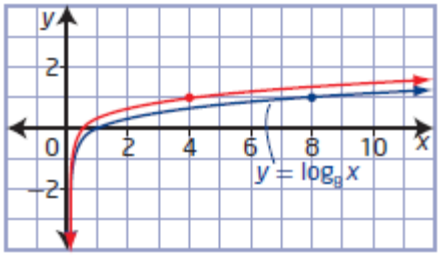
iv) Substitute $y = 0$. Then, solve for x.
$y = -3 \log_2 (x + 1) - 6$
$0 = -3 \log_2 (x + 1) - 6$
$6 = -3 \log_2 (x + 1)$
$-2 = \log_2 (x + 1)$
$2^{-2} = x + 1$
$\frac{1}{4} = x + 1$
$x = -\frac{3}{4}$
The x-intercept is $-\frac{3}{4}$, or -0.75.

Section 8.2 Page 390 Question 6

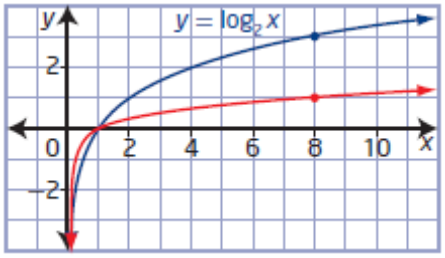
a) The key point (10, 1) on the graph of $y = \log x$ has become the image point (10, 5) on the red graph. Thus, the red graph can be generated by vertically stretching the graph of $y = \log x$ about the x-axis by a factor of 5. The red graph can be described by the equation $y = 5 \log x$.



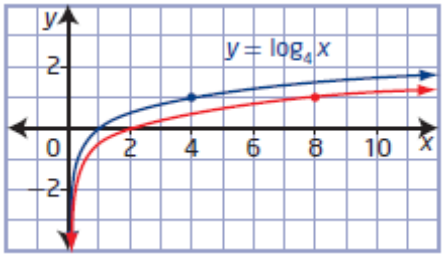
b) The key point (8, 1) on the graph of $y = \log_8 x$ has become the image point (4, 1) on the red graph. Thus, the red graph can be generated by horizontally stretching the graph of $y = \log_8 x$ about the y-axis by a factor of $\frac{1}{2}$. The red graph can be described by the equation $y = \log_8 2x$.



c) The key point (8, 3) on the graph of $y = \log_2 x$ has become the image point (8, 1) on the red graph. Thus, the red graph can be generated by vertically stretching the graph of $y = \log_2 x$ about the x-axis by a factor of $\frac{1}{3}$. The red graph can be described by the equation $y = \frac{1}{3} \log_2 x$.



d) The key point (4, 1) on the graph of $y = \log_4 x$ has become the image point (8, 1) on the red graph. Thus, the red graph can be generated by horizontally stretching the graph of $y = \log_4 x$ about the y-axis by a factor of 2. The red graph can be described by the equation $y = \log_4 \left(\frac{1}{2}x\right)$.



Section 8.2 Page 390 Question 7

a) For $y = \log_7 (4(x + 5)) + 6$, $b = 4$, $h = -5$, and $k = 6$. To obtain the graph of $y = \log_7 (4(x + 5)) + 6$, the graph of $y = \log_7 x$ must be horizontally stretched about the y-axis by a factor of $\frac{1}{4}$ and translated 5 units to the left and 6 units up.

b) For $y = 2\log_7\left(-\frac{1}{3}(x-1)\right)-4$, $a = 2$, $b = -\frac{1}{3}$, $h = 1$, and $k = -4$. To obtain the graph of $y = 2\log_7\left(-\frac{1}{3}(x-1)\right)-4$, the graph of $y = \log_7 x$ must be horizontally stretched about the y-axis by a factor of 3, reflected in the y-axis, vertically stretched about the x-axis by a factor of 2, and translated 1 unit to the right and 4 units down.

Section 8.2 Page 390 Question 8

a) For a reflection in the x-axis and a translation of 6 units left and 3 units up, $a = -1$, $h = -6$, and $k = 3$. The equation of the transformed function is $y = -\log_3 (x + 6) + 3$.

b) For a vertical stretch by a factor of 5 about the x-axis and a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$, $a = 5$, $b = 3$, $h = 0$, and $k = 0$. The equation of the transformed function is $y = 5 \log_3 3x$.

c) For a vertical stretch about the x-axis by a factor of $\frac{3}{4}$, a horizontal stretch about the y-axis by a factor of 4, a reflection in the y-axis, and a translation of 2 units right and 5 units down, $a = \frac{3}{4}$, $b = -\frac{1}{4}$, $h = 2$, and $k = -5$. The equation of the transformed function is $y = \frac{3}{4}\log_3\left(-\frac{1}{4}(x-2\right)-5$.

Section 8.2 Page 390 Question 9

a) $y = 5 \log_3 (-4x + 12) - 2$
$y = 5 \log_3 (-4(x - 3)) - 2$
For $y = 5 \log_3 (-4(x - 3)) - 2$, $a = 5$, $b = -4$, $h = 3$, and $k = -2$. To obtain the graph of $y = 5 \log_3 (-4(x - 3)) - 2$, the graph of $y = \log_3 x$ must be horizontally stretched about the y-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, vertically stretched about the x-axis by a factor of 5, and translated 3 units to the right and 2 units down.

b) $y = -\frac{1}{4}\log_3 (6 - x) + 1$
$y = -\frac{1}{4}\log_3 (-(x - 6)) + 1$
For $y = -\frac{1}{4}\log_3 (-(x - 6)) + 1$, $a = -\frac{1}{4}$, $b = -1$, $h = 6$, and $k = 1$. To obtain the graph of $y = -\frac{1}{4}\log_3 (-(x - 6)) + 1$, the graph of $y = \log_3 x$ must be reflected in the y-axis,

vertically stretched about the x-axis by a factor of $\frac{1}{4}$, reflected in the x-axis, and translated 6 units to the right and 1 unit up.

Section 8.2 Page 390 Question 10

a) For a vertical translation, compare the point on the graph of $y = \log_3 x$ with the same x-coordinate as the given point on the transformed function, (9, –4).
(9, 2) → (9, –4)
So, $k = -6$ and the equation of the transformed image is $\log_3 x - 6$.

b) For a horizontal stretch, compare the point on the graph of $y = \log_2 x$ with the same y-coordinate as the given point on the transformed function, (8, 1).
(2, 1) → (8, 1)
So, $b = \frac{1}{4}$ and the equation of the transformed image is $\log_2 \left(\frac{1}{4}x\right)$.

Section 8.2 Page 391 Question 11

$$\frac{1}{3}(y + 2) = \log_6 (x - 4)$$
$$y + 2 = 3 \log_6 (x - 4)$$
$$y = 3 \log_6 (x - 4) - 2$$

For $y = 3 \log_6 (x - 4) - 2$, $a = 3$, $h = 4$, and $k = -2$. To obtain the graph of $y = 3 \log_6 (x - 4) - 2$, the graph of $y = \log_6 x$ must be vertically stretched about the x-axis by a factor of 3 and translated 4 units to the right and 2 units down.

Section 8.2 Page 391 Question 12

a) For $R = 0.67 \log 0.36E + 1.46$, $a = 0.67$, $b = 0.36$, and $k = 1.46$. The function is transformed from $R = \log E$ by a horizontal stretch about the y-axis by a factor of $\frac{1}{0.36}$ or $\frac{25}{9}$, vertically stretched about the x-axis by a factor of 0.67, and translated 1.46 units up.

b) Substitute $R = 7.0$.
$$R = 0.67 \log 0.36E + 1.46$$
$$7.0 = 0.67 \log 0.36E + 1.46$$
$$5.54 = 0.67 \log 0.36E$$
$$\frac{5.54}{0.67} = \log 0.36E$$

$$10^{\frac{5.54}{0.67}} = 0.36E$$

$$E = \frac{10^{\frac{5.54}{0.67}}}{0.36}$$

$$E = 515\,649\,042.5$$

The equivalent amount of energy released, to the nearest kilowatt-hour, is 515 649 043 kWh.

Section 8.2 Page 391 Question 13

a) Substitute $P = 110$.

$V = 0.23 + 0.35 \log (P - 56.1)$
$= 0.23 + 0.35 \log (110 - 56.1)$
$= 0.23 + 0.35 \log 53.9$
$= 0.836\ldots$

To the nearest tenth of a microlitre, the vessel volume is 0.8 µL.

b) Substitute $V = 0.7$.

$V = 0.23 + 0.35 \log (P - 56.1)$
$0.7 = 0.23 + 0.35 \log (P - 56.1)$
$0.47 = 0.35 \log (P - 56.1)$

$$\frac{0.47}{0.35} = \log (P - 56.1)$$

$$10^{\frac{0.47}{0.35}} = P - 56.1$$

$$P = 10^{\frac{0.47}{0.35}} + 56.1$$

$$P = 78.122...$$

To the nearest millimetre of mercury, the arterial blood pressure is 78 mmHg.

Section 8.2 Page 391 Question 14

a) Substitute $m = 60$.

$\log m = 0.008h + 0.4$
$\log 60 = 0.008h + 0.4$
$\log 60 - 0.4 = 0.008h$

$$h = \frac{\log 60 - 0.4}{0.008}$$

$h = 172.268\ldots$

The height of the child, to the nearest centimetre, is 172 cm.

b) Substitute $h = 150$.

$\log m = 0.008h + 0.4$
$\log m = 0.008(150) + 0.4$
$\log m = 1.6$
$m = 10^{1.6}$
$m = 39.810\ldots$

The mass of the child, to the nearest kilogram, is 40 kg.

Section 8.2 Page 391 Question 15

For example, the point (8, 1) is on the graph of $f(x) = \log_8 a$.
Determine the value of a such that the point (8, 1) is on the graph of $g(x) = a \log_2 x$.
$a \log_2 8 = 1$

$$\log_2 8 = \frac{1}{a}$$

$$2^{\frac{1}{a}} = 8$$

$$2^{\frac{1}{a}} = 2^3$$

$$\frac{1}{a} = 3$$

$$a = \frac{1}{3}$$

Section 8.2 Page 391 Question 16

a) The graph of $y = 2 \log_5 x - 7$ is reflected in the x-axis and translated 6 units up.
For a base function being transformed: $(x, y) \rightarrow (x, -y + 6)$
Using the given function as the base function:
$(x, 2y - 7) \rightarrow (x, -(2y - 7) + 6)$
$\rightarrow (x, -2y + 13)$
The equation of the transformed image is $y = -2 \log_5 x + 13$.

b) The graph of $y = \log (6(x - 3))$ is stretched horizontally about the y-axis by a factor of 3 and translated 9 units left.
For a base function being transformed: $(x, y) \rightarrow (3x - 9, y)$
Using the given function as the base function:

$$\left(\frac{x}{6} + 3, y\right) \rightarrow \left(3\left(\frac{x}{6} + 3\right) - 9, y\right)$$

$$\rightarrow \left(\frac{x}{2}, y\right)$$

The equation of the transformed image is $y = \log 2x$.

Section 8.2 Page 391 Question 17

The graph of $f(x) = \log_2 x$ has been transformed to $g(x) = a \log_2 x + k$: $(x, y) \rightarrow (x, ay + k)$.
Given points on the transformed image: $\left(\frac{1}{4}, -9\right)$ and $(16, -6)$.
Use the mapping to create a system of equations.
$\left(\frac{1}{4}, -2\right) \rightarrow \left(\frac{1}{4}, -9\right)$: $-2a + k = -9$ ①
$(16, 4) \rightarrow (16, -6)$: $4a + k = -6$ ②

$2 \times ①$: $-4a + 2k = -18$
$+ ②$: $\underline{4a + k = -6}$
$3k = -24$
$k = -8$
Substitute $k = -8$ into ①.
$-2a + k = -9$
$-2a + -8 = -9$
$-2a = -1$
$a = \frac{1}{2}$

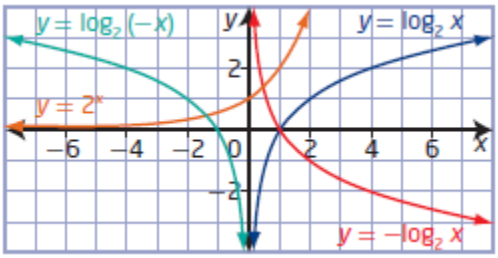
Section 8.2 Page 391 Question C1

The graph of $f(x) = 5^x$ is
• reflected in the line $y = x$: $g(x) = \log_5 x$
• vertically stretched about the x-axis by a factor of $\frac{1}{4}$: $a = \frac{1}{4}$
• horizontally stretched about the y-axis by a factor of 3: $b = \frac{1}{3}$
• translated 4 units right and 1 unit down: $h = 4$ and $k = -1$
The equation of transformed image is $g(x) = \frac{1}{4} \log_5 \frac{1}{3}(x - 4) - 1$.

Section 8.2 Page 391 Question C2

a) For $f(x) = \log_2 x$,
$y = -f(x) = -\log_2 x$
$y = f(-x) = \log_2 (-x)$
$y = f^{-1}(x) = 2^x$

b) The graph of $y = -\log_2 x$ is a reflection in the x-axis of the graph of $f(x) = \log_2 x$.
The graph of $y = \log_2 (-x)$ is a reflection in the y-axis of the graph of $f(x) = \log_2 x$.
The graph of $y = 2^x$ is a reflection in the line $y = x$ of the graph of $f(x) = \log_2 x$.



Section 8.2 Page 391 Question C3

a) The graph of $y = 3(7^{2x-1}) + 5$ is reflected in the line $y = x$. Determine the equation of the inverse of the function.

$$y = 3(7^{2x-1}) + 5$$
$$x = 3(7^{2y-1}) + 5$$
$$x - 5 = 3(7^{2y-1})$$
$$\frac{1}{3}(x-5) = 7^{2y-1}$$
$$2y - 1 = \log_7 \frac{1}{3}(x-5)$$
$$2y = \log_7 \frac{1}{3}(x-5) + 1$$
$$y = \frac{1}{2}\log_7 \frac{1}{3}(x-5) + \frac{1}{2}$$

b) Given $f(x) = 2\log3\,(x-1) + 8$, find $f^{-1}(x)$.

$f(x) = 2\log3\,(x-1) + 8$

$$y = 2\log3\,(x-1) + 8$$
$$x = 2\log3\,(y-1) + 8$$
$$x - 8 = 2\log3\,(y-1)$$
$$\frac{1}{2}(x-8) = \log3\,(y-1)$$
$$y - 1 = 3^{\frac{1}{2}(x-8)}$$
$$y = 3^{\frac{1}{2}(x-8)} + 1$$

Section 8.3 Laws of Logarithms

Section 8.3 Page 400 Question 1

a) $\log_7 xy^3\sqrt{z} = \log_7 x + \log_7 y^3 + \log_7 \sqrt{z}$
$= \log_7 x + 3\log_7 y + \frac{1}{2}\log_7 z$

b) $\log_5 (xyz)^8 = 8\log_5 xyz$
$= 8(\log_5 x + \log_5 y + \log_5 z)$

c) $\log\left(\frac{x^2}{y\sqrt[3]{z}}\right) = \log x^2 - \log y\sqrt[3]{z}$
$= 2\log x - (\log y + \log \sqrt[3]{z})$
$= 2\log x - \log y - \frac{1}{3}\log z$

d) $\log_3 x\sqrt{\frac{y}{z}} = \log_3 x + \log_3 \sqrt{\frac{y}{z}}$
$= \log_3 x + \frac{1}{2}\log_3 \frac{y}{z}$
$= \log_3 x + \frac{1}{2}(\log_3 y - \log_3 z)$

Section 8.3 Page 400 Question 2

a) $\log_{12} 24 - \log_{12} 6 + \log_{12} 36$

$= \log_{12} \frac{24(36)}{6}$

$= \log_{12} 144$

$= 2$

b) $3\log_5 10 - \frac{1}{2}\log_5 64$

$= \log_5 10^3 - \log_5 \sqrt{64}$

$= \log_5 \frac{1000}{8}$

$= \log_5 125$

$= 3$

c) $\log_3 27\sqrt{3} = \log_3 27 + \log_3 \sqrt{3}$

$= \log_3 27 + \frac{1}{2}\log_3 3$

$= 3 + \frac{1}{2}(1)$

$= \frac{7}{2}$

d) $\log_2 72 - \frac{1}{2}(\log_2 3 + \log_2 27)$

$= \log_2 72 - \log_2 \sqrt{81}$

$= \log_2 72 - \log_2 9$

$= \log_2 \frac{72}{9}$

$= \log_2 8$

$= 3$

Section 8.3 Page 400 Question 3

a) $\log_9 x - \log_9 y + 4\log_9 z$

$= \log_9 \frac{x}{y} + \log_9 z^4$

$= \log_9 \frac{xz^4}{y}$

b) $\frac{\log_3 x}{2} - 2\log_3 y$

$= \log_3 \sqrt{x} - \log_3 y^2$

$= \log_3 \frac{\sqrt{x}}{y^2}$

c) $\log_6 x - \frac{1}{5}(\log_6 x + 2\log_6 y)$

$= \log_6 x - \frac{1}{5}\log_6 xy^2$

$= \log_6 x - \log_6 \sqrt[5]{xy^2}$

$= \log_6 \frac{x}{\sqrt[5]{xy^2}}$

d) $\frac{\log x}{3} + \frac{\log y}{3}$

$= \frac{1}{3}(\log x + \log y)$

$= \frac{1}{3}\log xy$

$= \log \sqrt[3]{xy}$

Section 8.3 Page 400 Question 4

Given: log 1.44 ≈ 0.158 36, log 1.2 ≈ 0.079 18, and log 1.728 ≈ 0.237 54.
a) For 1.44 × 1.2,
log (1.44 × 1.2) = log 1.44 + log 1.2
= 0.158 36 + 0.079 18
= 0.237 54
= log 1.728
So, 1.44 × 1.2 = 1.728.

b) For 1.728 ÷ 1.2,
log (1.728 ÷ 1.2) = log 1.728 – log 1.2
= 0.237 54 – 0.079 18
= 0.158 36
= log 1.44
So, 1.728 ÷ 1.2 = 1.44.

b) For $\sqrt{1.44}$,
log $\sqrt{1.44}$ = 0.5 log 1.44
= 0.5(0.158 36)
= 0.079 18
= log 1.2
So, $\sqrt{1.44}$ = 1.2.

Section 8.3 Page 400 Question 5

a) Given: $k = \log_2 40 - \log_2 5$

$$3^k = 3^{\log_2 40 - \log_2 5}$$
$$= 3^{\log_2 \frac{40}{5}}$$
$$= 3^{\log_2 8}$$
$$= 3^3$$
$$= 27$$

b) Given: $n = 3 \log_8 4$

$$7^n = 7^{3\log_8 4}$$
$$= 7^{\log_8 4^3}$$
$$= 7^{\log_8 64}$$
$$= 7^2$$
$$= 49$$

Section 8.3 Page 400 Question 6

a) You need to apply a horizontal stretch about the y-axis by a factor of $\frac{1}{8}$ to the graph of $y = \log_2 x$ to result in the graph of $y = \log_2 8x$.

b) Using the product law of logarithms, the function $y = \log_2 8x$ can be written as $y = \log_2 8 + \log_2 x$, or $y = \log_2 x + 3$.
You need to apply a translation of 3 units up to the graph of $y = \log_2 x$ to result in the graph of $y = \log_2 8x$.

Section 8.3 Page 401 Question 7

a) The equation $\frac{\log_c x}{\log_c y} = \log_c x - \log_c y$ is false, as $\log_c \frac{x}{y} = \log_c x - \log_c y$.

b) The equation $\log_c (x + y) = \log_c x + \log_c y$ is false, as $\log_c xy = \log_c x + \log_c y$.

c) The equation $\log_c c^n = n$ is true, since

$$\begin{aligned}\log_c c^n &= n \log_c c \\ &= n(1) \\ &= n\end{aligned}$$

d) The equation $(\log_c x)^n = n \log_c x$ is false, as $\log_c x^n = n \log_c x$.

e) The equation $-\log_c \left(\frac{1}{x}\right) = \log_c x$ is true, since

$$\begin{aligned}-\log_c \left(\frac{1}{x}\right) &= \log_c \left(\frac{1}{x}\right)^{-1} \\ &= \log_c x\end{aligned}$$

Section 8.3 Page 401 Question 8

Given: $\log 3 = P$ and $\log 5 = Q$

a) $\log \frac{3}{5} = \log 3 - \log 5$
$= P - Q$

b) $\log 15 = \log 3(5)$
$= \log 3 + \log 5$
$= P + Q$

c) $\log 3\sqrt{5} = \log 3 + \log \sqrt{5}$
$= \log 3 + \frac{1}{2} \log 5$
$= P + \frac{1}{2} Q$

d) $\log \frac{25}{9} = \log \left(\frac{5}{3}\right)^2$
$= 2 \log \frac{5}{3}$
$= 2(\log 5 - \log 3)$
$= 2(Q - P)$

Section 8.3 Page 401 Question 9

Given: $\log_2 7 = K$

a) $\log_2 7^6 = 6 \log_2 7$
$= 6K$

b) $\log_2 14 = \log_2 7(2)$
$= \log_2 7 + \log_2 2$
$= K + 1$

c) $\log_2(49\times 4) = \log_2 49 + \log_2 4$
$= \log_2 7^2 + \log_2 4$
$= 2\log_2 7 + \log_2 4$
$= 2K + 2$

d) $\log_2 \frac{\sqrt[5]{7}}{8} = \log_2 \sqrt[5]{7} - \log_2 8$
$= \frac{1}{5}\log_2 7 - \log_2 8$
$= \frac{1}{5}K - 3$

Section 8.3 Page 401 Question 10

a) $\log_5 x + \log_5 \sqrt{x^3} - 2\log_5 x = \log_5 x\sqrt{x^3} - \log_5 x^2$
$= \log_5 \frac{x\sqrt{x^3}}{x^2}$
$= \log_5 \frac{x^2\sqrt{x}}{x^2}$
$= \log_5 \sqrt{x}$
$= \frac{1}{2}\log_5 x,\ x > 0$

b) $\log_{11} \frac{x}{\sqrt{x}} + \log_{11} \sqrt{x^5} - \frac{7}{3}\log_{11} x = \log_{11} \frac{x\sqrt{x^5}}{\sqrt{x}} - \log_{11} x^{\frac{7}{3}}$
$= \log_{11} \frac{x^3\sqrt{x}}{\sqrt{x}} - \log_{11} x^2\sqrt[3]{x}$
$= \log_{11} \frac{x^3}{x^2\sqrt[3]{x}}$
$= \log_{11} x^{\frac{2}{3}}$
$= \frac{2}{3}\log_{11} x,\ x > 0$

Section 8.3 Page 401 Question 11

a) $\log_2(x^2 - 25) - \log_2(3x - 15) = \log_2 \frac{x^2 - 25}{3x - 15}$
$= \log_2 \frac{(x-5)(x+5)}{3(x-5)}$
$= \log_2 \frac{x+5}{3},\ x < -5 \text{ or } x > 5$

b) $\log_7(x^2-16)-\log_7(x^2-2x-8)=\log_7\dfrac{x^2-16}{x^2-2x-8}$

$=\log_7\dfrac{(x-4)(x+4)}{(x-4)(x+2)}$

$=\log_7\dfrac{x+4}{x+2}, x<-4 \text{ or } x>4$

c) $2\log_8(x+3)-\log_8(x^2+x-6)=\log_8\dfrac{x+3}{x^2+x-6}$

$=\log_8\dfrac{x+3}{(x+3)(x-2)}$

$=\log_8\dfrac{1}{x-2}, x>2$

Section 8.3 Page 401 Question 12

a) Left Side: $\log_c 48-(\log_c 3+\log_c 2)$ Right Side: $\log_c 8$

$=\log_c 48-\log_c 3(2)$

$=\log_c\dfrac{48}{6}$

$=\log_c 8$

Left Side = Right Side

b) Left Side: $7\log_c 4$ Right Side: $14\log_c 2$

$=\log_c 4^7$

$=\log_c (2^2)^7$

$=\log_c 2^{14}$

$=14\log_c 2$

Left Side = Right Side

c) Left Side: $\dfrac{1}{2}(\log_c 2+\log_c 6)$ Right Side: $\log_c 2+\log_c\sqrt{3}$

$=\dfrac{1}{2}\log_c 12$

$=\log_c\sqrt{12}$

$=\log_c 2\sqrt{3}$

$=\log_c 2+\log_c\sqrt{3}$

Left Side = Right Side

d) Left Side

$\log_c (5c)^2$

$= 2 \log_c 5c$

$= 2(\log_c 5 + \log_c c)$

$= 2(\log_c 5 + 1)$

Right Side

$2(\log_c 5 + 1)$

Left Side = Right Side

Section 8.3 Page 401 Question 13

a) Substitute $I = 0.000\ 01$ and $I_0 = 10^{-12}$.

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

$$= 10 \log\left(\frac{0.000\ 01}{10^{-12}}\right)$$

$$= 10 \log\left(\frac{10^{-5}}{10^{-12}}\right)$$

$$= 10 \log 10^7$$

$$= 10(7)$$

$$= 70$$

The decibel level of the a hairdryer is 70 dB.

b) Let the decibel levels of two sounds be $\beta_1 = 10 \log \frac{I_1}{I_0}$ and $\beta_2 = 10 \log \frac{I_2}{I_0}$.

From Example 4 on page 398, comparing the two intensities results in the equation $\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$. Substitute $\beta_2 = 118$ and $\beta_1 = 85$.

$$\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$$

$$118 - 85 = 10\left(\log \frac{I_2}{I_1}\right)$$

$$33 = 10\left(\log \frac{I_2}{I_1}\right)$$

$$3.3 = \log \frac{I_2}{I_1}$$

$$10^{3.3} = \frac{I_2}{I_1}$$

$$1995.262\ldots = \frac{I_2}{I_1}$$

The fire truck siren is approximately 1995 times as loud as city traffic.

c) Substitute $\frac{I_2}{I_1} = 63$ and $\beta_1 = 80$ into $\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$.

$$\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$$

$$\beta_2 - 80 = 10 \log 63$$

$$\beta_2 = 10 \log 63 + 80$$

$$\beta_2 = 97.993\ldots$$

The decibel level of the farm tractor is approximately 98 dB.

Section 8.3 Page 401 Question 14

The decibel scale is logarithmic, not linear. So, a 20 dB sound is actually 10^1 times as loud as a 10 dB sound.

Section 8.3 Page 401 Question 15

Substitute $G = 24$ and $V_i = 0.2$.

$$G = 20 \log \frac{V}{V_i}$$

$$24 = 20 \log \frac{V}{0.2}$$

$$1.2 = \log \frac{V}{0.2}$$

$$10^{1.2} = \frac{V}{0.2}$$

$$V = 0.2(10^{1.2})$$

$$V = 3.169...$$

The voltage is 3.2 V, to the nearest tenth of a volt.

Section 8.3 Page 402 Question 16

a) Substitute pH = 7.0.

$$\text{pH} = -\log [\text{H}+]$$

$$7.0 = -\log [\text{H}+]$$

$$-7.0 = \log [\text{H}+]$$

$$[\text{H}+] = 10^{-7.0}$$

The hydrogen ion concentration is 10^{-7} mol/L.

b) Let the pH levels of two rains be $\text{pH}_1 = -\log [\text{H}_1+]$ and $\text{pH}_2 = -\log [\text{H}_2+]$.
Compare the two pH levels.

$$\text{pH}_2 - \text{pH}_1 = -\log [\text{H}_2+] - (-\log [\text{H}_1+])$$

$$\text{pH}_2 - \text{pH}_1 = \log [\text{H}_2+]^{-1} - \log [\text{H}_1+]^{-1}$$

$$pH_2 - pH_1 = \log \frac{[H_1+]}{[H_2+]}$$

Substitute $pH_2 = 5.6$ and $pH_1 = 4.5$.

$$5.6 - 4.5 = \log \frac{[H_1+]}{[H_2+]}$$

$$1.1 = \log \frac{[H_1+]}{[H_2+]}$$

$$10^{1.1} = \frac{[H_1+]}{[H_2+]}$$

$$12.589\ldots = \frac{[H_1+]}{[H_2+]}$$

Acid rain is approximately 12.6 times more acidic than normal rain.

c) Substitute $\frac{[H_1+]}{[H_2+]} = 500$ and $pH_2 = 6.1$ into $pH_2 - pH_1 = \log \frac{[H_1+]}{[H_2+]}$.

$$pH_2 - pH_1 = \log \frac{[H_1+]}{[H_2+]}$$

$$6.1 - pH_1 = \log 500$$

$$-pH_1 = \log 500 - 6.1$$

$$pH_1 = -\log 500 + 6.1$$

$$pH_1 = 3.401\ldots$$

The pH of the hair conditioner is approximately 3.4.

Section 8.3 Page 402 Question 17

Given: $\Delta v = \frac{3.1}{0.434}(\log m_0 - \log m_f)$ and $\frac{m_0}{m_f} = 1.06$

$$\Delta v = \frac{3.1}{0.434}(\log m_0 - \log m_f)$$

$$= \frac{3.1}{0.434}\log \frac{m_0}{m_f}$$

$$= \frac{3.1}{0.434}\log 1.06$$

$$0.180...$$

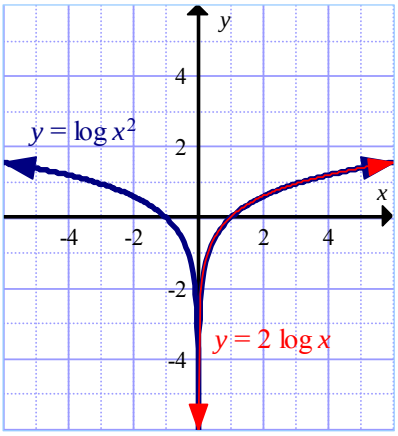
The change in velocity of the rocket is 0.18 km/s, to the nearest hundredth of a kilometre per second.

Section 8.3 Page 402 Question 18

a) The graphs are the same for $x > 0$. However, the graph of $y = \log x^2$ has a second branch for $x < 0$, which is the reflection in the y-axis of the branch for $x > 0$.

b) The graphs are not identical because the domains are different. For $y = \log x^2$, the domain is $\{x \mid x \neq 0, x \in \mathrm{R}\}$. The domain for $y = 2 \log x$ is $\{x \mid x > 0, x \in \mathrm{R}\}$.

c) For $\log x^2 = 2 \log x$, the restriction $x > 0$ is required.



Section 8.3 Page 402 Question 19

a)

$$y = \log_c x$$
$$c^y = x$$
$$\log_d c^y = \log_d x$$
$$y \log_d c = \log_d x$$
$$y = \frac{\log_d x}{\log_d c}$$

b) Use the change in base formula: $\log_c x = \dfrac{\log x}{\log c}$.

$$\log_2 9.5 = \frac{\log 9.5}{\log 2}$$
$$\approx 3.2479$$

c)

$$\varphi = -\log_2 D$$
$$= -\frac{\log D}{\log 2}$$

d)

Use the formula $\varphi = -\log_2 D$.
Let the φ-values be $\varphi_1 = -\log_2 D_1$ and $\varphi_2 = -\log_2 D_2$.
Compare the two values.

$$\varphi_2 - \varphi_1 = -\log_2 D_2 - (-\log_2 D_1)$$
$$\varphi_2 - \varphi_1 = \log_2 D_2^{-1} - \log_2 D_1^{-1}$$
$$\varphi_2 - \varphi_1 = \log_2 \frac{D_1}{D_2}$$

Substitute $\varphi_2 = 2$ and $\varphi_1 = -5.7$.

$$2 - (-5.7) = \log_2 \frac{D_1}{D_2}$$
$$7.7 = \log_2 \frac{D_1}{D_2}$$
$$2^{7.7} = \frac{D_1}{D_2}$$
$$207.936\ldots = \frac{D_1}{D_2}$$

Use the formula $\varphi = -\dfrac{\log D}{\log 2}$.

Let the φ-values be $\varphi_1 = -\dfrac{\log D_1}{\log 2}$ and $\varphi_2 = -\dfrac{\log D_2}{\log 2}$.

Compare the two values.

$$\varphi_2 - \varphi_1 = -\frac{\log D_2}{\log 2} - \left(-\frac{\log D_1}{\log 2}\right)$$
$$\varphi_2 - \varphi_1 = -\frac{1}{\log 2}(\log D_2 - \log D_1)$$
$$\varphi_2 - \varphi_1 = -\frac{1}{\log 2} \log \frac{D_2}{D_1}$$

Substitute $\varphi_2 = -5.7$ and $\varphi_1 = 2$.

$$-5.7 - 2 = -\frac{1}{\log 2} \log \frac{D_2}{D_1}$$

	$7.7 \log 2 = \log \frac{D_2}{D_1}$ $10^{7.7 \log 2} = \frac{D_2}{D_1}$ $207.936\ldots = \frac{D_2}{D_1}$

Using either form of the formula, the diameter of the pebble is approximately 207.9 times that of the medium sand.

Section 8.3 Page 402 Question 20

a) Left Side

$= \log_{q^3} p^3$

$= \frac{\log_q p^3}{\log_q q^3}$

$= \frac{3\log_q p}{3\log_q q}$

$= \frac{\log_q p}{1}$

= Right Side

b) Left Side

$= \frac{1}{\log_p 2} - \frac{1}{\log_q 2}$

$= \frac{1}{\frac{\log_2 2}{\log_2 p}} - \frac{1}{\frac{\log_2 2}{\log_2 q}}$

$= \frac{\log_2 p}{1} - \frac{\log_2 q}{1}$

$= \log_2 \frac{p}{q}$

= Right Side

c) Left Side

$= \frac{1}{\log_q p} + \frac{1}{\log_q p}$

$= \frac{1}{\frac{\log_{q^2} p}{\log_{q^2} q}} + \frac{1}{\frac{\log_{q^2} p}{\log_{q^2} q}}$

$= \frac{\log_{q^2} q}{\log_{q^2} p} + \frac{\log_{q^2} q}{\log_{q^2} p}$

$= \frac{1}{\log_{q^2} p}(\log_{q^2} q + \log_{q^2} q)$

$= \frac{1}{\log_{q^2} p} \log_{q^2} q^2$

$= \frac{1}{\log_{q^2} p}$

= Right Side

d) Left Side

$= \log_{\frac{1}{q}} p$

$= \frac{\log_q p}{\log_q \frac{1}{q}}$

$= \frac{\log_q p}{\log_q 1 - \log_q q}$

$= \frac{\log_q p}{0 - 1}$

$= -\log_q p$

$= \log_q p^{-1}$

$= \log_q \frac{1}{p}$

= Right Side

Section 8.3 Page 403 Question C1

a) The function $y = \log x^3$ can be written as $y = 3 \log x$. You need to apply a vertical stretch about the x-axis by a factor of 3 to the graph of $y = \log x$ to result in the graph of $y = \log x^3$.

b) The function $y = \log (x + 2)^5$ can be written as $y = 5 \log (x + 2)$. You need to apply a vertical stretch about the x-axis by a factor of 5 and a translation of 2 units to the left to the graph of $y = \log x$ to result in the graph of $y = \log (x + 2)^5$.

c) The function $y = \log \frac{1}{x}$ can be written as $y = -\log x$. You need to apply a reflection in the x-axis to the graph of $y = \log x$ to result in the graph of $y = \log \frac{1}{x}$.

d) The function $y = \log \frac{1}{\sqrt{x-6}}$ can be written as $y = -\frac{1}{2} \log (x - 6)$. You need to apply a vertical stretch about the x-axis by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a translation of 6 units to the right to the graph of $y = \log x$ to result in the graph of $y = \log \frac{1}{\sqrt{x-6}}$.

Section 8.3 Page 403 Question C2

$$\begin{aligned}
\log_2\left(\sin\frac{\pi}{4}\right) + \log_2\left(\sin\frac{3\pi}{4}\right) &= \log_2\frac{\sqrt{2}}{2} + \log_2\frac{\sqrt{2}}{2} \\
&= 2(\log_2\sqrt{2} - \log_2 2) \\
&= 2\left(\frac{1}{2}\log_2 2 - 1\right) \\
&= 2\left(\frac{1}{2} - 1\right) \\
&= -1
\end{aligned}$$

Section 8.3 Page 403 Question C3

a) $d = \log 4 - \log 2$

$= \log \frac{4}{2}$

$= \log 2$

The common difference in the arithmetic series log 2 + log 4 + log 8 + log 16 + log 32 is log 2.

b) Use the formula for the sum of an arithmetic series: $S_n = \frac{n}{2}(t_1 + t_n)$. Substitute $n = 5$, $t_1 = \log 2$, and $t_n = \log 32$.

$$S_n = \frac{n}{2}(t_1 + t_n)$$
$$S_5 = \frac{5}{2}(\log 2 + \log 32)$$
$$S_5 = \frac{5}{2}(\log 2 + \log 2^5)$$
$$S_5 = \frac{5}{2}(\log 2 + 5\log 2)$$
$$S_5 = \frac{5}{2}(6\log 2)$$
$$S_5 = 15\log 2$$

Section 8.3 Page 403 Question C4

Example:

	Product Law	Quotient Law	Power Law
Algebraic Representation	$\log_c MN = \log_c M + \log_c N$	$\log_c \frac{M}{N} = \log_c M - \log_c N$	$\log_c M^P = P \log_c M$
Written Description	The logarithm of a product of numbers is the sum of the logarithms of the numbers.	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.	The logarithm of a power of a number is the exponent times the logarithm of the number.
Example	$\log_2 5x = \log_2 5 + \log_2 x$	$\log \frac{x}{5} = \log x - \log 5$	$\log_3 x^2 = 2 \log_3 x$
Common Error	$\log_2 5 + \log_2 x \neq \log_2 (5 + x)$	$\log x - \log 5 \neq \log_2 (x - 5)$	$2 \log_3 x \neq \log_3 2x$

Section 8.4 Logarithmic and Exponential Equations

Section 8.4 Page 412 Question 1

a) $15 = 12 + \log x$
$3 = \log x$
$10^3 = x$
$1000 = x$

b) $\log_5 (2x - 3) = 2$
$5^2 = 2x - 3$
$28 = 2x$
$x = 14$

c) $4 \log_3 x = \log_3 81$
$\log_3 x^4 = \log_3 81$
$x^4 = 81$
$x^4 = 3^4$
$x = 3$

d) $2 = \log (x - 8)$
$x - 8 = 10^2$
$x = 108$

Section 8.4 Page 412 Question 2

a) $4(7^x) = 92$

$7^x = 23$

$\log 7^x = \log 23$

$x \log 7 = \log 23$

$x = \dfrac{\log 23}{\log 7}$

$x \approx 1.61$

b) $2^{\frac{x}{3}} = 11$

$\log 2^{\frac{x}{3}} = \log 11$

$\dfrac{x}{3}\log 2 = \log 11$

$x = \dfrac{3\log 11}{\log 2}$

$x \approx 10.38$

c) $6^{x-1} = 271$

$\log 6^{x-1} = \log 271$

$(x - 1) \log 6 = \log 271$

$x = \dfrac{\log 271}{\log 6} + 1$

$x \approx 4.13$

d) $4^{2x+1} = 54$

$\log 4^{2x+1} = \log 54$

$(2x + 1) \log 4 = \log 54$

$x = \dfrac{1}{2}\left(\dfrac{\log 54}{\log 4} - 1\right)$

$x \approx 0.94$

Section 8.4 Page 412 Question 3

I disagree with Hamdi's check. Neither $\log_3 (x - 8)$ nor $\log_3 (x - 6)$ are defined for $x = 5$.

Section 8.4 Page 413 Question 4

a) The equation $\log_7 x + \log_7 (x - 1) = \log_7 4x$ is defined for $x > 1$. So, the possible root $x = 0$ is extraneous.

b) The equation $\log_6 (x^2 - 24) - \log_6 x = \log_6 5$ is defined for $x > \sqrt{24}$, or approximately $x > 4.9$. So, both possible roots, $x = 3$ and $x = -8$, are extraneous.

c) The equation $\log_3 (x + 3) - \log_3 (x + 5) = 1$ is defined for $x > -3$. So, the possible root $x = -6$ is extraneous.

d) The equation $\log_2 (x - 2) = 2 - \log_2 (x - 5)$ is defined for $x > 5$. So, the possible root $x = 1$ is extraneous.

Section 8.4 Page 413 Question 5

a) $2 \log_3 x = \log_3 32 + \log_3 2$

$\log_3 x^2 = \log_3 64$

$x^2 = 64$

$x = 8$

b) $\frac{3}{2}\log_7 x = \log_7 125$

$\log_7 x^{\frac{3}{2}} = \log_7 125$

$x^{\frac{3}{2}} = 125$

$x = 125^{\frac{2}{3}}$

$x = 25$

c) $\log_2 x - \log_2 3 = 5$

$\log_2 \frac{x}{3} = 5$

$\frac{x}{3} = 2^5$

$x = 32(3)$

$x = 96$

d) $\log_6 x = 2 - \log_6 4$

$\log_6 x + \log_6 4 = 2$

$\log_6 4x = 2$

$6^2 = 4x$

$\frac{36}{4} = x$

$9 = x$

Section 8.4 Page 413 Question 6

a) Rubina subtracted the contents of the logarithmic expressions on the left side of the equation when she should have divided them.

Correct solution:

$\log_6 (2x + 1) - \log_6 (x - 1) = \log_6 5$

$\log_6 \frac{2x+1}{x-1} = \log_6 5$

$\frac{2x+1}{x-1} = 5$

$2x + 1 = 5(x - 1)$

$2x + 1 = 5x - 5$

$-3x = -6x$

$x = 2$

b) Ahmed's work is correct. However, he incorrectly concluded that there was no solution. The equation $2 \log_5 (x + 3) = \log_5 9$ is defined for $x > -3$. So, the solution is $x = 0$.

c) Jennifer incorrectly eliminated the logarithmic expression in the third line. The right side should have been 2^3, not 3.

Correct solution:

$\log_2 x + \log_2 (x + 2) = 3$

$\log_2 (x(x + 2)) = 3$

$\log_2 (x^2 + 2x) = 3$

$x^2 + 2x = 2^3$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4$ or $x = 2$

The solution is $x = 2$, since $x > 0$.

Section 8.4 Page 413 Question 7

a)

$7^{2x} = 2^{x+3}$

$\log 7^{2x} = \log 2^{x+3}$

$2x \log 7 = (x + 3) \log 2$

$2x \log 7 = x \log 2 + 3 \log 2$

$2x \log 7 - x \log 2 = 3 \log 2$

$x(2 \log 7 - \log 2) = 3 \log 2$

$$x = \frac{3 \log 2}{2 \log 7 - \log 2}$$

$x \approx 0.65$

b)

$1.6^{x-4} = 5^{3x}$

$\log 1.6^{x-4} = \log 5^{3x}$

$(x - 4) \log 1.6 = 3x \log 5$

$x \log 1.6 - 4 \log 1.6 = 3x \log 5$

$x \log 1.6 - 3x \log 5 = 4 \log 1.6$

$x(\log 1.6 - 3 \log 5) = 4 \log 1.6$

$$x = \frac{4 \log 1.6}{\log 1.6 - 3 \log 5}$$

$x \approx -0.43$

c)

$9^{2x-1} = 71^{x+2}$

$\log 9^{2x-1} = \log 71^{x+2}$

$(2x - 1) \log 9 = (x + 2) \log 71$

$2x \log 9 - \log 9 = x \log 71 + 2 \log 71$

$2x \log 9 - x \log 71 = 2 \log 71 + \log 9$

$x(2 \log 9 - \log 71) = 2 \log 71 + \log 9$

$$x = \frac{2 \log 71 + \log 9}{2 \log 9 - \log 71}$$

$x \approx 81.37$

d)

$4(7^{x+2}) = 9^{2x-3}$

$\log 4(7^{x+2}) = \log 9^{2x-3}$

$\log 4 + \log 7^{x+2} = \log 9^{2x-3}$

$\log 4 + (x + 2) \log 7 = (2x - 3) \log 9$

$\log 4 + x \log 7 + 2 \log 7 = 2x \log 9 - 3 \log 9$

$x \log 7 - 2x \log 9 = -3 \log 9 - \log 4 - 2 \log 7$

$x(\log 7 - 2 \log 9) = -3 \log 9 - \log 4 - 2 \log 7$

$$x = \frac{-3 \log 9 - \log 4 - 2 \log 7}{\log 7 - 2 \log 9}$$

$x \approx 4.85$

Section 8.4 Page 413 Question 8

a) $\log_5(x-18) - \log_5 x = \log_5 7$

$$\log_5 \frac{x-18}{x} = \log_5 7$$
$$\frac{x-18}{x} = 7$$
$$x-18 = 7x$$
$$-6x = 18$$
$$x = -3$$

Since the equation is defined for $x > 18$, there is no solution.

b) $\log_2(x-6) + \log_2(x-8) = 3$

$$\log_2((x-6)(x-8)) = 3$$
$$(x-6)(x-8) = 2^3$$
$$x^2 - 14x + 48 = 8$$
$$x^2 - 14x + 40 = 0$$
$$(x-10)(x-4) = 0$$

$x = 10$ or $x = 4$

Since the equation is defined for $x > 8$, the solution is $x = 10$.

c)

$$2\log_4(x+4) - \log_4(x+12) = 1$$
$$\log_4(x+4)^2 - \log_4(x+12) = 1$$
$$\log_4 \frac{(x+4)^2}{x+12} = 1$$
$$\frac{(x+4)^2}{x+12} = 4^1$$
$$(x+4)^2 = 4(x+12)$$
$$x^2 + 8x + 16 = 4x + 48$$
$$x^2 + 4x - 32 = 0$$
$$(x+8)(x-4) = 0$$

$x = -8$ or $x = 4$

Since the equation is defined for $x > -4$, the solution is $x = 4$.

d) $\log_3(2x-1) = 2 - \log_3(x+1)$

$\log_3(2x-1) + \log_3(x+1) = 2$

$\log_3((2x-1)(x+1)) = 2$

$(2x-1)(x+1) = 3^2$

$2x^2 + x - 1 = 9$

$2x^2 + x - 10 = 0$

$(2x+5)(x-2) = 0$

$x = -\frac{5}{2}$ or $x = 2$

Since the equation is defined for $x > \frac{1}{2}$, the solution is $x = 2$.

e) $\log_2 \sqrt{x^2+4x} = \frac{5}{2}$

$\frac{1}{2}\log_2(x^2+4x) = \frac{5}{2}$

$\log_2(x^2+4x) = 5$

$x^2 + 4x = 2^5$

$x^2 + 4x - 32 = 0$

$(x+8)(x-4) = 0$

$x = -8$ or $x = 4$

Since the equation is defined for $x < -4$ or $x > 0$, the solutions are $x = -8$ and $x = 4$.

Section 8.4 Page 413 Question 9

a) Substitute $m = -1.44$ and $M = 1.45$ into $m - M = 5 \log d - 5$.

$m - M = 5 \log d - 5$

$-1.44 - 1.45 = 5 \log d - 5$

$-2.89 = 5 \log d - 5$

$2.11 = 5 \log d$

$\frac{2.11}{5} = \log d$

$d = 10^{\frac{2.11}{5}}$

$d = 2.642\ldots$

Sirius is approximately 2.64 parsecs from Earth.

b) The distance 2.64 pc is equivalent to 2.64(3.26), or about 8.61 light years.

Section 8.4 Page 413 Question 10

Substitute $E = 24$ into $\log E = \log 10.61 + 0.1964 \log m$.

$\log E = \log 10.61 + 0.1964 \log m$

$\log 24 = \log 10.61 + 0.1964 \log m$

$\log 24 - \log 10.61 = 0.1964 \log m$

$$\log \frac{24}{10.61} = 0.1964 \log m$$

$$\frac{1}{0.1964} \log \frac{24}{10.61} = \log m$$

$$m = 10^{\frac{1}{0.1964} \log \frac{24}{10.61}}$$

$$m = 63.821...$$

The mass of the mountain goat is 64 kg, to the nearest kilogram.

Section 8.4 Page 414 Question 11

a) Substitute $t = 0$.

$P = 10\,000(1.035)^t$

$= 10\,000(1.035)^0$

$= 10\,000$

When the lake was stocked, 10 000 northern pike were put in the lake.

b) Since the base is 1.035, or 1 + 0.035, the annual growth rate as a percent is 3.5%.

c) Substitute $P = 20\,000$.

$P = 10\,000(1.035)^t$

$20\,000 = 10\,000(1.035)^t$

$2 = (1.035)^t$

$\log 2 = \log (1.035)^t$

$\log 2 = t \log 1.035$

$$\frac{\log 2}{\log 1.035} = t$$

$20.148\ldots = t$

It will take approximately 20.1 years for the number of northern pike in the lake to double.

Section 8.4 Page 414 Question 12

a) Substitute $d = 5906$ into $\log T = \frac{3}{2} \log d - 3.263$.

$$\log T = \frac{3}{2} \log d - 3.263$$

$$\log T = \frac{3}{2} \log 5906 - 3.263$$

$$T = 10^{\frac{3}{2}\log 5906 - 3.263}$$

$$T = 247.708\ldots$$

To the nearest Earth year, it takes Pluto 248 Earth years to revolve around the sun.

b) Substitute $T = 1.88$ into $\log T = \frac{3}{2} \log d - 3.263$.

$$\log T = \frac{3}{2} \log d - 3.263$$

$$\log 1.88 = \frac{3}{2} \log d - 3.263$$

$$\log 1.88 + 3.263 = \frac{3}{2} \log d$$

$$\frac{2}{3}(\log 1.88 + 3.263) = \log d$$

$$d = 10^{\frac{2}{3}(\log 1.88 + 3.263)}$$

$$d = 228.089\ldots$$

Mars is 228 million kilometres from the sun, to the nearest million kilometres.

Section 8.4 Page 414 Question 13

a) Substitute $P = 10\,000$, $i = \frac{0.06}{2}$, or 0.03, and $A = 11\,000$.

$$A = P(1 + i)^n$$

$$11\,000 = 10\,000(1 + 0.03)^n$$

$$1.1 = 1.03^n$$

$$\log 1.1 = \log 1.03^n$$

$$\log 1.1 = n \log 1.03$$

$$\frac{\log 1.1}{\log 1.03} = n$$

$$3.224\ldots = n$$

Since $n = 3$ results in \$10 927.27, and interest is compounded at the end of each 6 months, it will take 2 years for the GIC to be worth \$11 000.

b) Substitute $P = 1200$, $i = \frac{0.28}{365}$, and $A = 1241.18$.

$$A = P(1 + i)^n$$
$$1241.18 = 1200\left(1 + \frac{0.28}{365}\right)^n$$
$$\frac{1241.18}{1200} = \left(1 + \frac{0.28}{365}\right)^n$$
$$\log\frac{1241.18}{1200} = \log\left(1 + \frac{0.28}{365}\right)^n$$
$$\log\frac{1241.18}{1200} = n\log\left(1 + \frac{0.28}{365}\right)$$
$$n = \frac{\log\frac{1241.18}{1200}}{\log\left(1 + \frac{0.28}{365}\right)}$$
$$n = 44.000...$$

Linda's payment is 44 days overdue.

c) Substitute $A = 3P$ and $i = \frac{0.055}{2}$, or 0.0275.

$$A = P(1 + i)^n$$
$$3P = P(1 + 0.0275)^n$$
$$3 = 1.0275^n$$
$$\log 3 = \log 1.0275^n$$
$$\log 3 = n \log 1.0275$$
$$\frac{\log 3}{\log 1.0275} = n$$
$$40.496\ldots = n$$

Since $n = 40$ results in $A = \$2.96$ for $P = \$1$, and compound interest is added at the end of each 6 months, it will take $41 \div 2$, or 20.5 years for the money to triple in value.

Section 8.4 Page 414 Question 14

Substitute $PV = 250\ 000$, $i = \frac{0.074}{2}$, or 0.037, and $R = 10\ 429.01$.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$$250\,000 = \frac{10\,429.01\,[1-(1+0.037)^{-n}]}{0.037}$$

$$\frac{9250}{10\,429.01} = 1 - 1.037^{-n}$$

$$\frac{9250}{10\,429.01} - 1 = -1.037^{-n}$$

$$\frac{1179.01}{10\,429.01} = 1.037^{-n}$$

$$\log\frac{1179.01}{10\,429.01} = \log 1.037^{-n}$$

$$\log\frac{1179.01}{10\,429.01} = -n\log 1.037$$

$$n = -\frac{\log\frac{1179.01}{10\,429.01}}{\log 1.037}$$

$$n = 60.000...$$

The mortgage will be completely paid off after 60 ÷ 2, or 30 years.

Section 8.4 Page 414 Question 15

Substitute $m(t) = 0.315m_0$ into $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

$$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0.315m_0 = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0.315 = 0.5^{\frac{t}{5730}}$$

$$\log 0.315 = \log 0.5^{\frac{t}{5730}}$$

$$\log 0.315 = \frac{t}{5730}\log 0.5$$

$$t = \frac{5730\log 0.315}{\log 0.5}$$

$$t = 9549.482...$$

The tree was almost 9550 years old when it was discovered.

Section 8.4 Page 415 Question 16

Substitute $m(t) = 274$, $m_0 = 280$, and $t = 6$ into $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where $m(t)$ and m_0 are measured in megabecquerels, t is time, in hours, and h is the half-life of I-131, in hours.

$$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$274 = 280\left(\frac{1}{2}\right)^{\frac{6}{h}}$$

$$\frac{274}{280} = 0.5^{\frac{6}{h}}$$

$$\log\frac{274}{280} = \log 0.5^{\frac{6}{h}}$$

$$\log\frac{274}{280} = \frac{6}{h}\log 0.5$$

$$h = \frac{6\log 0.5}{\log\frac{274}{280}}$$

$$h = 191.994...$$

The half-life of I-131 is 192 ÷ 24, or 8 days, to the nearest day.

Section 8.4 Page 415 Question 17

Let the light intensity, $l(d)$, below the water's surface be represented by $l(d) = l_0(0.96)^d$, where l_0 is the intensity at the surface and d is the depth, in metres.

Substitute $l(d) = 0.25l_0$.

$$l(d) = l_0(0.96)^d$$

$$0.25l_0 = l_0(0.96)^d$$

$$0.25 = 0.96^d$$

$$\log 0.25 = \log 0.96^d$$

$$\log 0.25 = d \log 0.96$$

$$d = \frac{\log 0.25}{\log 0.96}$$

$$d = 33.959\ldots$$

To the nearest tenth of a metre, at 34.0 m the light intensity is 25% of the intensity at the surface.

Section 8.4 Page 415 Question 18

Solve the system of equations, $\log_3 81 = x - y$ and $\log_2 32 = x + y$, by elimination.

$\log_3 81 = x - y$

$\underline{\log_2 32 = x + y}$

$\log_3 81 + \log_2 32 = 2x$

$\log_3 3^4 + \log_2 2^5 = 2x$

$4 + 5 = 2x$

$x = \frac{9}{2}$

Substitute $x = \frac{9}{2}$ into $\log_3 81 = x - y$.

$\log_3 81 = x - y$

$\log_3 81 = \frac{9}{2} - y$

$4 = \frac{9}{2} - y$

$y = \frac{1}{2}$

Section 8.4 Page 415 Question 19

a) The first line, $\log 0.1 < 3 \log 0.1$, is not true.

b) Since $\log x < 0$, for $0 < x < 1$, the inequality symbol in the last line should be reversed. In other words, from line 4 to line 5 you are dividing by a negative quantity.

Section 8.4 Page 415 Question 20

a) $x^{\frac{2}{\log x}} = x$

$\frac{2}{\log x} = 1$

$2 = \log x$

$x = 10^2$

$x = 100$

b) $\log x^{\log x} = 4$

$\log x(\log x) = 4$

$(\log x)^2 = 4$

$\log x = \pm 2$

$x = 10^{-2}$ or $x = 10^2$

$= \frac{1}{100}$ $= 100$

c) $(\log x)^2 = \log x^2$

$(\log x)^2 = 2 \log x$

$(\log x)^2 - 2 \log x = 0$

$\log x(\log x - 2) = 0$

$\log x = 0$ or $\log x - 2 = 0$

$x = 1$ $\log x = 2$

$x = 100$

Section 8.4 Page 415 Question 21

a)

$$\log_4 x + \log_2 x = 6$$
$$\frac{\log_2 x}{\log_2 4} + \log_2 x = 6$$
$$\frac{\log_2 x}{2} + \log_2 x = 6$$
$$\frac{3}{2}\log_2 x = 6$$
$$\log_2 x = 4$$
$$x = 2^4$$
$$x = 16$$

b)

$$\log_3 x - \log_{27} x = \frac{4}{3}$$
$$\log_3 x - \frac{\log_3 x}{\log_3 27} = \frac{4}{3}$$
$$\log_3 x - \frac{\log_3 x}{3} = \frac{4}{3}$$
$$\frac{2}{3}\log_3 x = \frac{4}{3}$$
$$\log_3 x = 2$$
$$x = 3^2$$
$$x = 9$$

Section 8.4 Page 415 Question 22

$$(x^2 + 3x - 9)^{2x-8} = 1$$
$$\log (x^2 + 3x - 9)^{2x-8} = \log 1$$
$$(2x - 8)\log (x^2 + 3x - 9) = 0$$

$2x - 8 = 0$ or $\log (x^2 + 3x - 9) = 0$

$2x = 8$ $x^2 + 3x - 9 = 1$

$x = 4$ $x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = -5$ or $x = 2$

Section 8.4 Page 415 Question C1

a) $8(2^x) = 512$

$\log 8(2^x) = \log 512$

$\log 8 + \log 2^x = \log 512$

$\log 8 + x \log 2 = \log 512$

$x \log 2 = \log 512 - \log 8$

$$x = \frac{\log 64}{\log 2}$$

$x = 6$

b) Example: Fatima could have divided both sides of the equation by 8 to avoid taking the logarithm of each side.

$8(2^x) = 512$

$2^x = 64$

$2^x = 2^6$

$x = 6$

c) Example: I prefer the approach in part b). It is much shorter.

Section 8.4 Page 415 Question C2

For the sequence 4, 12, 36, …, 708 588, $t_1 = 4$ and $r = 3$.
Substitute $t_n = 708\ 588$, $t_1 = 4$, and $r = 3$ into $t_n = t_1 r^{n-1}$.

$$t_n = t_1 r^{n-1}$$
$$708\ 588 = 4(3)^{n-1}$$
$$177\ 147 = 3^{n-1}$$
$$\log 177\ 147 = \log 3^{n-1}$$
$$\log 177\ 147 = (n-1)\log 3$$
$$\frac{\log 177\ 147}{\log 3} = n - 1$$
$$\frac{\log 177\ 147}{\log 3} + 1 = n$$
$$12 = n$$

Section 8.4 Page 415 Question C3

For the series 8192 + 4096 + 2048 + …, $t_1 = 8192$ and $r = 0.5$.

Substitute $S_n = 16\ 383$, $t_1 = 8192$, and $r = 0.5$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
$$16\ 383 = \frac{8192(0.5^n - 1)}{0.5 - 1}$$
$$-\frac{8191.5}{8192} = 0.5^n - 1$$
$$1 - \frac{8191.5}{8192} = 0.5^n$$
$$\frac{0.5}{8192} = 0.5^n$$
$$\log \frac{0.5}{8192} = \log 0.5^n$$
$$\log \frac{0.5}{8192} = n \log 0.5$$
$$n = \frac{\log \frac{0.5}{8192}}{\log 0.5}$$
$$n = 14$$

Section 8.4 Page 415 Question C4

a) $2 \log_2 (\cos x) + 1 = 0$

$$\log_2 (\cos x) = -\frac{1}{2}$$

$$2^{-\frac{1}{2}} = \cos x$$

$$\frac{1}{\sqrt{2}} = \cos x$$

$$x = \frac{\pi}{4} \text{ or } x = \frac{7\pi}{4}$$

b) $\log (\sin x) + \log (2 \sin x - 1) = 0$

$$\log ((\sin x)(2 \sin x - 1)) = 0$$

$$(\sin x)(2 \sin x - 1) = 1$$

$$2 \sin^2 x - \sin x - 1 = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

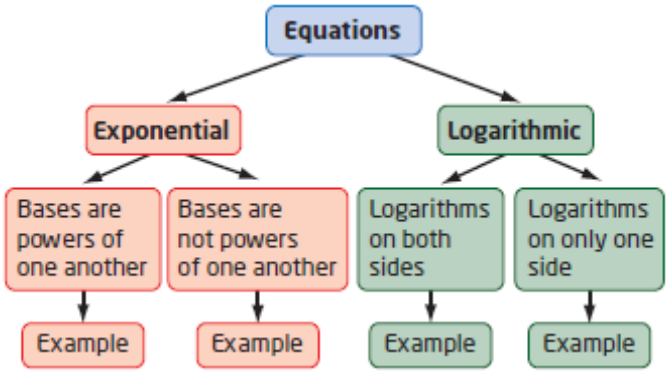
$2 \sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 1$

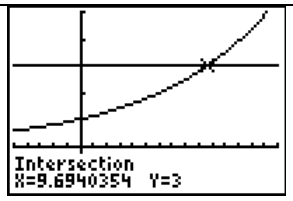
$x = \frac{4\pi}{3}, \frac{5\pi}{3}$ $x = \frac{\pi}{2}$

Since the equation is defined for $\sin x > \frac{1}{2}$, the solution is $x = \frac{\pi}{2}$.

Section 8.4 Page 415 Question C5



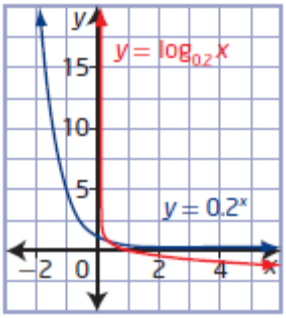
Exponential Equations		Logarithmic Equations	
Example	**Example**	**Example**	**Example**
$3^{5x} = 27^{x-1}$ $3^{5x} = (3^3)^{x-1}$ $3^{5x} = 3^{3x-3}$ Equate the exponents.	$3 = 1.12^x$ Graph $y = 3$ and $y = 1.12^x$ and find the point of intersection.	$4 \log_3 x = \log_3 81$ $\log_3 x^4 = \log_3 81$ $x^4 = 81$ $x^4 = 3^4$ $x = 3$	$\log_5 (2x - 3) = 2$ $5^2 = 2x - 3$ $28 = 2x$ $x = 14$

$5x = 3x - 3$ $2x = -3$ $x = -\frac{3}{2}$	 The solution is $x \approx 9.7$.		

Chapter 8 Review

Chapter 8 Review Page 416 Question 1

a)



b) The graph of $y = \log_{0.2} x$ has the following characteristics
i) domain: $\{x \mid x > 0, x \in \mathrm{R}\}$ and range: $\{y \mid y \in \mathrm{R}\}$
ii) x-intercept: 1
iii) no y-intercept
iv) equation of the asymptote: $x = 0$

c) Since $f(x) = 0.2^x$, the equation of inverse is $f^{-1}(x) = \log_{0.2} x$.

Chapter 8 Review Page 416 Question 2

Use the given point (2, 16) on the graph of the inverse of $y = \log_c x$, or $y = c^x$ to determine the value of c.

$y = c^x$
$16 = c^2$
$4^2 = c^2$
$c = 4$

Chapter 8 Review Page 416 Question 3

Write $a < \log_2 24 < b$ in exponential form: $2^a < 24 < 2^b$.
Since $2^4 = 16$ and $2^5 = 32$, then $a = 4$ and $b = 5$.
So, the value of $\log_2 24$ must be between 4 and 5.

Chapter 8 Review Page 366 Question 4

a) $\log_{125} x = \frac{2}{3}$

$125^{\frac{2}{3}} = x$

$x = 25$

b) $\log_9 \frac{1}{81} = x$

$9^x = \frac{1}{81}$

$9^x = 9^{-2}$

$x = -2$

c) $\log_3 27\sqrt{3} = x$

$3^x = 27\sqrt{3}$

$3^x = 3^{\frac{7}{2}}$

$x = \frac{7}{2}$

d) $\log_x 8 = \frac{3}{4}$

$x^{\frac{3}{4}} = 8$

$x = 8^{\frac{4}{3}}$

$x = 16$

e) $6^{\log x} = \frac{1}{36}$

$6^{\log x} = 6^{-2}$

$\log x = -2$

$10^{-2} = x$

$x = \frac{1}{100}$

Chapter 8 Review Page 416 Question 5

Determine the amplitude of each earthquake.

Japan earthquake:

$M = \log \frac{A}{A_0}$

$9.0 = \log \frac{A}{A_0}$

$10^{9.0} = \frac{A}{A_0}$

$A = 10^{9.0} A_0$

Japan aftershock:

$M = \log \frac{A}{A_0}$

$7.4 = \log \frac{A}{A_0}$

$10^{7.4} = \frac{A}{A_0}$

$A = 10^{7.4} A_0$

Compare the amplitudes.

$\frac{10^{9.0}}{10^{7.4}} = 10^{1.6}$

$= 39.810\ldots$

The seismic shaking of the Japan earthquake was approximately 40 times that of the aftershock.

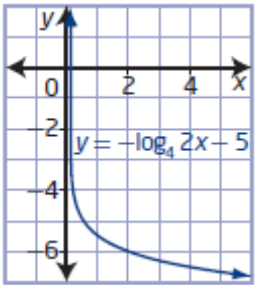
Chapter 8 Review Page 416 Question 6

a) Given: $y = \log_4 x$

• Stretch horizontally about the y-axis by a factor of $\frac{1}{2}$: $b = 2$, $y = \log_4 2x$

• Reflect in the x-axis: $a = -1$, $y = -\log_4 2x$

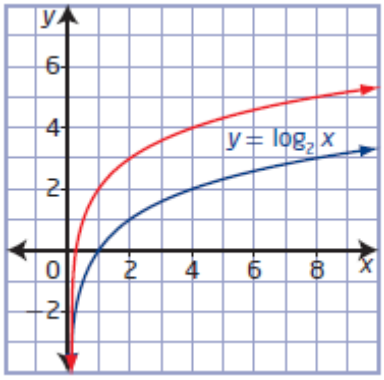
• Translate 5 units down: $k = -5$, $y = -\log_4 2x - 5$



b) The equation of the transformed image in the form $y = a \log_c (b(x - h)) + k$ is $y = -\log_4 2x - 5$. So, $a = -1$, $b = 2$, $c = 4$, $h = 0$, and $k = -5$.

Chapter 8 Review Page 416 Question 7

Choose a key points on the blue graph, say (4, 2) and (8, 3).
The key points (4, 2) and (8, 3) on the graph of $y = \log_2 x$ have become the image points (1, 2) and (2, 3) on the red graph. Thus, the red graph can be generated by horizontally stretching the graph of $y = \log_2 x$ about the y-axis by a factor of $\frac{1}{4}$. The red graph can be described by the equation $y = \log_2 4x$.



Chapter 8 Review Page 417 Question 8

a) For $y = -\log_5 (3(x - 12)) + 2$, $a = -1$, $b = 3$, $h = 12$, and $k = 2$. To obtain the graph of $y = -\log_5 (3(x - 12)) + 2$, the graph of $y = \log_5 x$ must be reflected in the x-axis, horizontally stretched about the y-axis by a factor of $\frac{1}{3}$, and translated 12 units to the right and 2 units up.

b) For $y + 7 = \frac{\log_5 (6 - x)}{4}$, or $y = \frac{1}{4} \log_5 (-(x - 6)) - 7$, $a = \frac{1}{4}$, $b = -1$, $h = 6$, and $k = -7$. To obtain the graph of $y + 7 = \frac{\log_5 (6 - x)}{4}$, the graph of $y = \log_5 x$ must be vertically stretched about the x-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, and translated 6 units to the right and 7 units down.

Chapter 8 Review Page 417 Question 9

Given: $y = 3 \log_2 (x + 8) + 6$

a) The equation of the vertical asymptote occurs when $x + 8 = 0$. Therefore, the equation of the vertical asymptote is $x = -8$.

b) The domain is $\{x \mid x > -8, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

c) Substitute $x = 0$. Then, solve for y.

$y = 3 \log_2 (x + 8) + 6$
$= 3 \log_2 (0 + 8) + 6$
$= 3 \log_2 8 + 6$
$= 3(3) + 6$
$= 15$

The y-intercept is 15.

d) Substitute $y = 0$. Then, solve for x.

$y = 3 \log_2 (x + 8) + 6$
$0 = 3 \log_2 (x + 8) + 6$
$-6 = 3 \log_2 (x + 8)$
$-2 = \log_2 (x + 8)$
$2^{-2} = x + 8$
$\frac{1}{4} = x + 8$
$x = -\frac{31}{4}$

The x-intercept is $-\frac{31}{4}$, or -7.75.

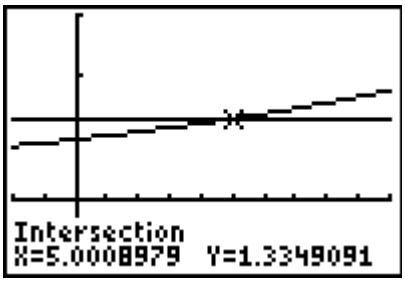
Chapter 8 Review Page 417 Question 10

a) For $n = 12 \log_2 \frac{f}{440}$, $a = 12$ and $b = \frac{1}{440}$. The function is transformed from $n = \log_2 f$ by a horizontal stretch about the y-axis by a factor of 440 and vertically stretched about the x-axis by a factor of 12.

b) Substitute $f = 587.36$.

$n = 12 \log_2 \frac{f}{440}$
$n = 12 \log_2 \frac{587.36}{440}$
$\frac{n}{12} = \log_2 \frac{587.36}{440}$
$2^{\frac{n}{12}} = \frac{587.36}{440}$

Graph $y = 2^{\frac{x}{12}}$ and $y = \frac{587.36}{440}$ and determine the point of intersection.



The note D is 5 notes above A.

c) Substitute $n = 8$.

$$n = 12 \log_2 \frac{f}{440}$$

$$8 = 12 \log_2 \frac{f}{440}$$

$$\frac{8}{12} = \log_2 \frac{f}{440}$$

$$\frac{2}{3} = \log_2 \frac{f}{440}$$

$$2^{\frac{2}{3}} = \frac{f}{440}$$

$$f = 440\left(2^{\frac{2}{3}}\right)$$

$$f = 698.456\ldots$$

The frequency of F is 698.46 Hz, to the nearest hundredth of a hertz.

Chapter 8 Review Page 417 Question 11

a) $\log_5 \left(\frac{x^5}{y\sqrt[3]{z}}\right) = \log_5 x^5 - \log_5 y\sqrt[3]{z}$

$$= 5\log_5 x - (\log_5 y + \log_5 \sqrt[3]{z})$$

$$= 5\log_5 x - \log_5 y - \frac{1}{3}\log_5 z$$

b) $\log\sqrt{\frac{xy^2}{z}} = \frac{1}{2}\log\frac{xy^2}{z}$

$$= \frac{1}{2}\left(\log xy^2 - \log z\right)$$

$$= \frac{1}{2}\left(\log x + \log y^2 - \log z\right)$$

$$= \frac{1}{2}\left(\log x + 2\log y - \log z\right)$$

Chapter 8 Review Page 417 Question 12

a) $\log x - 3\log y + \frac{2}{3}\log z$

$= \log x - \log y^3 + \log \sqrt[3]{z^2}$

$= \log \frac{x\sqrt[3]{z^2}}{y^3}$

b) $\log x - \frac{1}{2}(\log y + 3\log z)$

$= \log x - \frac{1}{2}(\log y + \log z^3)$

$= \log x - \frac{1}{2}\log yz^3$

$= \log x - \log \sqrt{yz^3}$

$= \log \frac{x}{\sqrt{yz^3}}$

Chapter 8 Review Page 417 Question 13

a) $2\log x + 3\log \sqrt{x} - \log x^3 = \log x^2 + \log \sqrt{x^3} - \log x^3$

$= \log \frac{x^2\sqrt{x^3}}{x^3}$

$= \log \frac{x^3\sqrt{x}}{x^3}$

$= \log \sqrt{x}$

$= \frac{1}{2}\log x,\ x > 0$

b) $\log(x^2 - 25) - 2\log(x+5) = \log(x^2 - 25) - \log(x+5)^2$

$= \log \frac{x^2 - 25}{(x+5)^2}$

$= \log \frac{(x-5)(x+5)}{(x+5)(x+5)}$

$= \log \frac{x-5}{x+5},\ x < -5 \text{ or } x > 5$

Chapter 8 Review Page 417 Question 14

a) $\log_6 18 - \log_6 2 + \log_6 4 = \log_6 \frac{18(4)}{2}$

$= \log_6 36$

$= 2$

b) $\log_4 \sqrt{12} + \log_4 \sqrt{9} - \log_4 \sqrt{27} = \log_4 \dfrac{\sqrt{12}\sqrt{9}}{\sqrt{27}}$

$= \log_4 \dfrac{2\sqrt{3}(3)}{3\sqrt{3}}$

$= \log_4 2$

$= 0.5$

Chapter 8 Review Page 417 Question 15

Let the pH levels of two berries be $pH_1 = -\log [H_1+]$ and $pH_2 = -\log [H_2+]$.
Compare the two pH levels.

$pH_2 - pH_1 = -\log [H_2+] - (-\log [H_1+])$

$pH_2 - pH_1 = \log [H_2+]^{-1} - \log [H_1+]^{-1}$

$pH_2 - pH_1 = \log \dfrac{[H_1+]}{[H_2+]}$

Substitute $pH_2 = 4.0$ and $pH_1 = 3.2$.

$4.0 - 3.2 = \log \dfrac{[H_1+]}{[H_2+]}$

$0.8 = \log \dfrac{[H_1+]}{[H_2+]}$

$10^{0.8} = \dfrac{[H_1+]}{[H_2+]}$

$6.309\ldots = \dfrac{[H_1+]}{[H_2+]}$

The blueberry is approximately 6.3 times more acidic than the Saskatoon berry.

Chapter 8 Review Page 417 Question 16

Substitute $m_2 = -26.74$ and $m_1 = -12.74$ in $m_2 - m_1 = -2.5 \log \left(\dfrac{F_2}{F_1} \right)$.

$m_2 - m_1 = -2.5 \log \left(\dfrac{F_2}{F_1} \right)$

$-26.74 - (-12.74) = -2.5 \log \left(\dfrac{F_2}{F_1} \right)$

$-14 = -2.5 \log \left(\dfrac{F_2}{F_1} \right)$

$5.6 = \log \left(\dfrac{F_2}{F_1} \right)$

$$10^{5.6} = \frac{F_2}{F_1}$$

$$398\ 107.170\ldots = \frac{I_2}{I_1}$$

The sun appears to be approximately 398 107 times brighter than the moon.

Chapter 8 Review Page 418 Question 17

Substitute $\frac{I_2}{I_1} = 20$ and $\beta_1 = 80$ into $\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$.

$$\beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_1}\right)$$

$$\beta_2 - 80 = 10 \log 20$$

$$\beta_2 = 10 \log 20 + 80$$

$$\beta_2 = 93.010\ldots$$

The decibel level at which the police can issue a fine to a motorcycle operator is approximately 93 dB.

Chapter 8 Review Page 418 Question 18

a)

$$3^{2x+1} = 75$$

$$\log 3^{2x+1} = \log 75$$

$$(2x + 1) \log 3 = \log 75$$

$$2x \log 3 + \log 3 = \log 75$$

$$2x \log 3 = \log 75 - \log 3$$

$$x = \frac{\log 25}{2 \log 3}$$

$$x \approx 1.46$$

b)

$$7^{x+1} = 4^{2x-1}$$

$$\log 7^{x+1} = \log 4^{2x-1}$$

$$(x + 1) \log 7 = (2x - 1) \log 4$$

$$x \log 7 + \log 7 = 2x \log 4 - \log 4$$

$$x \log 7 - 2x \log 4 = -(\log 4 + \log 7)$$

$$x (\log 7 - 2 \log 4) = -(\log 28)$$

$$x = -\frac{\log 28}{\log 7 - 2 \log 4}$$

$$x \approx 4.03$$

Chapter 8 Review Page 418 Question 19

a)

$$2\log_5(x-3) = \log_5 4$$

$$\log_5(x-3)^2 = \log_5 4$$

$$(x-3)^2 = 4$$

$$x^2 - 6x + 9 = 4$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$x = 5$ or $x = 1$

Since the equation is defined for $x > 3$, the solution is $x = 5$.

b)

$$\log_4(x+2) - \log_4(x-4) = \frac{1}{2}$$

$$\log_4 \frac{x+2}{x-4} = \frac{1}{2}$$

$$\frac{x+2}{x-4} = 4^{\frac{1}{2}}$$

$$x + 2 = 2(x-4)$$

$$x = 10$$

Since the equation is defined for $x > 4$, the solution is $x = 10$.

c)

$$\log_2(3x+1) = 2 - \log_2(x-1)$$
$$\log_2(3x+1) + \log_2(x-1) = 2$$
$$\log_2((3x+1)(x-1)) = 2$$
$$(3x+1)(x-1) = 2^2$$
$$3x^2 - 2x - 1 = 4$$
$$3x^2 - 2x - 5 = 0$$
$$(3x-5)(x+1) = 0$$

$x = \frac{5}{3}10$ or $x = -1$

Since the equation is defined for $x > 1$, the solution is $x = \frac{5}{3}$.

d)

$$\log\sqrt{x^2 - 21x} = 1$$
$$\frac{1}{2}\log(x^2 - 21x) = 1$$
$$\log(x^2 - 21x) = 2$$
$$x^2 - 21x = 10^2$$
$$x^2 - 21x - 100 = 0$$
$$(x-25)(x+4) = 0$$

$x = 25$ or $x = -4$

Since the equation is defined for $x < 0$ or $x > 21$, the solutions are $x = 25$ and $x = -4$.

Chapter 8 Review Page 418 Question 20

Let the value of the computer, $v(t)$, be represented by $v(t) = v_0(0.68)^t$, where v_0 is the initial value of the computer and t is the time, in years.

Substitute $v(t) = 100$ and $v_0 = 1200$.

$$v(t) = v_0(0.68)^t$$
$$100 = 1200(0.68)^t$$
$$\frac{100}{1200} = 0.68^t$$
$$\log\frac{1}{12} = \log 0.68^t$$
$$\log\frac{1}{12} = t\log 0.68$$
$$t = \frac{\log\frac{1}{12}}{\log 0.68}$$
$$t = 6.443\ldots$$

Since $t = 6.4$ results in a value of \$101.68, the computer will be worth less than \$100 in approximately 6.5 years.

Chapter 8 Review Page 418 Question 21

Substitute $R = 1050$ into $\log R = \log 73.3 + 0.75 \log m$.

$$\log R = \log 73.3 + 0.75 \log m$$
$$\log 1050 = \log 73.3 + 0.75 \log m$$
$$\log 1050 - \log 73.3 = 0.75 \log m$$
$$\frac{1}{0.75}\log\frac{1050}{73.3} = \log m$$

$$10^{\frac{1}{0.75}\log\frac{1050}{73.3}} = m$$
$$34.789\ldots = m$$

The mass of the wolf is 35 kg, to the nearest kilogram.

Chapter 8 Review Page 418 Question 22

Substitute $m(t) = 600$, $m_0 = 800$, and $h = 6$ into $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where $m(t)$ and m_0 are measured in megabecquerels, t is time, in hours, and h is the half-life of Tc-99m, in hours.

$$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$
$$600 = 800\left(\frac{1}{2}\right)^{\frac{t}{6}}$$
$$0.75 = 0.5^{\frac{t}{6}}$$
$$\log 0.75 = \log 0.5^{\frac{t}{6}}$$
$$\log 0.75 = \frac{t}{6}\log 0.5$$
$$h = \frac{6\log 0.75}{\log 0.5}$$
$$h = 2.490...$$

The radioactivity of the Tc-99m in the patient's body will be 600 MBq in 2.5 h, to the nearest tenth of an hour.

Chapter 8 Review Page 418 Question 23

a) Substitute $P = 500$, $i = \frac{0.05}{4}$, or 0.0125, and $A = 1000$.

$$A = P(1 + i)^n$$
$$1000 = 500(1 + 0.0125)^n$$
$$2 = 1.0125^n$$
$$\log 2 = \log 1.0125^n$$
$$\log 2 = n \log 1.0125$$
$$\frac{\log 2}{\log 1.0125} = n$$
$$55.797\ldots = n$$

It will take approximately 56 ÷ 4, or 14 years for the GIC to be worth $11 000.

b) Substitute $FV = 100\,000$, $i = \frac{0.048}{2}$, or 0.012, and $R = 500$.

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$100\,000 = \frac{500[(1+0.012)^n - 1]}{0.012}$$

$$\frac{0.012(100\,000)}{500} = 1.012^n - 1$$

$$2.4 = 1.012^n - 1$$

$$3.4 = 1.012^n$$

$$\log 3.4 = \log 1.012^n$$

$$\log 3.4 = n \log 1.012$$

$$n = \frac{\log 3.4}{\log 1.012}$$

$$n = 102.591...$$

It will take about 103 ÷ 4, or 25.75 years for Mahal's investment to be worth $100 000.

Chapter 8 Practice Test

Chapter 8 Practice Test Page 419 Question 1

The inverse of $y = \left(\frac{1}{4}\right)^x$ is $y = \log_{\frac{1}{4}} x$, which is represented by the graph in choice **D**.

Chapter 8 Practice Test Page 419 Question 2

The exponential form of $k = -\log_h 5$, or $k = \log_h 5^{-1}$, is $h^k = \frac{1}{5}$: choice **A**.

Chapter 8 Practice Test Page 419 Question 3

The function $y = \log_3 \sqrt{x+7}$ can be written as $y = \frac{1}{2} \log_3 (x + 7)$. Then, the graph of $y = \log_3 x$ must be vertically stretched about the x-axis by a factor of $\frac{1}{2}$ and translated 7 units to the left to obtain the graph of $y = \log_3 \sqrt{x+7}$: choice **B**.

Chapter 8 Practice Test Page 419 Question 4

$$\log_3 \frac{x^p}{x^q} = \log_3 x^p - \log_3 x^q$$
$$= p \log_3 x - q \log_3 x$$
$$= (p - q) \log_3 x$$

Choice **A**.

Chapter 8 Practice Test Page 419 Question 5

Given $x = \log_2 3$

$$\log_2 8\sqrt{3} = \log_2 8 + \log_2 \sqrt{3}$$
$$= \log_2 8 + \frac{1}{2} \log_2 3$$
$$= 3 + \frac{1}{2} x$$

Choice **C**.

Chapter 8 Practice Test Page 419 Question 6

Let the pH levels of two acids be $pH_1 = -\log [H_1+]$ and $pH_2 = -\log [H_2+]$.
Compare the two pH levels.

$pH_2 - pH_1 = -\log [H_2+] - (-\log [H_1+])$

$pH_2 - pH_1 = \log [H_2+]^{-1} - \log [H_1+]^{-1}$

$$pH_2 - pH_1 = \log \frac{[H_1+]}{[H_2+]}$$

Substitute $\frac{[H_1+]}{[H_2+]} = 4$ and $pH_2 = 2.9$.

$2.9 - pH_1 = \log 4$

$pH_1 = 2.9 - \log 4$

$pH_1 = 2.297\ldots$

The pH of formic acid is approximately 2.3: choice **B**.

Chapter 8 Practice Test Page 420 Question 7

a) $\log_9 x = -2$

$9^{-2} = x$

$$\frac{1}{81} = x$$

b) $\log_x 125 = \frac{3}{2}$

$x^{\frac{3}{2}} = 125$

$x = 25$

c) $\log_3 (\log_x 125) = 1$

$\log_x 125 = 3$

$x^3 = 125$

$x = 5$

d) $7^{\log_7 3} = x$

$3 = x$

e) $\log_2 8^{x-3} = 4$

$(x-3)\log_2 8 = 4$

$(x-3)3 = 4$

$3x - 9 = 4$

$3x = 13$

$x = \frac{13}{3}$

Chapter 8 Practice Test Page 420 Question 8

Given: $5^{m+n} = 125$ and $\log_{m-n} 8 = 3$

$5^{m+n} = 125$

$5^{m+n} = 5^3$

$m + n = 3$ ①

$\log_{m-n} 8 = 3$

$(m-n)^3 = 8$

$(m-n)^3 = 2^3$

$m - n = 2$ ②

Solve the system of equations.

$m + n = 3$

$\underline{m - n = 2}$

$2m = 5$ ① + ②

$m = \frac{5}{2}$

Substitute $m = \frac{5}{2}$ into ①.

$m + n = 3$

$\frac{5}{2} + n = 3$

$n = \frac{1}{2}$

Chapter 8 Practice Test Page 420 Question 9

For $y = -5 \log_2 (8(x-1))$, $a = -5$, $b = 8$, and $h = 1$.

Examples:

To obtain the graph of $y = -5 \log_2 (8(x-1))$, the graph of $y = \log_2 x$ must be reflected in the x-axis, vertically stretched about the x-axis by a factor of 5, horizontally stretched about the y-axis by a factor of $\frac{1}{8}$, and translated 1 unit to the right.

OR

To obtain the graph of $y = -5 \log_2 (8(x-1))$, the graph of $y = \log_2 x$ must be horizontally stretched about the y-axis by a factor of $\frac{1}{8}$, vertically stretched about the x-axis by a factor of 5, reflected in the x-axis, and translated 1 unit to the right.

Chapter 8 Practice Test Page 420 Question 10

Given: $y = 2 \log_5 (x + 5) + 6$

a) The equation of the vertical asymptote occurs when $x + 5 = 0$. Therefore, the equation of the vertical asymptote is $x = -5$.

b) The domain is $\{x \mid x > -5, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

c) Substitute $x = 0$. Then, solve for y.

$$\begin{aligned} y &= 2 \log_5 (x + 5) + 6 \\ &= 2 \log_5 (0 + 5) + 6 \\ &= 2 \log_5 5 + 6 \\ &= 2(1) + 6 \\ &= 8 \end{aligned}$$

The y-intercept is 8.

d) Substitute $y = 0$. Then, solve for x.

$$\begin{aligned} y &= 2 \log_5 (x + 5) + 6 \\ 0 &= 2 \log_5 (x + 5) + 6 \\ -6 &= 2 \log_5 (x + 5) \\ -3 &= \log_5 (x + 5) \\ 5^{-3} &= x + 5 \\ \frac{1}{125} &= x + 5 \\ x &= -\frac{624}{125} \end{aligned}$$

The x-intercept is $-\frac{624}{125}$, or -4.992.

Chapter 8 Practice Test Page 420 Question 11

a) $\log_2 (x-4) - \log_2 (x+2) = 4$

$$\begin{aligned} \log_2 \frac{x-4}{x+2} &= 4 \\ \frac{x-4}{x+2} &= 2^4 \\ x - 4 &= 16(x+2) \\ -15x &= 36 \\ x &= -\frac{36}{15} \\ x &= -\frac{12}{5} \end{aligned}$$

Since the equation is defined for $x > 4$, there is no solution.

b) $\log_2(x-4) = 4 - \log_2(x+2)$

$\log_2(x-4) + \log_2(x+2) = 4$

$\log_2((x-4)(x+2)) = 4$

$(x-4)(x+2) = 2^4$

$x^2 - 2x - 8 = 16$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x = 6$ or $x = -4$

Since the equation is defined for $x > 4$, the solution is $x = 6$.

c) $\log_2(x^2 - 2x)^7 = 21$

$7\log_2(x^2 - 2x) = 21$

$\log_2(x^2 - 2x) = 3$

$x^2 - 2x = 2^3$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

Since the equation is defined for $x < 0$ or $x > 2$, the solutions are $x = 4$ and $x = -2$.

Chapter 8 Practice Test Page 420 Question 12

a) $3^{2x+1} = 75$

$\log 3^{2x+1} = \log 75$

$(2x + 1)\log 3 = \log 75$

$2x \log 3 + \log 3 = \log 75$

$2x \log 3 = \log 75 - \log 3$

$$x = \frac{\log 25}{2\log 3}$$

$x \approx 1.46$

b) $12^{x-2} = 3^{2x+1}$

$\log 12^{x-2} = \log 3^{2x+1}$

$(x - 2)\log 12 = (2x + 1)\log 3$

$x \log 12 - 2 \log 12 = 2x \log 3 + \log 3$

$x \log 12 - 2x \log 3 = 2 \log 12 + \log 3$

$x(\log 12 - 2 \log 3) = 2 \log 12 + \log 3$

$$x = \frac{2\log 12 + \log 3}{\log 12 - 2\log 3}$$

$x \approx 21.09$

Chapter 8 Practice Test Page 420 Question 13

Substitute $PV = 1\ 000\ 000$, $i = \frac{0.06}{2}$, or 0.03, and $R = 35\ 000$.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$$1\ 000\ 000 = \frac{35\ 000[1-(1+0.03)^{-n}]}{0.03}$$

$$\frac{0.03(1\ 000\ 000)}{35\ 000} = 1-1.03^{-n}$$

$$\frac{6}{7} = 1-1.03^{-n}$$

$$-\frac{1}{7} = -1.03^{-n}$$

$$\frac{1}{7} = 1.03^{-n}$$

$$\log\frac{1}{7} = \log 1.03^{-n}$$

$$\log\frac{1}{7} = -n\log 1.03$$

$$n = -\frac{\log\frac{1}{7}}{\log 1.03}$$

$$n = 65.831...$$

Holly can make semi-annual withdrawals for about 66 ÷ 2, or 33 years.

Chapter 8 Practice Test Page 420 Question 14

Substitute $\Delta G = 4200$ into $\Delta G = 1427.6(\log C_2 - \log C_1)$.

$$\Delta G = 1427.6(\log C_2 - \log C_1)$$

$$4200 = 1427.6(\log C_2 - \log C_1)$$

$$\frac{4200}{1427.6} = \log\frac{C_2}{C_1}$$

$$\frac{C_2}{C_1} = 10^{\frac{4200}{1427.6}}$$

$$\frac{C_2}{C_1} = 874.984...$$

The glucose concentration outside the cell is approximately 875 times as great as inside the cell.

Chapter 8 Practice Test Page 420 Question 15

Substitute $\frac{I_2}{I_1} = 2$ and $\beta_1 = 45$ into $\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$.

$$\beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_1}\right)$$

$$\beta_2 - 45 = 10\log 2$$

$$\beta_2 = 10\log 2 + 45$$

$$\beta_2 = 48.010\ldots$$

Since the decibel level with two refrigerators running is about 48 dB, the owner should not be worried. For comparison, this decibel level is between quiet and normal conversation on the decibel scale.

Chapter 8 Practice Test Page 420 Question 16

Substitute $c(t) = 12.8$, $c_0 = 4.0$, and $t = 8$ into $c(t) = c_0(2)^{\frac{t}{d}}$, where $c(t)$ and c_0 are measured in grams per litre, t is time, in hours, and d is the doubling time of the yeast cells, in hours.

$$c(t) = c_0(2)^{\frac{t}{d}}$$

$$12.8 = 4.0(2)^{\frac{8}{d}}$$

$$3.2 = 2^{\frac{8}{d}}$$

$$\log 3.2 = \log 2^{\frac{8}{d}}$$

$$\log 3.2 = \frac{8}{d}\log 2$$

$$d = \frac{8\log 2}{\log 3.2}$$

$$h = 4.767...$$

The doubling time of the yeast cells is 4.8 h, to the nearest tenth of an hour.

Chapter 8 Practice Test Page 420 Question 17

Let the CPI, $C(t)$, be represented by $C(t) = C_0(2)^{\frac{t}{d}}$, where C_0 is the CPI in 1992, t is the number of years since 1992, and d is the doubling time, in years.
Substitute $C_0 = 1$, $t = 14$, and $l(d) = 1.299$.

$$C(t) = C_0(2)^{\frac{t}{d}}$$

$$1.299 = 1(2)^{\frac{14}{d}}$$

$$1.299 = 2^{\frac{14}{d}}$$

$$\log 1.299 = \log 2^{\frac{14}{d}}$$

$$\log 1.299 = \frac{14}{d}\log 2$$

$$d = \frac{14\log 2}{\log 1.299}$$

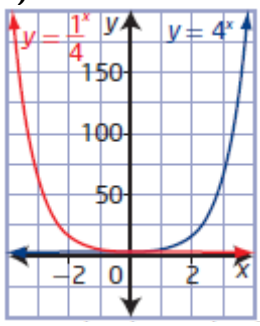
$$d = 37.095...$$

If the CPI continues to grow at the same rate, in the year 2029 the price of the basket will be twice the 1992 price.

Cumulative Review, Chapters 7-8

Cumulative Review, Chapters 7-8 Page 422 Question 1

a)



b) The two functions have the same domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, and equation of the asymptote $y = 0$.

c) The function $y = 4^x$ is increasing, since $c > 1$. The function $y = \left(\frac{1}{4}\right)^x$ is decreasing, since $0 < c < 1$.

Cumulative Review, Chapters 7-8 Page 422 Question 2

a) For $y = 5(2^x) + 1$, $c > 1$ so the graph is increasing. The graph will pass through the point (0, 6). Graph **B**.

b) For $y = \left(\frac{1}{2}\right)^{x+5}$, $c < 1$ so the graph is decreasing. The graph will pass through the point (0, 0.031 25). Graph **D**.

c) For $y + 1 = 2^{5-x}$ or $y = \left(\frac{1}{2}\right)^{x-5}$, $c < 1$ so the graph is decreasing. The graph will pass through the point (0, 31). Graph **A**.

d) For $y = 5\left(\frac{1}{2}\right)^{-x}$ or $y = 5(2^x)$, $c > 1$ so the graph is increasing. The graph will pass through the approximate point (0, 5). Graph **C**.

Cumulative Review, Chapters 7-8 Page 422 Question 3

a) Substitute $t = 0$.

$B(t) = 1000\left(2^{\frac{t}{3}}\right)$

$B(0) = 1000\left(2^{\frac{0}{3}}\right)$

$B(0) = 1000$

There were 1000 bacteria initially.

b) The doubling period is 3 h.

c) Substitute $t = 24$.

$B(t) = 1000\left(2^{\frac{t}{3}}\right)$

$B(24) = 1000\left(2^{\frac{24}{3}}\right)$

$B(24) = 256\ 000$

There were 256 000 bacteria after 24 h.

d) Substitute $B(t) = 128\ 000$.

$B(t) = 1000\left(2^{\frac{t}{3}}\right)$

$128\ 000 = 1000\left(2^{\frac{t}{3}}\right)$

$128 = 2^{\frac{t}{3}}$

$2^7 = 2^{\frac{t}{3}}$

$7 = \frac{t}{3}$

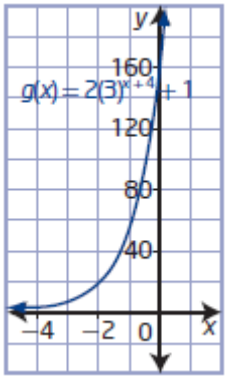
$t = 21$

There will be 128 000 bacteria in 21 h.

Cumulative Review, Chapters 7-8 Page 422 Question 4

a) For $g(x) = 2(3^{x+4}) + 1$, $a = 2$, $h = -4$, and $k = 1$. The graph of $f(x) = 3^x$ must be vertically stretch by a factor of 2 and translated 4 units to the left and 1 unit up.

b)



c) The domain remains the same. The range changes from $\{y \mid y > 0, y \in \mathrm{R}\}$ to $\{y \mid y > 1, y \in \mathrm{R}\}$ because of the vertical translation. The equation of the asymptote changes from $y = 0$ to $y = 1$ also because of the vertical translation. The y-intercept changes from 1 to 163 because of the vertical stretch and vertical translation.

Cumulative Review, Chapters 7-8 Page 422 Question 5

a) 2^{3x+6} and $8^{x-5} = (2^3)^{x-5}$
$= 2^{3x-15}$

b) 27^{4-x} and $\left(\frac{1}{9}\right)^{2x} = (3^{-2})^{2x}$
$= (3^3)^{4-x}$ $= 3^{-4x}$
$= 3^{12-3x}$

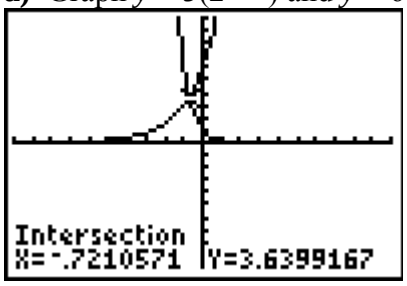
Cumulative Review, Chapters 7-8 Page 422 Question 6

a)
$5 = 2^{x+4} - 3$
$8 = 2^{x+4}$
$2^3 = 2^{x+4}$
$3 = x + 4$
$x = -1$

b)
$$\frac{25^{x+3}}{625^{x-4}} = 125^{2x+7}$$
$$\frac{(5^2)^{x+3}}{(5^4)^{x-4}} = (5^3)^{2x+7}$$
$$\frac{5^{2x+6}}{5^{4x-16}} = 5^{6x+21}$$
$$5^{-2x+22} = 5^{6x+21}$$
$$-2x + 22 = 6x + 21$$
$$-8x = -1$$
$$x = \frac{1}{8}$$

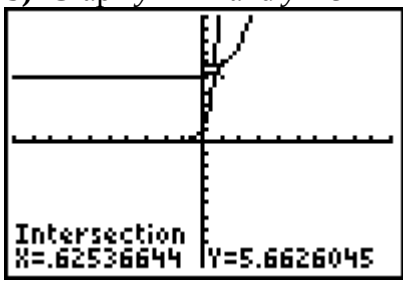
Cumulative Review, Chapters 7-8 Page 422 Question 7

a) Graph $y = 3(2^{x+1})$ and $y = 6^{-x}$ and identify the point of intersection.



The solution is $x = -0.72$, to two decimal places.

b) Graph $y = 4^{2x}$ and $y = 3^{x-1} + 5$ and identify the point of intersection.



The solution is $x = 0.63$, to two decimal places.

Cumulative Review, Chapters 7-8 Page 422 Question 8

a) Substitute $t = 5$.

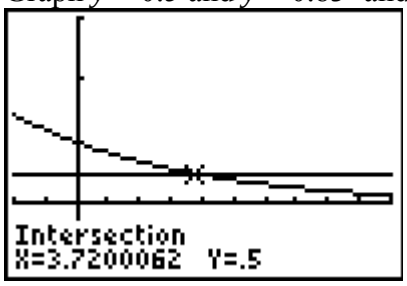
$p = 100(0.83^t)$
$= 100(0.83^5)$
$= 39.390\ldots$

The percent air pressure in the tank is approximately 39%.

b) Substitute $p = 50$.

$p = 100(0.83^t)$
$50 = 100(0.83^t)$
$0.5 = 0.83^t$

Graph $y = 0.5$ and $y = 0.83^x$ and identify the point of intersection.



The air pressure will be 50% of the starting pressure in approximately 3.7 s.

Cumulative Review, Chapters 7-8 Page 422 Question 9

a) In logarithmic form, $y = 3^x$ is $x = \log_3 y$.

b) In logarithmic form, $m = 2^{a+1}$ is $a + 1 = \log_2 m$.

Cumulative Review, Chapters 7-8 Page 422 Question 10

a) In exponential form, $\log_x 3 = 4$ is $x^4 = 3$.

b) In exponential form, $\log_a (x + 5) = b$ is $a^b = x + 5$.

Cumulative Review, Chapters 7-8 Page 423 Question 11

a) $\log_3 \frac{1}{81} = \log_3 3^{-4}$
$= -4$

b) $\log_2 \sqrt{8} + \frac{1}{3}\log_2 512 = \frac{1}{2}\log_2 8 + \frac{1}{3}\log_2 512$
$= \frac{1}{2}(3) + \frac{1}{3}(9)$
$= \frac{3}{2} + 3$
$= \frac{9}{2}$

c) $\log_2(\log_5 \sqrt{5}) = \log_2\left(\frac{1}{2}\log_5 5\right)$
$= \log_2\left(\frac{1}{2}\right)$
$= -1$

d) Use the inverse property $c^{\log_c x} = x$. For $k = \log_7 49$,
$7^k = 7^{\log_7 49}$
$= 49$

Cumulative Review, Chapters 7-8 Page 423 Question 12

a) $\log_x 16 = 4$
$x^4 = 16$
$x^4 = 2^4$
$x = 2$

b) $\log_2 x = 5$
$2^5 = x$
$32 = x$

c) $5^{\log_5 x} = \frac{1}{125}$
$x = \frac{1}{125}$

d) $\log_x(\log_3 \sqrt{27}) = \frac{1}{5}$

$$\log_x\left(\frac{1}{2}\log_3 27\right) = \frac{1}{5}$$

$$\log_x\left(\frac{1}{2}(3)\right) = \frac{1}{5}$$

$$x^{\frac{1}{5}} = \frac{3}{2}$$

$$x = \frac{243}{32}$$

Cumulative Review, Chapters 7-8 Page 423 Question 13

For $y = \frac{\log_6(2x-8)}{3} + 5$ or $y = \frac{1}{3}\log_6(2(x-4)) + 5$, $a = \frac{1}{3}$, $b = 2$, $h = 4$, and $k = 4$. The graph of $y = \log_6 x$ must be transformed by a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a vertical stretch about the x-axis by a factor of $\frac{1}{3}$, and translated by 4 units to the right and 5 units up.

Cumulative Review, Chapters 7-8 Page 423 Question 14

a) For a vertical stretch about the x-axis by a factor of 3 and a horizontal translation of 5 units left, $a = 3$ and $h = 5$. The equation of the transformed function is $y = 3\log(x + 5)$.

b) For a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a vertical translation of 2 units down, $a = -1$, $b = 2$, and $k = -2$. The equation of the transformed function is $y = -\log 2x - 2$.

Cumulative Review, Chapters 7-8 Page 423 Question 15

a) Substitute pH = 6.2.
pH = –log [H+]
6.2 = –log [H+]
–6.2 = log [H+]
$[H+] = 10^{-6}$
$[H+] \approx 6.3 \times 10^{-7}$

Substitute pH = 7.8.
pH = –log [H+]
7.8 = –log [H+]
–7.8 = log [H+]
$[H+] = 10^{-7.8}$
$[H+] \approx 1.6 \times 10^{-8}$

The range of the concentration of hydrogen ions that is best for alfalfa is 1.6×10^{-8} mol/L to 6.3×10^{-7} mol/L.

b) Substitute $[H+] = 3.0 \times 10^{-6}$.

$pH = -\log [H+]$

$= -\log (3.0 \times 10^{-6})$

$= 5.522\ldots$

Since the pH level is above 5.5, nitrogen is available to plants.

Cumulative Review, Chapters 7-8 Page 423 Question 16

a) $2\log m - (\log \sqrt{n} + 3\log p) = \log m^2 - (\log \sqrt{n} + \log p^3)$

$= \log m^2 - \log\left(p^3\sqrt{n}\right)$

$= \log \dfrac{m^2}{p^3\sqrt{n}}, m > 0, n > 0, p > 0$

b) $\dfrac{1}{3}(\log_a x - \log_a \sqrt{x}) + \log_a 3x^2 = \dfrac{1}{3}\log_a \dfrac{x}{\sqrt{x}} + \log_a 3x^2$

$= \dfrac{1}{3}\log_a \sqrt{x} + \log_a 3x^2$

$= \log_a \sqrt[6]{x} + \log_a 3x^2$

$= \log_a \sqrt[6]{x}(3x^2)$

$= \log_a 3x^{\frac{13}{6}}, x > 0$

c) $2\log(x+1) + \log(x-1) - \log(x^2-1) = \log(x+1)^2 + \log(x-1) - \log(x^2-1)$

$= \log \dfrac{(x+1)^2(x-1)}{x^2-1}$

$= \log \dfrac{(x+1)(x+1)(x-1)}{(x+1)(x-1)}$

$= \log(x+1), x > 1$

d) $\log_2 27^x - \log_2 3^x = \log_2 \dfrac{27^x}{3^x}$

$= \log_2 9^x, x \in \mathrm{R}$ or

$= \log_2 3^{2x}, x \in \mathrm{R}$

Cumulative Review, Chapters 7-8 Page 423 Question 17

Zack incorrectly factored $x^2 - 8x - 65$ as $(x + 13)(x - 5)$. The correct factored form is $(x - 13)(x + 5)$. So, the solutions are $x = 13$ and $x = -5$.

Cumulative Review, Chapters 7-8 Page 423 Question 18

a)

$$4^{2x+1} = 9(4^{1-x})$$
$$\frac{4^{2x+1}}{4^{1-x}} = 9$$
$$4^{3x} = 9$$
$$\log 4^{3x} = \log 9$$
$$3x \log 4 = \log 9$$
$$x = \frac{\log 9}{3 \log 4}$$
$$x \approx 0.53$$

b)

$$\log_3 x + 3\log_3 x^2 = 14$$
$$\log_3 x + \log_3 (x^2)^3 = 14$$
$$\log_3 x(x^6) = 14$$
$$\log_3 x^7 = 14$$
$$x^7 = 3^{14}$$
$$x = 3^2$$
$$x = 9$$

c)

$$\log(2x-3) = \log(4x-3) - \log x$$
$$\log(2x-3) - \log(4x-3) + \log x = 0$$
$$\log \frac{(2x-3)x}{4x-3} = 0$$
$$10^0 = \frac{(2x-3)x}{4x-3}$$
$$1 = \frac{2x^2 - 3x}{4x-3}$$
$$4x - 3 = 2x^2 - 3x$$
$$0 = 2x^2 - 7x + 3$$
$$0 = (2x-1)(x-3)$$
$$x = \frac{1}{2} \text{ or } x = 3$$

Since the equation is defined for $x > \frac{3}{2}$, the solution is $x = 3$.

d)

$$\log_2 x + \log_2 (x+6) = 4$$
$$\log_2 (x(x+6)) = 4$$
$$x(x+6) = 2^4$$
$$x^2 + 6x = 16$$
$$x^2 + 6x - 16 = 0$$
$$(x+8)(x-2) = 0$$
$$x = -8 \text{ or } x = 2$$

Since the equation is defined for $x > 0$, the solution is $x = 2$.

Cumulative Review, Chapters 7-8 Page 423 Question 19

a) Substitute $M = 4$.

$\log E = 4.4 + 1.4M$

$\log E = 4.4 + 1.4(4)$

$\log E = 10$

$E = 10^{10}$

Substitute $M = 5$.

$\log E = 4.4 + 1.4M$

$\log E = 4.4 + 1.4(5)$

$\log E = 11.4$

$E = 10^{11.4}$

The energy of earthquakes with magnitudes 4 and 5 are 10^{10} J and $10^{11.4}$ J, respectively.

b) For each increase in M of 1, E changes by a factor of $10^{1.4}$, or about 25.1 times.

Cumulative Review, Chapters 7-8 Page 423 Question 20

Substitute $FV = 1\ 000\ 000$, $i = \frac{0.06}{2}$, or 0.015, and $R = 625$.

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$1\ 000\ 000 = \frac{625[(1+0.015)^n - 1]}{0.015}$$

$$\frac{0.015(1\ 000\ 000)}{625} = 1.015^n - 1$$

$$24 = 1.015^n - 1$$

$$25 = 1.015^n$$

$$\log 25 = \log 1.015^n$$

$$\log 25 = n \log 1.015$$

$$n = \frac{\log 25}{\log 1.015}$$

$$n = 216.197...$$

Since $n = 216$ results in only \$996 946.64, it will take about 217 ÷ 4, or 54.25 years for Aaron's investment to be worth \$1 000 000.

Unit 2 Test

Unit 2 Test Page 424 Question 1

Use the given points, (3, –6) and (6, –12), to determine the value of a on the graph of $y = a(2^{bx})$.

For (3, –6),

$y = a(2^{bx})$

$-6 = a(2^{b3})$

$a = -\frac{6}{2^{3b}}$

Then, use (6, –12) and $a = -\frac{6}{2^{3b}}$,

$$y = a(2^{bx})$$
$$-12 = -\frac{6}{2^{3b}}(2^{b6})$$
$$2 = 2^{3b}$$
$$1 = 3b$$
$$b = \frac{1}{3}$$

Substitute $b = \frac{1}{3}$ into $a = -\frac{6}{2^{3b}}$.

$$a = -\frac{6}{2^{3b}}$$
$$= -\frac{6}{2^{3\left(\frac{1}{3}\right)}}$$
$$= -3$$

Choice **D**.

Unit 2 Test Page 424 Question 2

For $y = 3(b^{x+1}) - 2$, $a = 3$, $h = -1$, and $k = -2$. The graph of $y = b^x$ must be vertically stretched by a factor of 2 and translated 1 unit to the left and 2 units down to obtain the graph of $y = 3(b^{x+1}) - 2$. The domain stays the same, $\{x \mid x \in \mathrm{R}\}$, but the range changes from $\{y \mid y > 0, y \in \mathrm{R}\}$ to from $\{y \mid y > -2, y \in \mathrm{R}\}$. The x-intercept changes from none to one. The y-intercept changes from 1 to $3b - 2$.
Choice **B**.

Unit 2 Test Page 424 Question 3

The mass, m, of C-14 remaining at time t can be found using the relationship $m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$. If a bone has lost 40% of its carbon-14, then 60% remains. An equation that can be used to determine its age is $60 = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$: choice **A**.

Unit 2 Test Page 424 Question 4

$$2x = \log_3 (y - 1)$$
$$y - 1 = 3^{2x}$$
$$y = 3^{2x} + 1$$
$$y = 9^x + 1$$

An equivalent form for $2x = \log_3 (y - 1)$ is $y = 9^x + 1$: choice **C**.

Unit 2 Test Page 424 Question 5

The function $f(x) = -\log_2 (x + 3)$ is defined for $x + 3 > 0$, or $x > -3$. So, the domain is $\{x \mid x > -3, x \in \mathrm{R}\}$: choice **A**.

Unit 2 Test Page 424 Question 6

Given: $\log_2 5 = x$

$$\log_2 \sqrt[4]{25^3} = \frac{3}{4}\log_2 25$$
$$= \frac{3}{4}\log_2 5^2$$
$$= \frac{3}{2}\log_2 5$$
$$= \frac{3}{2}x$$

Choice **A**.

Unit 2 Test Page 424 Question 7

Given: $\log_4 16 = x + 2y$ and $\log 0.0001 = x - y$

$\log_4 16 = x + 2y$ $\qquad$ $\log 0.0001 = x - y$

$2 = x + 2y$ ① $\qquad$ $-4 = x - y$ ②

Solve the system of equations.

$$\begin{array}{r} 2 = x + 2y \\ \underline{-4 = x - y} \\ 6 = 3y \qquad ① - ② \\ 2 = y \end{array}$$

Choice **D**.

Unit 2 Test Page 424 Question 8

For a vertical stretch about the x-axis by a factor of 2, a reflection about the x-axis, and a horizontal translation of 3 units right, $a = -2$ and $h = 3$.

The graph of the function $f(x) = \left(\frac{1}{4}\right)^x$ is transformed by a vertical stretch about the x-axis by a factor of 2, a reflection about the x-axis, and a horizontal translation of 3 units right.

The equation of the transformed function is $g(x) = -2\left(\frac{1}{4}\right)^{x-3}$.

Unit 2 Test **Page 424** **Question 9**

$$\frac{9^{\frac{1}{2}}}{27^{\frac{2}{3}}} = \frac{(3^2)^{\frac{1}{2}}}{(3^3)^{\frac{2}{3}}}$$
$$= \frac{3}{3^2}$$
$$= 3^{-1}$$

The quotient $\frac{9^{\frac{1}{2}}}{27^{\frac{2}{3}}}$ expressed as a single power of 3 is 3^{-1}.

Unit 2 Test **Page 425** **Question 10**

For a function that is reflected in the x-axis and translated 1 unit down, the mapping is $(x, y) \rightarrow (x, -y - 1)$.

The point P(2, 1) is on the graph of the logarithmic function $y = \log_2 x$. When the function is reflected in the x-axis and translated 1 unit down, the coordinates of the image of P are $(2, -2)$.

Unit 2 Test **Page 425** **Question 11**

$$\log 10^x = 0.001$$
$$10^{0.001} = 10^x$$
$$x = 0.001$$

The solution to the equation $\log 10^x = 0.001$ is $x = 0.001$.

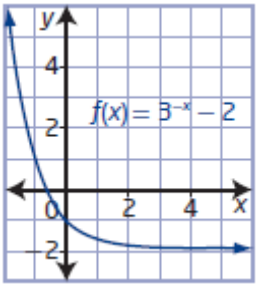
Unit 2 Test **Page 425** **Question 12**

$$\log_5 40 - 3\log_5 10 = \log_5 40 - \log_5 10^3$$
$$= \log_5 \frac{40}{1000}$$
$$= \log_5 \frac{1}{25}$$
$$= -2$$

Evaluating $\log_5 40 - 3 \log_5 10$ results in -2.

Unit 2 Test Page 425 Question 13

a)



b) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y > -2, y \in \mathrm{R}\}$.

c) Solve $f(x) = 0$.

$f(x) = 3^{-x} - 2$

$0 = 3^{-x} - 2$

$2 = 3^{-x}$

$\log 2 = \log 3^{-x}$

$\log 2 = -x \log 3$

$$x = -\frac{\log 2}{\log 3}$$

$x \approx -0.6$

Unit 2 Test Page 425 Question 14

a)

$$9^{\frac{1}{4}}\left(\frac{1}{3}\right)^{\frac{x}{2}} = \sqrt[3]{27^4}$$

$$(3^2)^{\frac{1}{4}}(3^{-1})^{\frac{x}{2}} = (3^3)^{\frac{4}{3}}$$

$$3^{\frac{1}{2}-\frac{x}{2}} = 3^4$$

$$\frac{1}{2} - \frac{x}{2} = 4$$

$$1 - x = 8$$

$$x = -7$$

b)

$$5(2^{x-1}) = 10^{2x-3}$$

$$\log 5(2^{x-1}) = \log 10^{2x-3}$$

$$\log 5 + \log 2^{x-1} = (2x-3)\log 10$$

$$\log 5 + (x-1)\log 2 = (2x-3)(1)$$

$$\log 5 + x\log 2 - \log 2 = 2x - 3$$

$$x\log 2 - 2x = -3 - \log 5 + \log 2$$

$$x(\log 2 - 2) = -3 - \log 5 + \log 2$$

$$x = \frac{-3 - \log 5 + \log 2}{\log 2 - 2}$$

$$x = 2$$

Unit 2 Test Page 425 Question 15

a) For the function $f(x) = 1 - \log(x - 2)$, or $f(x) = -\log(x - 2) + 1$, $a = -1$, $h = 2$, and $k = 1$. The function is defined for $x - 2 > 0$ or $x > 2$. So, the domain is $\{x \mid x > 2, x \in \mathrm{R}\}$, the range is $\{y \mid y \in \mathrm{R}\}$, and the equation of the asymptote is $x = 2$.

b) $f(x) = 1 - \log(x - 2)$
$y = 1 - \log(x - 2)$
$x = 1 - \log(y - 2)$
$x - 1 = -\log(y - 2)$
$-(x - 1) = \log(y - 2)$
$y - 2 = 10^{-(x-1)}$
$y = 10^{-(x-1)} + 2$
$f^{-1}(x) = 10^{-(x-1)} + 2$

c) Substitute $x = 0$.
$f^{-1}(x) = 10^{-(x-1)} + 2$
$f^{-1}(0) = 10^{-(0-1)} + 2$
$f^{-1}(0) = 10 + 2$
$f^{-1}(0) = 12$

Unit 2 Test Page 425 Question 16

a) $\log 4 = \log x + \log(13 - 3x)$
$\log 4 = \log(x(13 - 3x))$
$\log 4 = \log(13x - 3x^2)$
$4 = 13x - 3x^2$
$0 = -3x^2 + 13x - 4$
$0 = 3x^2 - 13x + 4$
$0 = (3x - 1)(x - 4)$
$x = \frac{1}{3}$ or $x = 4$

Since the equation is defined for $0 < x < \frac{13}{3}$, the solutions are $x = \frac{1}{3}$ and $x = 4$.

b) $\log_3(3x + 6) - \log_3(x - 4) = 2$
$\log_3 \frac{3x + 6}{x - 4} = 2$
$\frac{3x + 6}{x - 4} = 3^2$
$3x + 6 = 9(x - 4)$
$3x + 6 = 9x - 36$
$-6x = -42$
$x = 7$

Since the equation is defined for $x > 4$, the solution is $x = 7$.

Unit 2 Test Page 425 Question 17

Giovanni's first error occurs in line 2. He multiplied the base by 2 when he should have divided both sides by 2. His next error occurs in line six, where he incorrectly applied the quotient law of logarithms: $\frac{\log 8}{\log 6} \neq \log 8 - \log 6$. The correct solution is

$2(3^x) = 8$
$3^x = 4$
$\log 3^x = \log 4$

$$x \log 3 = \log 4$$
$$x = \frac{\log 4}{\log 3}$$
$$x \approx 1.26$$

Unit 2 Test Page 425 Question 18

Determine the amplitude of the Tofino earthquake. Substitute $M = 5.6$.

$$M = \log \frac{A}{A_0}$$
$$5.6 = \log \frac{A}{A_0}$$
$$10^{5.6} = \frac{A}{A_0}$$
$$A = 10^{5.6} A_0$$

Then, the amplitude of the aftershock is $\frac{1}{4}A$, or $\frac{1}{4}10^{5.6}A_0$.

$$M = \log \frac{A}{A_0}$$
$$= \log \frac{\frac{1}{4}10^{5.6} A_0}{A_0}$$
$$= \log \frac{1}{4}10^{5.6}$$
$$= 4.997...$$

The magnitude of the aftershock is 5.0, to the nearest tenth.

Unit 2 Test Page 425 Question 19

a) Let the world population, $P(t)$, in billions, be represented by $P(t) = 6(1.03)^t$, where t is the number of years since 2000.

b) Substitute $P(t) = 10$.

$$P(t) = 6(1.013)^t$$
$$10 = 6(1.013)^t$$
$$\frac{5}{3} = 1.013^t$$
$$\log \frac{5}{3} = \log 1.013^t$$
$$\log \frac{5}{3} = t \log 1.013$$

$$t = \frac{\log \frac{5}{3}}{\log 1.013}$$
$$t = 39.549\ldots$$

The population will reach at least 10 billion by 2040.

Unit 2 Test Page 425 Question 20

Substitute $FV = 150\ 000$, $i = \frac{0.05}{2}$, or 0.025, and $R = 11\ 500$.

$$FV = \frac{R[(1+i)^n - 1]}{i}$$
$$150\ 000 = \frac{11\ 500[(1+0.025)^n - 1]}{0.025}$$
$$\frac{0.025(150\ 000)}{11\ 500} = 1.025^n - 1$$
$$\frac{15}{46} = 1.025^n - 1$$
$$\frac{61}{46} = 1.025^n$$
$$\log \frac{61}{46} = \log 1.025^n$$
$$\log \frac{61}{46} = n \log 1.025$$
$$n = \frac{\log \frac{61}{46}}{\log 1.025}$$
$$n = 11.429...$$

Since $n = 11$ results in only \$143 559.86, it will take 12 deposits for the account to contain at least \$150 000.

Chapter 9 Rational Functions

Section 9.1 Exploring Rational Functions Using Transformations

Section 9.1 Page 442 Question 1

Compare each graph to the function form $y = \frac{a}{x-h} + k$.

a) Since the graph of the rational function has a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 0$, $h = -1$ and $k = 0$. Then, the function is of the form $y = \frac{a}{x+1}$, which is $B(x) = \frac{2}{x+1}$.

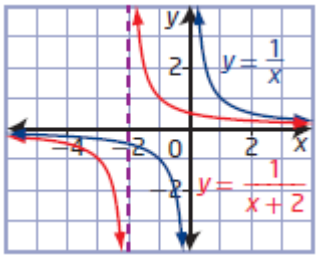
b) Since the graph of the rational function has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = -1$, $h = 0$ and $k = -1$. Then, the function is of the form $y = \frac{a}{x} - 1$, which is $A(x) = \frac{2}{x} - 1$.

c) Since the graph of the rational function has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 1$, $h = 0$ and $k = 1$. Then, the function is of the form $y = \frac{a}{x} + 1$, which is $D(x) = \frac{2}{x} + 1$.

d) Since the graph of the rational function has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 0$, $h = 1$ and $k = 0$. Then, the function is of the form $y = \frac{a}{x-1}$, which is $C(x) = \frac{2}{x-1}$.

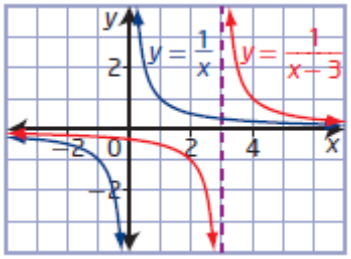
Section 9.1 Page 442 Question 2

a) The base function for $y = \frac{1}{x+2}$ is $y = \frac{1}{x}$. Compare the function to the form $y = \frac{a}{x-h} + k$: $a = 1$, $h = -2$, and $k = 0$.



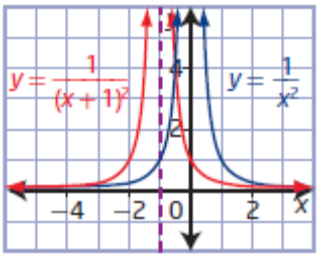
The graph of the base function must be translated 2 units to the left. So, the vertical asymptote is at $x = -2$ and the horizontal asymptote is still at $y = 0$.

b) The base function for $y = \frac{1}{x-3}$ is $y = \frac{1}{x}$. Compare the function to the form $y = \frac{a}{x-h} + k$: $a = 1$, $h = 3$, and $k = 0$.

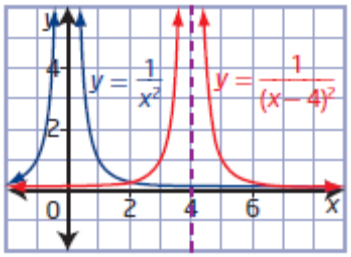


The graph of the base function must be translated 3 units to the right. So, the vertical asymptote is at $x = 3$ and the horizontal asymptote is still at $y = 0$.

c) The base function for $y = \frac{1}{(x+1)^2}$ is $y = \frac{1}{x^2}$. Compare the function to the form $y = \frac{a}{(x-h)^2} + k$: $a = 1$, $h = -1$, and $k = 0$. The graph of the base function must be translated 1 unit to the left. So, the vertical asymptote is at $x = -1$ and the horizontal asymptote is still at $y = 0$.

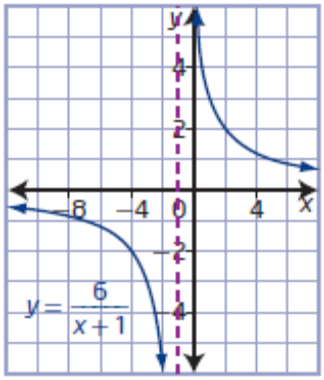


d) The base function for $y = \frac{1}{(x-4)^2}$ is $y = \frac{1}{x^2}$. Compare the function to the form $y = \frac{a}{(x-h)^2} + k$: $a = 1$, $h = 4$, and $k = 0$. The graph of the base function must be translated 4 units to the right. So, the vertical asymptote is at $x = 4$ and the horizontal asymptote is still at $y = 0$.

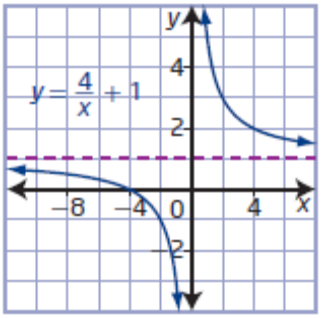


Section 9.1 Page 442 Question 3

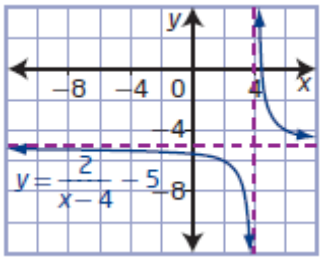
a) For $y = \frac{6}{x+1}$, $a = 6$, $h = -1$, and $k = 0$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 6 and translated 1 unit to the left. The domain is $\{x \mid x \neq -1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$. There is no x-intercept. The y-intercept is 6. The vertical asymptote is at $x = -1$ and the horizontal asymptote is at $y = 0$.



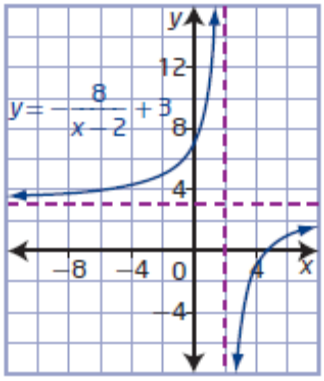
b) For $y = \frac{4}{x} + 1$, $a = 4$, $h = 0$, and $k = 1$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 4 and translated 1 unit up. The domain is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 1, y \in \mathrm{R}\}$. The x-intercept is -4. There is no y-intercept. The vertical asymptote is at $x = 0$ and the horizontal asymptote is at $y = 1$.



c) For $y = \frac{2}{x-4} - 5$, $a = 2$, $h = 4$, and $k = -5$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 2 and translated 4 units to the right and 5 units down. The domain is $\{x \mid x \neq 4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -5, y \in \mathrm{R}\}$. The x-intercept is $\frac{22}{5}$, or 4.4. The y-intercept is $-\frac{11}{2}$, or –5.5. The vertical asymptote is at $x = 4$ and the horizontal asymptote is at $y = -5$.

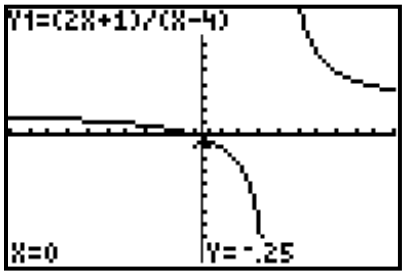


d) For $y = -\frac{8}{x-2} + 3$, $a = -8$, $h = 2$, and $k = 3$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 8, reflected in the x-axis, and translated 2 units to the right and 3 units up. The domain is $\{x \mid x \neq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 3, y \in \mathrm{R}\}$. The x-intercept is $\frac{14}{3}$. The y-intercept is 7. The vertical asymptote is at $x = 2$ and the horizontal asymptote is at $y = 3$.

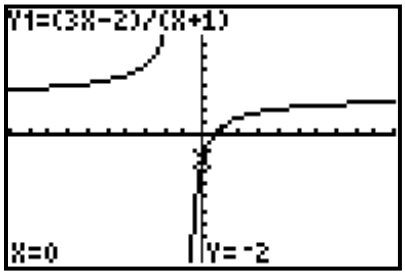


Section 9.1 Page 442 Question 4

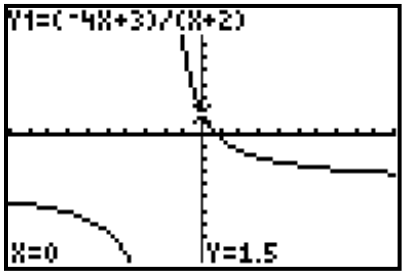
a) For $y = \frac{2x+1}{x-4}$, the vertical asymptote is at $x = 4$, the horizontal asymptote is at $y = 2$, the x-intercept is –0.5, and the y-intercept is –0.25.



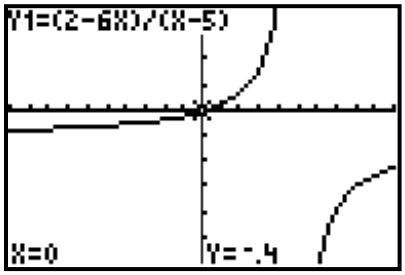
b) For $y = \frac{3x-2}{x+1}$, the vertical asymptote is at $x = -1$, the horizontal asymptote is at $y = 3$, the x-intercept is about 0.67, and the y-intercept is –2.



c) For $y = \dfrac{-4x+3}{x+2}$, the vertical asymptote is at $x = -2$, the horizontal asymptote is at $y = -4$, the x-intercept is 0.75, and the y-intercept is 1.5.



d) For $y = \dfrac{2-6x}{x-5}$, the vertical asymptote is at $x = 5$, the horizontal asymptote is at $y = -6$, the x-intercept is about 0.33, and the y-intercept is –0.4.



Section 9.1 Page 442 Question 5

a) $y = \dfrac{11x+12}{x}$

$= 11 + \dfrac{12}{x}$

$= \dfrac{12}{x} + 11$

For $y = \dfrac{12}{x} + 11$, $a = 12$, $h = 0$, and $k = 11$. The vertical asymptote is at $x = 0$, and the horizontal asymptote is at $y = 11$.

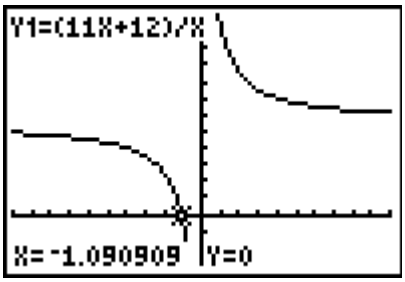
Substitute $y = 0$.

$y = \dfrac{12}{x} + 11$

$0 = \dfrac{12}{x} + 11$

$-11 = \dfrac{12}{x}$

$x = -\dfrac{12}{11}$



The x-intercept is $-\dfrac{12}{11}$, or about –1.09, and there is no y-intercept.

b) $y = \dfrac{x}{x+8}$

$= \dfrac{x+8-8}{x+8}$

$= \dfrac{x+8}{x+8} - \dfrac{8}{x+8}$

$= 1 - \dfrac{8}{x+8}$

$= -\dfrac{8}{x+8} + 1$

For $y = -\dfrac{8}{x+8} + 1$, $a = -8$, $h = -8$, and $k = 1$. The vertical asymptote is at $x = -8$, and the horizontal asymptote is at $y = 1$.

Substitute $y = 0$.

$y = -\dfrac{8}{x+8} + 1$

$0 = -\dfrac{8}{x+8} + 1$

$\dfrac{8}{x+8} = 1$

$x = 0$

Substitute $x = 0$.

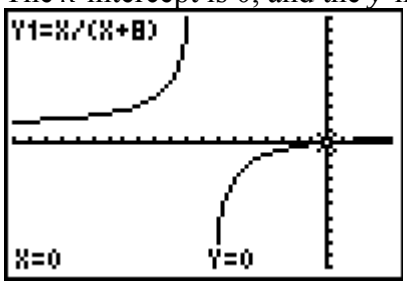
$y = -\dfrac{8}{x+8} + 1$

$= -\dfrac{8}{0+8} + 1$

$= -1 + 1$

$= 0$

The x-intercept is 0, and the y-intercept is 0.



c) $y = \dfrac{-x-2}{x+6}$

$= \dfrac{-x-6+6-2}{x+6}$

$= \dfrac{-(x+6)+4}{x+6}$

$= \dfrac{-(x+6)}{x+6} + \dfrac{4}{x+6}$

$= -1 + \dfrac{4}{x+6}$

For $y = \frac{4}{x+6} - 1$, $a = 4$, $h = -6$, and $k = -1$. The vertical asymptote is at $x = -6$, and the horizontal asymptote is at $y = -1$.

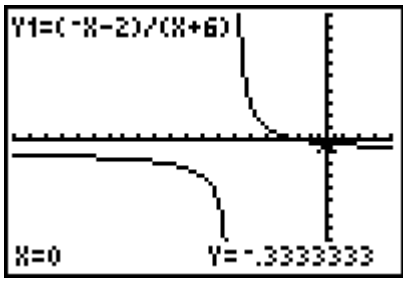
Substitute $y = 0$.

$$y = \frac{4}{x+6} - 1$$
$$0 = \frac{4}{x+6} - 1$$
$$1 = \frac{4}{x+6}$$
$$x + 6 = 4$$
$$x = -2$$

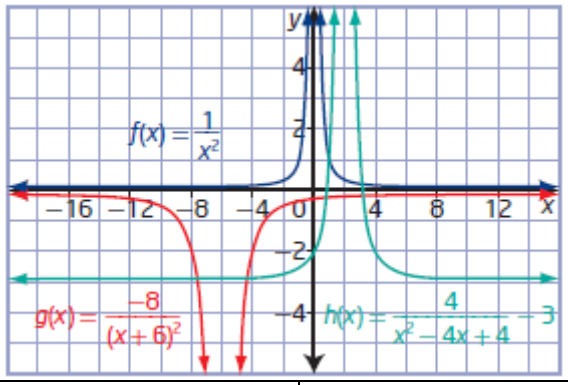
Substitute $x = 0$.

$$y = \frac{4}{x+6} - 1$$
$$= \frac{4}{0+6} - 1$$
$$= \frac{2}{3} - 1$$
$$= -\frac{1}{3}$$

The x-intercept is -2, and the y-intercept is $-\frac{1}{3}$, or about -0.33.



Section 9.1 Page 442 Question 6



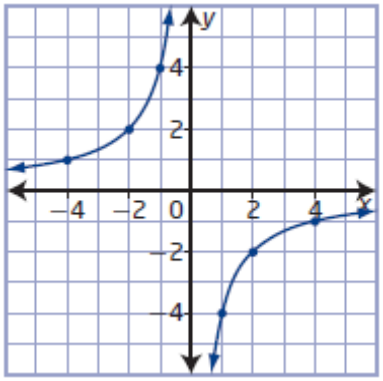
Characteristic	$f(x) = \frac{1}{x^2}$	$g(x) = \frac{-8}{(x+6)^2}$	$h(x) = \frac{4}{x^2-4x+4} - 3$
Non-permissible value	$x = 0$	$x = -6$	$x = -2$
Behaviour near non-permissible value	As x approaches 0, $\lvert y \rvert$ becomes very large.	As x approaches -6, $\lvert y \rvert$ becomes very large.	As x approaches -2, $\lvert y \rvert$ becomes very large.
End behaviour	As $\lvert x \rvert$ becomes very large, y approaches 0.	As $\lvert x \rvert$ becomes very large, y approaches 0.	As $\lvert x \rvert$ becomes very large, y approaches -3.

Domain	$\{x \mid x \neq 0, x \in \mathrm{R}\}$	$\{x \mid x \neq -6, x \in \mathrm{R}\}$	$\{x \mid x \neq -2, x \in \mathrm{R}\}$
Range	$\{y \mid y > 0, y \in \mathrm{R}\}$	$\{y \mid y < 0, y \in \mathrm{R}\}$	$\{y \mid y > -3, y \in \mathrm{R}\}$
Equation of vertical asymptote	$x = 0$	$x = -6$	$x = -2$
Equation of horizontal asymptote	$y = 0$	$y = 0$	$y = -3$

Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consists of all real numbers except for a single value. The range of each function consists of a restricted set of the real numbers. $|y|$ becomes very large for each function when the values of x approach the non-permissible value for the function.

Section 9.1 Page 443 Question 7

a) From the graph, the vertical asymptote is at $x = 0$ and the horizontal asymptote is at $y = 0$. So, $h = 0$ and $k = 0$.



Then, the equation of the function is of the form $y = \frac{a}{x}$.

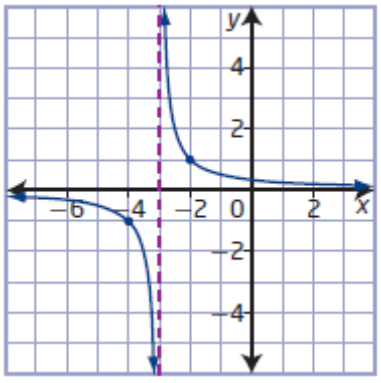
Use one of the given points, say (2, –2), to determine the value of a.

$$-2 = \frac{a}{2}$$

$$a = -4$$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = -\frac{4}{x}$.

b) From the graph, the vertical asymptote is at $x = -3$ and the horizontal asymptote is at $y = 0$. So, $h = -3$ and $k = 0$. Then, the equation of the function is of the form $y = \frac{a}{x+3}$.



Use one of the given points, say (–2, 1), to determine the value of a.

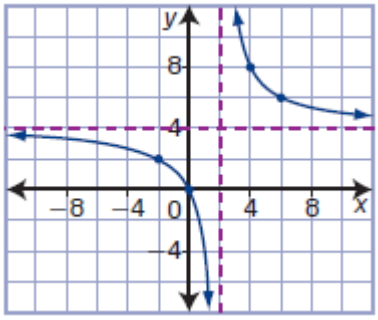
$$1 = \frac{a}{-2+3}$$

$$a = 1$$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = \frac{1}{x+3}$.

c) From the graph, the vertical asymptote is at $x = 2$ and the horizontal asymptote is at $y = 4$. So, $h = 2$ and $k = 4$. Then, the equation of the function is of the form

$y = \frac{a}{x-2} + 4$.



Use one of the given points, say (4, 8), to determine the value of a.

$8 = \frac{a}{4-2} + 4$

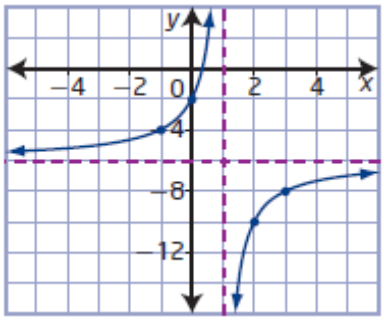
$4 = \frac{a}{2}$

$a = 8$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = \frac{8}{x-2} + 4$.

d) From the graph, the vertical asymptote is at $x = 1$ and the horizontal asymptote is at $y = –6$. So, $h = 1$ and $k = –6$. Then, the equation of the function is of the form

$y = \frac{a}{x-1} - 6$.



Use one of the given points, say (0, –2), to determine the value of a.

$–2 = \frac{a}{0-1} - 6$

$4 = –a$

$a = –4$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = -\frac{4}{x-1} - 6$.

Section 9.1 Page 443 Question 8

a) Given: $y = \frac{a}{x-7} + k$ passing through points (10, 1) and (2, 9)

For (10, 1),

$y = \frac{a}{x-7} + k$

$1 = \frac{a}{10-7} + k$

$1 = \frac{a}{3} + k$

$3 = a + 3k$ ①

For (2, 9),

$y = \frac{a}{x-7} + k$

$9 = \frac{a}{2-7} + k$

$9 = \frac{a}{-5} + k$

$–45 = a – 5k$ ②

Solve the system of equations.

$$\begin{array}{r} 3 = a + 3k \\ -45 = a - 5k \\ \hline 48 = 8k \qquad ① - ② \\ k = 6 \end{array}$$

Substitute $k = 6$ into ①.

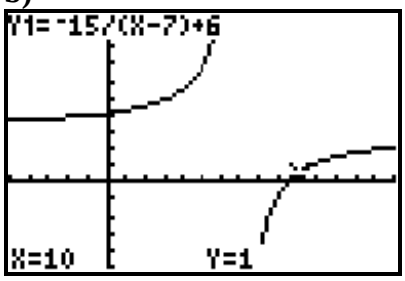
$3 = a + 3k$

$3 = a + 3(6)$

$a = -15$

The equation of the function is $y = -\dfrac{15}{x-7} + 6$.

b)



Section 9.1 Page 443 Question 9

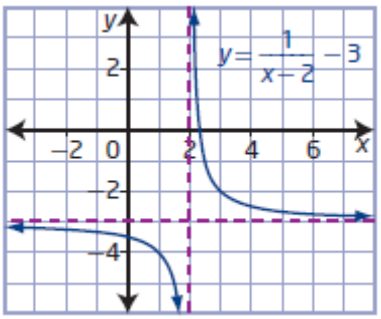
Examples:

a) For asymptotes at $x = 2$ and $y = -3$, $h = 2$ and $k = -3$. Choose $a = 1$, then the equation of the function in the form $y = \dfrac{a}{x-h} + k$ is $y = \dfrac{1}{x-2} - 3$.

Then, rewrite in the form $y = \dfrac{p(x)}{q(x)}$:

$$\begin{aligned} y &= \frac{1}{x-2} - 3 \\ &= \frac{1}{x-2} - \frac{3(x-2)}{x-2} \\ &= \frac{1-3x+2}{x-2} \\ &= \frac{3-3x}{x-2} \end{aligned}$$

b)



The domain is $\{x \mid x \neq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -3, y \in \mathrm{R}\}$.

c) There are many possible functions that meet the given criteria, since any value of a (other than 0) will result in the same equations for the asymptotes.

Section 9.1 Page 443 Question 10

a) In the fourth line, Mira incorrectly factored –3 from $-3x - 21$. She should have grouped $-3x + 21$. The corrected solution is

$$y = \frac{2-3x}{x-7}$$
$$y = \frac{-3x+2}{x-7}$$
$$y = \frac{-3x+21-21+2}{x-7}$$
$$y = \frac{-3(x-7)-19}{x-7}$$
$$y = \frac{-3(x-7)}{x-7} - \frac{19}{x-7}$$
$$y = -3 - \frac{19}{x-7}$$
$$y = -\frac{19}{x-7} - 3$$

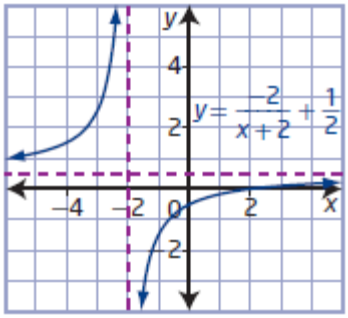
b) Example: Without technology, Mira could have discovered her error by substituting the same value of x into each form of the function. With technology, Mira could have graphed the two functions to see if they were the same.

Section 9.1 Page 443 Question 11

a)

$$y = \frac{x-2}{2x+4}$$
$$y = \frac{x-2}{2(x+2)}$$
$$y = \frac{x+2-4}{2(x+2)}$$
$$y = \frac{x+2}{2(x+2)} - \frac{4}{2(x+2)}$$
$$y = \frac{1}{2} - \frac{2}{x+2}$$
$$y = -\frac{2}{x+2} + \frac{1}{2}$$

b) For $y = -\frac{2}{x+2} + \frac{1}{2}$, $a = -2$, $h = -2$, and $k = \frac{1}{2}$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 2, reflected in the x-axis, and translated 2 units to the left and $\frac{1}{2}$ unit up.



Section 9.1 Page 443 Question 12

Determine the intercepts.

Substitute $y = 0$.

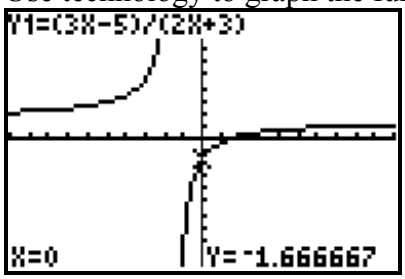
$$y = \frac{3x-5}{2x+3}$$
$$0 = \frac{3x-5}{2x+3}$$
$$0 = 3x - 5$$
$$x = \frac{5}{3}$$

Substitute $x = 0$.

$$y = \frac{3x-5}{2x+3}$$
$$= \frac{3(0)-5}{2(0)+3}$$
$$= -\frac{5}{3}$$

The x-intercept is $\frac{5}{3}$, and the y-intercept is $-\frac{5}{3}$.

Use technology to graph the function.



The asymptotes are located at $x = -\frac{3}{2}$ and $y = \frac{3}{2}$.

Section 9.1 Page 443 Question 13

For the function $N(p) = \frac{500\,000}{p}$, as the value of p increases, the value of N decreases. This means that as the average price of a home increases, the number of buyers looking to buy a home decreases.

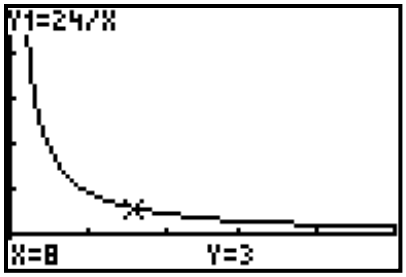
Section 9.1 Page 444 Question 14

a) For a rectangle with constant area of 24 cm^2,

$A = lw$

$24 = lw$

$l = \dfrac{24}{w}$

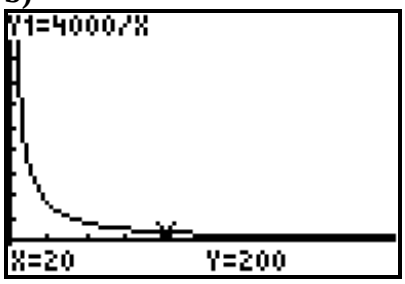


b) As the width increases, the length decreases to maintain an area of 24 cm^2.

Section 9.1 Page 444 Question 15

a) Let x represent the number of students who contribute. Let y represent the average amount required per student to meet the goal. Then, a function to model this situation is $xy = 4000$, or $y = \dfrac{4000}{x}$.

b)



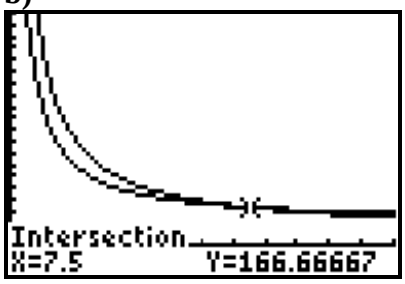
c) As the number of students that contribute increases, the average amount required by student decreases.

d) If the student council also received a $1000 donation from a local business, the function becomes $y = \dfrac{4000}{x} + 1000$. This represents a vertical translation of 1000 units up of the original function graph.

Section 9.1 Page 444 Question 16

a) Let C represent the average cost per year. Let t represent the time, in years. Then, a function to model freezer one is $C = \dfrac{500 + 100t}{t}$ and a function to model freezer two is $C = \dfrac{800 + 60t}{t}$.

b)



c) Both graphs have a vertical asymptote at $x = 0$, but different horizontal asymptotes. The average cost for freezer one approaches \$100/year, while the average cost for freezer two approaches \$60/year. The graph shows that the more years you run the freezer, the less the average cost per year is. Freezer one is cheaper to run for a short amount of time, while freezer two is cheaper if you run it for a longer period of time.

d) The point of intersection of the two graphs can help Hanna decide which model to choose. If Hanna wants to run the freezer for more than 7.5 years, she should choose the second model. Otherwise, she is better off with the first one.

Section 9.1 Page 444 Question 17

a) An equation for the current in the given circuit is $I = \frac{12}{15 + x}$.

b) Since the variable resistor can be set anywhere from 0 Ω to 100 Ω, an appropriate domain is $\{x \mid 0 \leq x \leq 100, x \in \mathrm{R}\}$. The graph does not have a vertical asymptote for this domain.

c) Substitute $I = 0.2$.

$$I = \frac{12}{15 + x}$$

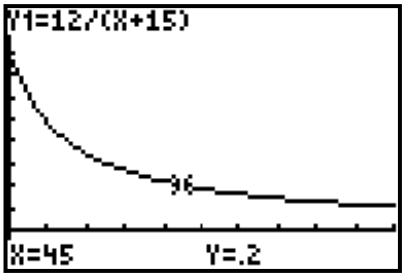
$$0.2 = \frac{12}{15 + x}$$

$$0.2(15 + x) = 12$$

$$15 + x = 60$$

$$x = 45$$

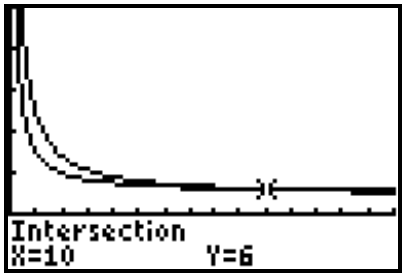
A setting of 45 Ω is required.



d) An equation for the current in the circuit without the bulb is $I = \frac{12}{x}$. The vertical asymptote is at $x = 0$, so the domain must change to $\{x \mid 0 < x \leq 100, x \in \mathrm{R}\}$.

Section 9.1 Page 445 Question 18

a) Let C represent the average cost per hour. Let t represent the rental time, in hours. Then, a function to model renting from store one is $C = \frac{20 + 4t}{t}$ and a function to model renting from store two is $C = \frac{10 + 5t}{t}$.



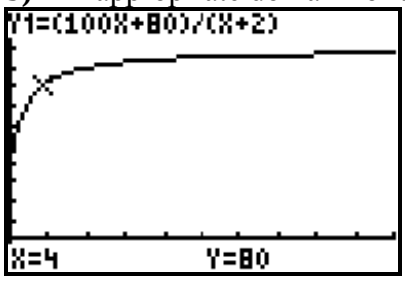
b) Both graphs have a vertical asymptote at $x = 0$, but different horizontal asymptotes. The average cost for renting from store one approaches \$4/h, while the average cost for renting from store two approaches \$5/h. The graph shows that the longer you rent the bike, the less the average cost per hour is. Store two is cheaper to rent for a short amount of time, while store one is cheaper if you rent for a longer period of time.

c) No. The point of intersection of the two graphs indicates that if you rent a bike for less than 10 h, then you should choose store two. Otherwise, choose store one.

Section 9.1 Page 445 Question 19

a) Let v represent the average speed, in kilometres per hour, over the entire trip and t represent the time, in hours, since leaving the construction zone. Then, an equation for v as a function of t for this situation is $v = \frac{80 + 100t}{2 + t}$.

b) An appropriate domain for this situation is $\{t \mid t > 0, t \in \mathrm{R}\}$.



c) The equation of the vertical asymptote is $t = -2$ and the equation of the horizontal asymptote is $v = 100$. The vertical asymptote does not mean anything in this context, since time cannot be negative. The horizontal asymptote means that the average speed gets closer and closer to 100 km/h but never reaches it.

d) Substitute $v = 80$.

$$v = \frac{80 + 100t}{2 + t}$$

$$80 = \frac{80 + 100t}{2 + t}$$

$$80(2 + t) = 80 + 100t$$

$$160 + 80t = 80 + 100t$$

$$-20t = -80$$

$$t = 4$$

The truck will have to drive 4 h after leaving the construction zone before its average speed is 80 km/h.

e) Example: By including an application on a GPS unit that calculates the average speed over the entire trip, a driver could adjust his/her speed accordingly to maintain a target speed that provides the best fuel economy for his/her vehicle.

Section 9.1 Page 445 Question 20

Given: vertical asymptote at $x = 6$, horizontal asymptote at $y = -4$, and x-intercept of -1

For a function of the form $y = \frac{a}{x - h} + k$, $h = 6$ and $k = -4$. Then, the equation becomes $y = \frac{a}{x - 6} - 4$. Use the point $(-1, 0)$ to determine the value of a.

$$0 = \frac{a}{-1 - 6} - 4$$

$$4 = -\frac{a}{7}$$

$$a = -28$$

Rewrite the function $y = -\frac{28}{x - 6} - 4$ in the form $y = \frac{ax + b}{cx + d}$.

$$y = -\frac{28}{x - 6} - 4$$

$$= -\frac{28}{x - 6} - \frac{4(x - 6)}{x - 6}$$

$$= \frac{-28 - 4x + 24}{x - 6}$$

$$= \frac{-4x - 4}{x - 6}$$

Section 9.1 Page 445 Question 21

a)

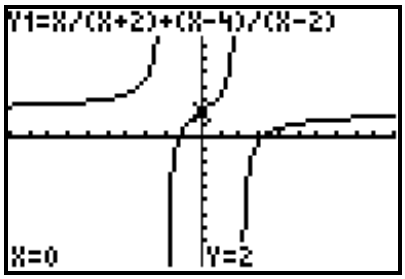
$$f(x) = \frac{x-3}{x+1}$$
$$y = \frac{x-3}{x+1}$$
$$x = \frac{y-3}{y+1}$$
$$x(y+1) = y-3$$
$$xy + x = y-3$$
$$xy - y = -x-3$$
$$y(x-1) = -x-3$$
$$y = \frac{-x-3}{x-1}$$
$$f^{-1}(x) = \frac{-x-3}{x-1}$$

b)

$$f(x) = \frac{2x}{x-5} + 4$$
$$y = \frac{2x}{x-5} + 4$$
$$x = \frac{2y}{y-5} + 4$$
$$(x-4)(y-5) = 2y$$
$$xy - 4y - 5x + 20 = 2y$$
$$xy - 6y = 5x - 20$$
$$y(x-6) = 5x - 20$$
$$y = \frac{5x-20}{x-6}$$
$$f^{-1}(x) = \frac{5x-20}{x-6}$$

Section 9.1 Page 445 Question 22

Use graphing technology to graph $y = \frac{x}{x+2} + \frac{x+4}{x-2}$.



The graph has three branches separated by vertical asymptotes at $x = -2$ and $x = 2$. It also appears to have a horizontal asymptote at $y = 2$ for $x < -2$ and $x > 2$.

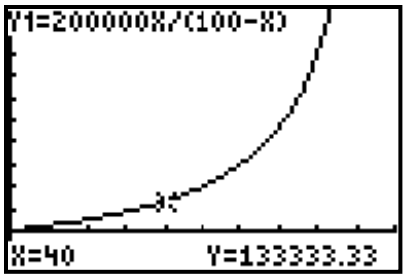
Section 9.1 Page 445 Question C1

Example: If the equation of the rational function is given in the form $y = \frac{a}{x-h} + k$, then using transformations is no different than with any other function. However, if the equation of the function is not in this form, it is more difficult to manipulate the equation to determine the transformations that have been applied.

Section 9.1 Page 445 Question C2

a) For the function $C(p) = \dfrac{200\,000p}{100 - p}$, an appropriate domain is $\{p \mid 0 \leq p < 100, p \in \mathrm{R}\}$. The function is not defined at $p = 100$, meaning that 100% of the emissions can never be eliminated.

b) The shape of the graph indicates that as the percent of emissions eliminated increases, so does the cost.



c) For $p = 80$,

$$C(p) = \frac{200\,000p}{100 - p}$$

$$C(80) = \frac{200\,000(80)}{100 - 80}$$

$$C(80) = 800\,000$$

For $p = 40$,

$$C(p) = \frac{200\,000p}{100 - p}$$

$$C(40) = \frac{200\,000(40)}{100 - 40}$$

$$C(40) = 133\,333.33$$

It costs 6 times as much to eliminate 80% as it does to eliminate 40%.

d) Is it not possible to completely eliminate all of the emissions according to this model because the graph of the function has a vertical asymptote at $p = 100$.

Section 9.1 Page 445 Question C3

For $y = \dfrac{2}{x - 3} + 4$, $a = 2$, $h = 3$, and $k = 4$. For $y = 2\sqrt{x - 3} + 4$, $a = 2$, $h = 3$, and $k = 4$.

Example: Both functions are vertically stretched by a factor of 2 and translated 3 units right and 4 units up. In the case of the rational function, the values of the parameters h and k represent the locations of asymptotes. For the square root function, the point (h, k) gives the location of the endpoint of the graph.

Section 9.2 Analysing Rational Functions

Section 9.2 Page 451 Question 1

a)

Characteristic	$y = \frac{x-4}{x^2-6x+8}$
Non-permissible value(s)	$x = 2, x = 4$
Feature exhibited at each non-permissible value	vertical asymptote, point of discontinuity
Behaviour near each non-permissible value	As x approaches 2, $\lvert y\rvert$ becomes very large. As x approaches 4, y approaches 0.5.
Domain	$\{x \mid x \neq 2, 4, x \in R\}$
Range	$\{y \mid y \neq 0, 0.5, y \in R\}$

b) In factored form, $y = \frac{x-4}{x^2-6x+8}$ is $y = \frac{x-4}{(x-4)(x-2)}$. There is a vertical asymptote at $x = 2$ because $x - 2$ is factor of the denominator only. There is a point of discontinuity at (4, 0.5) because $x - 4$ is a factor of both the numerator and the denominator.

Section 9.2 Page 451 Question 2

Examples:

a) In factored form, $y = \frac{x^2-3x}{x}$ is $y = \frac{x(x-3)}{x}$. The non-permissible value is $x = 0$.

x	y
−1.5	−4.5
−1.0	−4.0
−0.5	−3.5
0.5	−2.5
1.0	−2.0
1.5	−1.5

Since the function does not increase or decrease drastically as x approaches the non-permissible value, it must be a point of discontinuity.

b) In factored form, $y = \frac{x^2-3x-10}{x-2}$ is $y = \frac{(x-5)(x+2)}{x-2}$. The non-permissible value is $x = 2$.

x	y
1.7	40.7
1.8	60.8
1.9	120.9
2.1	−118.9
2.2	−58.8
2.3	−38.7

Since the function changes sign at the non-permissible value and $\lvert y\rvert$ increases, it must be a vertical asymptote.

c) In factored form, $y = \dfrac{3x^2 + 4x - 4}{x + 4}$ is $y = \dfrac{(3x - 2)(x + 2)}{x + 4}$. The non-permissible value is $x = -4$.

x	y
–3.7	74.23
–3.8	120.6
–3.9	260.3
–4.1	–300.3
–4.2	–160.6
–4.3	–114.23

Since the function changes sign at the non-permissible value and $|y|$ increases, it must be a vertical asymptote.

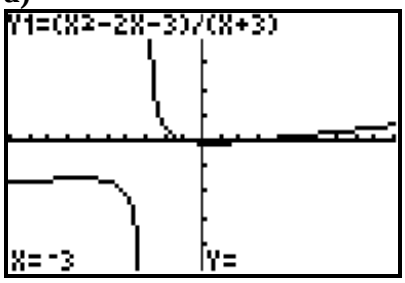
d) In factored form, $y = \dfrac{5x^2 + 4x - 1}{5x - 1}$ is $y = \dfrac{(5x - 1)(x + 1)}{5x - 1}$. The non-permissible value is $x = 0.2$.

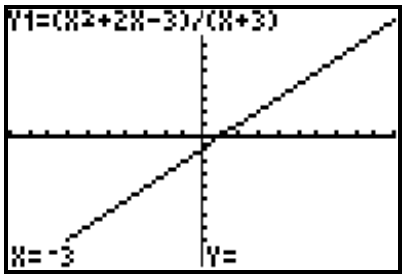
x	y
0.17	1.17
0.18	1.18
0.19	1.19
0.21	1.21
0.22	1.22
0.23	1.23

Since the function does not increase or decrease drastically as x approaches the non-permissible value, it must be a point of discontinuity.

Section 9.2 Page 451 Question 3

a)





Both of the functions have a non-permissible value of –3. However, the graph of $f(x)$ has a vertical asymptote, while the graph of $g(x)$ has a point of discontinuity.

b) In factored form, $f(x) = \dfrac{x^2 - 2x - 3}{x + 3}$ is $f(x) = \dfrac{(x - 3)(x + 1)}{x + 3}$. In factored form $g(x) = \dfrac{x^2 + 2x - 3}{x + 3}$ is $g(x) = \dfrac{(x + 3)(x - 1)}{x + 3}$.

The graph of $f(x)$ has a vertical asymptote at $x = -3$, since $x + 3$ is a factor of only the denominator.

Use the simplified form, $g(x) = x - 1$, $x \neq -3$, to determine the coordinates of the point of discontinuity. Substituting $x = -3$, the graph of $g(x)$ has a point of discontinuity at $(-3, -4)$. This is because $x + 3$ is a factor of both the numerator and the denominator.

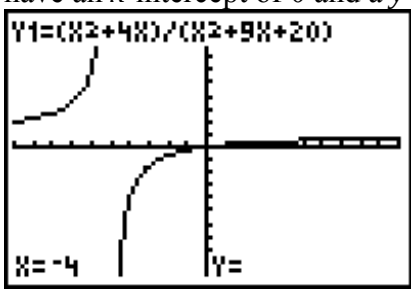
Section 9.2 Page 452 Question 4

a) $y = \dfrac{x^2 + 4x}{x^2 + 9x + 20}$

$= \dfrac{x(x+4)}{(x+4)(x+5)}$

$= \dfrac{x}{x+5}, x \neq -4, 5$

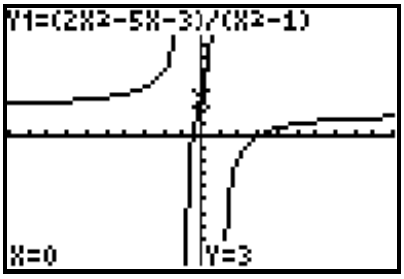
The graph will have a vertical asymptote at $x = -5$. Substituting $x = -4$, the graph will have a point of discontinuity at $(-4, -4)$. By substituting $y = 0$ and $x = 0$, the graph will have an x-intercept of 0 and a y-intercept of 0, respectively.



b) $y = \dfrac{2x^2 - 5x - 3}{x^2 - 1}$

$= \dfrac{(2x+1)(x-3)}{(x+1)(x-1)}, x \neq -1, 1$

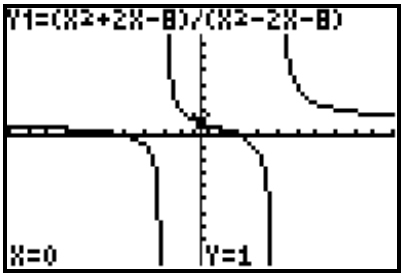
The graph will have vertical asymptotes at $x = -1$ and $x = 1$. The graph will have no points of discontinuity. By substituting $y = 0$ and $x = 0$, the graph will have x-intercepts of -0.5 and 3 and a y-intercept of 3, respectively.



c) $y = \dfrac{x^2 + 2x - 8}{x^2 - 2x - 8}$

$= \dfrac{(x-2)(x+4)}{(x-4)(x+2)}, x \neq -2, 4$

The graph will have vertical asymptotes at $x = -2$ and $x = 4$. The graph will have no points of discontinuity. By substituting $y = 0$ and $x = 0$, the graph will have x-intercepts of -4 and 2 and a y-intercept of 1, respectively.

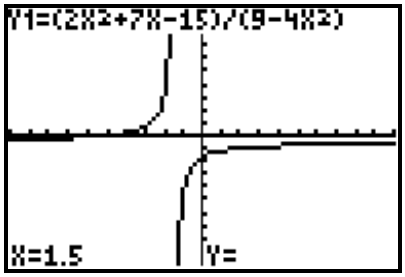


d) $y = \dfrac{2x^2 + 7x - 15}{9 - 4x^2}$

$= \dfrac{(2x-3)(x+5)}{(3-2x)(3+2x)}$

$= \dfrac{(2x-3)(x+5)}{-(2x-3)(2x+3)}$

$= \dfrac{(x+5)}{-(2x+3)}, x \neq -\dfrac{3}{2}, \dfrac{3}{2}$



The graph will have a vertical asymptote at $x = -\dfrac{3}{2}$. Substituting $x = \dfrac{3}{2}$, the graph will have a point of discontinuity at $\left(\dfrac{3}{2}, -\dfrac{13}{12}\right)$, or about (1.5, –1.083). By substituting $y = 0$ and $x = 0$, the graph will have an x-intercept of –5 and a y-intercept of $-\dfrac{5}{3}$, or about –1.67, respectively.

Section 9.2 Page 452 Question 5

a) In factored form, $A(x) = \dfrac{x^2 + 2x}{x^2 + 4}$ is $A(x) = \dfrac{x(x+2)}{x^2 + 4}$. The graph has no vertical asymptotes, no points of discontinuity, and x-intercepts of 0 and –2: graph **C**.

b) In factored form, $B(x) = \dfrac{x-2}{x^2 - 2x}$ is $B(x) = \dfrac{x-2}{x(x-2)}$. The graph has a vertical asymptote at $x = 0$, a point of discontinuity at (2, 0.5), and no x-intercepts: graph **A**.

c) In factored form, $C(x) = \dfrac{x+2}{x^2 - 4}$ is $C(x) = \dfrac{x+2}{(x+2)(x-2)}$. The graph has a vertical asymptote at $x = 2$, a point of discontinuity at (–2, –0.25), and no x-intercepts: graph **D**.

d) In factored form, $D(x) = \dfrac{2x}{x^2 + 2x}$ is $D(x) = \dfrac{2x}{x(x+2)}$. The graph has a vertical asymptote at $x = -2$, a point of discontinuity at (0, 1), and no x-intercepts: graph **B**.

Section 9.2 Page 452 Question 6

a) Since the graph has vertical asymptotes at $x = 1$ and $x = 4$, the denominator has factors $x - 1$ and $x - 4$. Since the graph has x-intercepts of 2 and 3, the numerator has factors $x - 2$ and $x - 3$. Then, the function is of the form

$$y = \frac{(x-2)(x-3)}{(x-1)(x-4)}$$

$$y = \frac{x^2-5x+6}{x^2-5x+4}$$

Choice **C**.

b) Since the graph has vertical asymptotes at $x = -1$ and $x = 2$, the denominator has factors $x + 1$ and $x - 2$. Since the graph has x-intercepts of 1 and 4, the numerator has factors $x - 1$ and $x - 4$. Then, the function is of the form

$$y = \frac{(x-1)(x-4)}{(x+1)(x-2)}$$

$$y = \frac{x^2-5x+4}{x^2-x-2}$$

Choice **B**.

c) Since the graph has vertical asymptotes at $x = -2$ and $x = 5$, the denominator has factors $x + 2$ and $x - 5$. Since the graph has x-intercepts of -4 and 3, the numerator has factors $x + 4$ and $x - 3$. Then, the function is of the form

$$y = \frac{(x+4)(x-3)}{(x+2)(x-5)}$$

$$y = \frac{x^2+x-12}{x^2-3x-10}$$

Choice **D**.

d) Since the graph has vertical asymptotes at $x = -5$ and $x = 4$, the denominator has factors $x + 5$ and $x - 4$. Since the graph has x-intercepts of -2 and 1, the numerator has factors $x + 2$ and $x - 1$. Then, the function is of the form

$$y = \frac{(x+2)(x-1)}{(x+5)(x-4)}$$

$$y = \frac{x^2+x-2}{x^2+x-20}$$

Choice **A**.

Section 9.2 Page 453 Question 7

a) Since the graph has a vertical asymptote at $x = -2$, the denominator has factor $x + 2$. Since the graph has an x-intercept of -6, the numerator has factor $x + 6$. Since the graph has a point of discontinuity at (0, 3), both the numerator and denominator have factor x. Then, the function is of the form

$$y = \frac{x(x+6)}{x(x+2)}$$

$$y = \frac{x^2+6x}{x^2+2x}$$

b) Since the graph has a vertical asymptote at $x = 1$, the denominator has factor $x - 1$. Since the graph has an x-intercept of 7, the numerator has factor $x - 7$. Since the graph has a point of discontinuity at (–3, 2.5), both the numerator and denominator have factor $x + 3$. Then, the function is of the form

$$y = \frac{(x-7)(x+3)}{(x-1)(x+3)}$$

$$y = \frac{x^2 - 4x - 21}{x^2 + 2x - 3}$$

Section 9.2 Page 453 Question 8

a) For vertical asymptotes at $x = \pm 5$ and x-intercepts of –10 and 4,
• the denominator has factors $x + 5$ and $x - 5$
• the numerator has factors $x + 10$ and $x - 4$
Then, the function is of the form $y = \frac{(x+10)(x-4)}{(x+5)(x-5)}$.

b) For a vertical asymptote at $x = -4$, a point of discontinuity at $\left(-\frac{11}{2}, 9\right)$, and an x-intercept of 8,
• the denominator has factor $x + 4$
• both the numerator and denominator have factor $2x + 11$
• the numerator has factor $x - 8$
Then, the function is of the form $y = \frac{(2x+11)(x-8)}{(x+4)(2x+11)}$.

c) For a point of discontinuity at $\left(-2, \frac{1}{5}\right)$ a vertical asymptote at $x = 3$, a, and an x-intercept of –1,
• both the numerator and denominator have factor $x + 2$
• the denominator has factor $x - 3$
• the numerator has factor $x + 1$
Then, the function is of the form $y = \frac{(x+2)(x+1)}{(x+2)(x-3)}$.

d) For vertical asymptotes at $x = 3$ and $x = \frac{6}{7}$, and x-intercepts of $-\frac{1}{4}$ and 0,
• the denominator has factors $x - 3$ and $7x - 6$
• the numerator has factors $4x + 1$ and x
Then, the function is of the form $y = \frac{(4x+1)x}{(x-3)(7x-6)}$.

Section 9.2 Page 453 Question 9

a) $f(x) = \dfrac{x-3}{x^2-5x-6}$

$= \dfrac{x-3}{(x-6)(x+1)}, x \neq -1, 6$

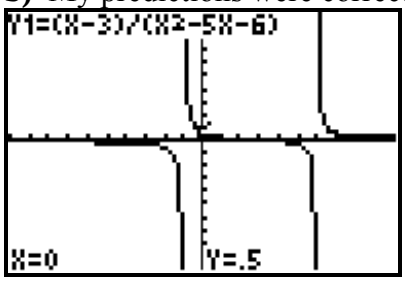
$g(x) = \dfrac{x-3}{x^2-5x+6}$

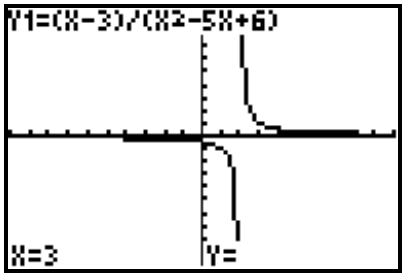
$= \dfrac{x-3}{(x-2)(x-3)}$

$= \dfrac{1}{x-2}, x \neq 2, 3$

The graphs will be different. The graph of $f(x)$ will have two vertical asymptotes, no points of discontinuity, and one x-intercept. The graph of $g(x)$ will have one vertical asymptote, one point of discontinuity, and no x-intercepts.

b) My predictions were correct.





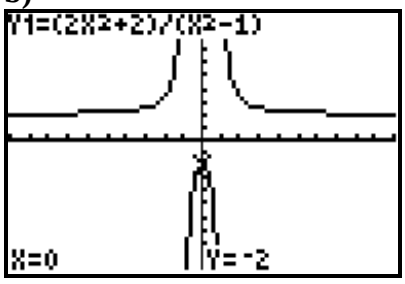
Section 9.2 Page 453 Question 10

Since the graph has points of discontinuity at (–3, –3) and (2, –3), both the numerator and the denominator have factors $x + 3$ and $x – 2$. Then, the function is of the form $y = \dfrac{-3(x+3)(x-2)}{(x+3)(x-2)}$.

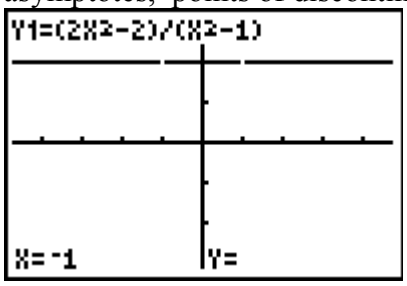
Section 9.2 Page 453 Question 11

a) In factored form, $y = \dfrac{2x^2+2}{x^2-1}$ is $y = \dfrac{2(x^2+1)}{(x-1)(x+1)}$. The graph has vertical asymptotes at $x = 1$ and $x = –1$, no points of discontinuity, and no x-intercepts.

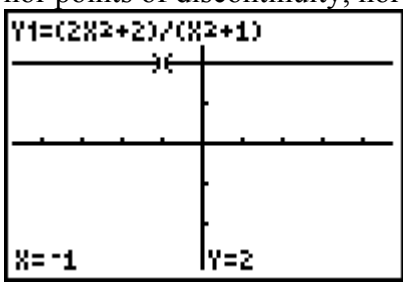
b)



c) i) In factored form, $y = \frac{2x^2 - 2}{x^2 - 1}$ is $y = \frac{2(x-1)(x+1)}{(x-1)(x+1)}$. The graph has no vertical asymptotes, points of discontinuity at (1, 2) and (–1, 2), and no *x*-intercepts.



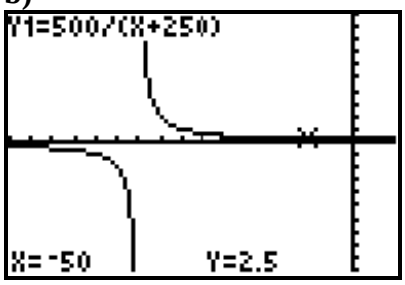
ii) In factored form, $y = \frac{2x^2 + 2}{x^2 + 1}$ is $y = \frac{2(x^2+1)}{(x^2+1)}$. The graph has no vertical asymptotes, nor points of discontinuity, nor *x*-intercepts.



Section 9.2 Page 453 Question 12

a) Let *w* represent the speed of the wind, in kilometres per hour, and *t* represent the time, in hours, then an equation that represents *t* as a function of *w* is $t = \frac{500}{250 + w}$, $w \neq -250$.

b)



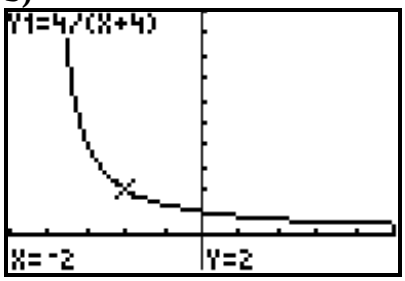
c) When the headwind reaches the speed of the aircraft, theoretically it will come to a standstill, so it will take an infinite amount of time for the aircraft to reach its destination.

d) Example: The realistic part of the graph would be in the range of normal wind speeds for whichever area the aircraft is in.

Section 9.2 Page 454 Question 13

a) The function that relates the time, t, in hours, it will take them to travel 4 km along the channel as a function of the speed, w, in kilometres per hour, of the current is $t = \frac{4}{4+w}$. The domain in this context is $\{w \mid -4 < w \leq 4, w \in \mathrm{R}\}$.

b)

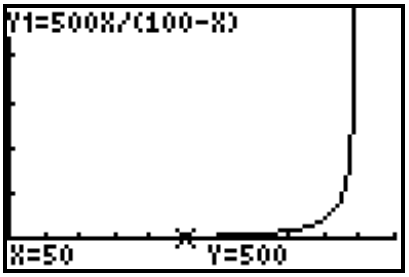


c) When the current against Ryan and Kandra reaches their kayaking speed, theoretically they will come to a standstill, so it will take an infinite amount of time for the them to reach their destination.

Section 9.2 Page 454 Question 14

a) There will be a vertical asymptote at $p = 100$, since $100 - p$ is a factor of the denominator only.

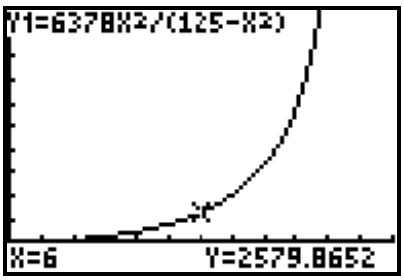
b) The graph shows that the percent of the population vaccinated will never reach 100%.



c) Yes, this is a good model for the estimated cost of vaccinating the population. The vaccination process will become more difficult after the major urban centres have been vaccinated. It will be much more costly to find every single person in the rural areas.

Section 9.2 Page 454 Question 15

a) The portion of the graph applicable is for a domain of $\{v \mid 0 \leq v < \sqrt{125}, v \in \mathrm{R}\}$.



b) As the initial velocity increase, so does the maximum height, but at a greater rate.

c) There will be a vertical asymptote at $v = \sqrt{125}$, since $\sqrt{125} - v$ is a factor of the denominator only. When the initial velocity reaches the non-permissible value, the object will leave Earth and never return. This models the escape velocity.

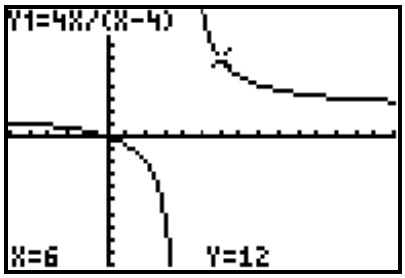
Section 9.2 Page 454 Question 16

Since the graph has vertical asymptotes at $x = -2$ and $x = 3$, the denominator has factors $x + 2$ and $x - 3$. Since the graph has x-intercepts of -6 and 2, the numerator has factors $x + 6$ and $x - 2$. Then, the function is of the form $y = \dfrac{(x+6)(x-2)}{(x+2)(x-3)}$.

Check the y-intercept. Substituting $x = 0$, gives a y-intercept of 2. Since the y-intercept of the graph is -1, the function becomes $y = -\dfrac{(x+6)(x-2)}{2(x+2)(x-3)}$.

Section 9.2 Page 455 Question 17

a) Substitute $f = 4$, then $I = \dfrac{4b}{b-4}$.



b) The image distance decreases while the object distance is still less than the focal length. The image distance starts to increase once the object distance is more than the focal length.

c) The non-permissible value results in a vertical asymptote. This relates to the fact that as the object distance approaches the focal length, it gets harder to resolve the image.

Section 9.2 Page 455 Question 18

a) Example: Functions $f(x)$ and $h(x)$ will have similar graphs since they are the same except for a point of discontinuity in the graph of $h(x)$.

b) All three graphs will have a vertical asymptote at $x = -b$, since $x + b$ is a factor of only the denominators. All three graphs will also have an x-intercept of $-a$, since $x + a$ is a factor of only the numerators.

Section 9.2 Page 455 Question 19

Given: $y = \dfrac{x^2 + bx + c}{4x^2 + 29x + c}$ with point of discontinuity at $\left(-8, \dfrac{11}{35}\right)$

From the point of discontinuity, the corresponding factor $x + 8$ is common to both the numerator and the denominator. Substitute $x = -8$ into the corresponding denominator equation and solve the c.

$$4x^2 + 29x + c = 0$$
$$4(-8)^2 + 29(-8) + c = 0$$
$$c = -24$$

Substitute $x = -8$ and $c = -24$ into the corresponding numerator equation and solve for b.

$$x^2 + bx + c = 0$$
$$(-8)^2 + b(-8) + (-24) = 0$$
$$-8b = -40$$
$$b = 5$$

Then, $y = \dfrac{x^2 + bx + c}{4x^2 + 29x + c}$ becomes $y = \dfrac{x^2 + 5x - 24}{4x^2 + 29x - 24}$. The factored form of $y = \dfrac{x^2 + 5x - 24}{4x^2 + 29x - 24}$ is $y = \dfrac{(x+8)(x-3)}{(x+8)(4x-3)}$. So, the x-intercept is 3 and the vertical asymptote occurs at $x = \dfrac{3}{4}$.

Section 9.2 Page 455 Question 20

For $f(x) = \dfrac{2x^2 - 4x}{x^2 + 3x - 28}$, find $y = \dfrac{1}{4} f(-(x-3))$.

$$f(x) = \frac{2x^2 - 4x}{x^2 + 3x - 28}$$

$$y = \frac{1}{4}\left[\frac{2(-(x-3))^2 - 4(-(x-3))}{(-(x-3))^2 + 3(-(x-3)) - 28}\right]$$

$$= \frac{1}{4}\left[\frac{2(x^2 - 6x + 9) + 4(x-3)}{x^2 - 6x + 9 - 3(x-3) - 28}\right]$$

$$= \frac{1}{4}\left[\frac{2x^2 - 12x + 18 + 4x - 12}{x^2 - 6x + 9 - 3x + 9 - 28}\right]$$

$$= \frac{1}{4}\left[\frac{2x^2 - 8x + 6}{x^2 - 9x - 10}\right]$$

$$= \frac{1}{2}\left[\frac{x^2 - 4x + 3}{x^2 - 9x - 10}\right]$$

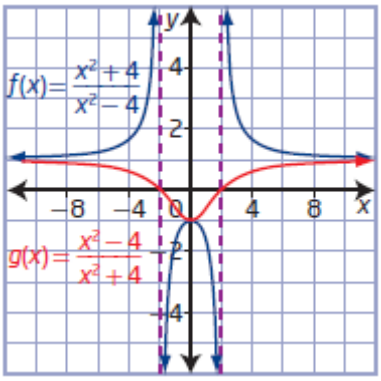
$$= \frac{x^2 - 4x + 3}{2x^2 - 18x - 20}$$

Section 9.2 Page 455 Question 21

a) Since the graph has points of discontinuity at (–4, –2) and (2, 2.5), the corresponding factors $x + 4$ and $x – 2$ are common to both the numerator and the denominator. Ignoring these points, the equation of the linear function is $y = \frac{3}{4}x + 1$ or $y = \frac{1}{4}(3x + 4)$. Putting these together, gives the function $y = \frac{(x+4)(x-2)(3x+4)}{4(x+4)(x-2)}$.

b) Since the graph has points of discontinuity at (–2, 0) and (1, –3), the corresponding factors $x + 2$ and $x – 1$ are common to both the numerator and the denominator. Ignoring these points, the equation of the quadratic function is $y = (x + 2)(x – 2)$. Putting these together, gives the function $y = \frac{(x+2)^2(x-1)(x-2)}{(x+2)(x-1)}$.

Section 9.2 Page 456 Question 22

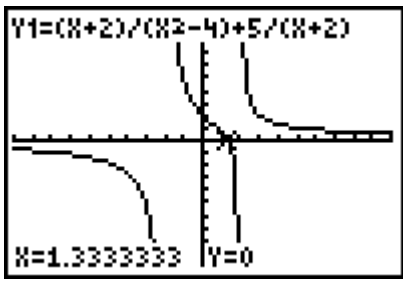


The functions are reciprocals since where the graph of $f(x)$ has vertical asymptotes, the graph of $g(x)$ has x-intercepts.

Section 9.2 Page 456 Question 23

a)
$$y = \frac{x+2}{x^2-4} + \frac{5}{x+2}$$
$$= \frac{x+2}{(x+2)(x-2)} + \frac{5}{x+2}$$
$$= \frac{x+2+5(x-2)}{(x+2)(x-2)}$$
$$= \frac{6x-8}{(x+2)(x-2)}, x \neq \pm 2$$

The graph will have vertical asymptotes at $x = –2$ and $x = 2$ and no points of discontinuity.

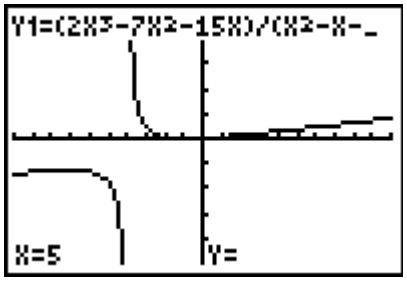


b) $y = \dfrac{2x^3 - 7x^2 - 15x}{x^2 - x - 20}$

$= \dfrac{x(2x+3)(x-5)}{(x-5)(x+4)}$

$= \dfrac{x(2x+3)}{x+4}, x \neq -4, 5$

The graph will have a vertical asymptote at $x = -4$ and a point of discontinuity at $\left(5, \dfrac{65}{9}\right)$.



Section 9.2 Page 456 Question C1

a) Example: No. Some rational functions have no points of discontinuity or asymptotes. For example, $g(x) = \dfrac{x^2 - 4}{x^2 + 4}$ and $h(x) = \dfrac{x^2 + 2x}{x^2 + 4}$ have no vertical asymptotes nor points of discontinuity since there are no values that make the denominator 0 nor factors common between the numerator and denominator.

b) Example: A rational function is a function that has a polynomial in the numerator and/or in the denominator. A factor of only the denominator corresponds to a vertical asymptote. A factor of both the numerator and the denominator corresponds to a point of discontinuity.

Section 9.2 Page 456 Question C2

Example: True. All polynomial functions are rational functions since it is possible to express a polynomial function as a rational function with a denominator of 1.

Section 9.2 Page 456 Question C3

Example: The graphs of rational functions in Section 9.1 all had asymptotes and two branches. The graphs of rational functions in Section 9.2 had
- no branches, two branches, or three branches
- asymptotes only
- points of discontinuity only
- asymptotes and points of discontinuity
- neither asymptotes nor points of discontinuity

Section 9.3 Connecting Graphs and Rational Equations

Section 9.3 Page 465 Question 1

a) The single function that can be used to solve $\frac{x}{x-2}+6=x$ is $y=\frac{x}{x-2}-x+6$: choice **B**.

b) The single function that can be used to solve $6-x=\frac{x}{x-2}+2$ is $y=\frac{x}{x-2}+x-4$: choice **D**.

c) The single function that can be used to solve $6-\frac{x}{x-2}=x-2$ is $y=\frac{x}{x-2}+x-8$: choice **A**.

d) The single function that can be used to solve $x+6=\frac{x}{x-2}$ is $y=\frac{x}{x-2}-x-6$: choice **C**.

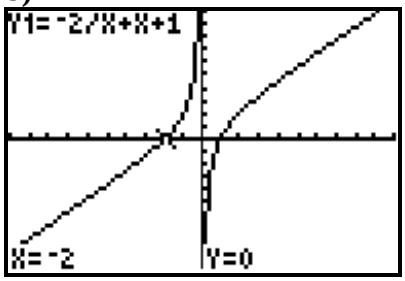
Section 9.3 Page 465 Question 2

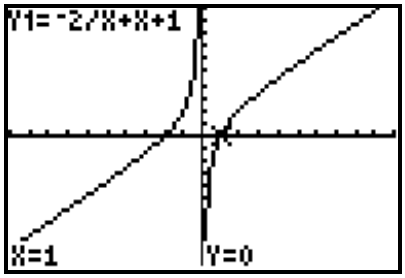
a) The equation has a non-permissible value of 0.

$$-\frac{2}{x}+x+1=0$$
$$-2+x^2+x=0$$
$$x^2+x-2=0$$
$$(x+2)(x-1)=0$$
$$x=-2 \text{ or } x=1$$

By inspection, both values check. The equation has two solutions, $x = –2$ and $x = 1$.

b)





The equation has two solutions, $x = –2$ and $x = 1$.

c) The value of the function is 0 when the value of x is –2 or 1. The x-intercepts of the graph of the corresponding function are the same as the roots of the equation.

Section 9.3 Page 465 Question 3

a) The equation has a non-permissible value of $-\frac{4}{3}$.

$$\frac{5x}{3x+4} = 7$$
$$5x = 7(3x+4)$$
$$5x = 21x + 28$$
$$-16x = 28$$
$$x = -\frac{28}{16}$$
$$x = -\frac{7}{4}$$

b) The equation has a non-permissible value of 0.

$$2 = \frac{20-3x}{x}$$
$$2x = 20 - 3x$$
$$5x = 20$$
$$x = 4$$

c) The equation has a non-permissible value of 2.

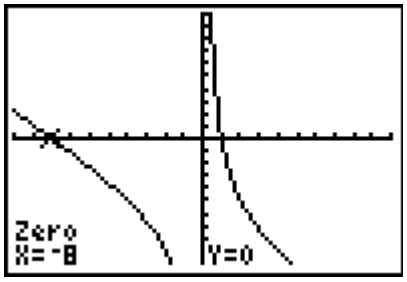
$$\frac{x^2}{x-2} = x - 6$$
$$x^2 = (x-6)(x-2)$$
$$x^2 = x^2 - 8x + 12$$
$$8x = 12$$
$$x = \frac{12}{8}$$
$$x = \frac{3}{2}$$

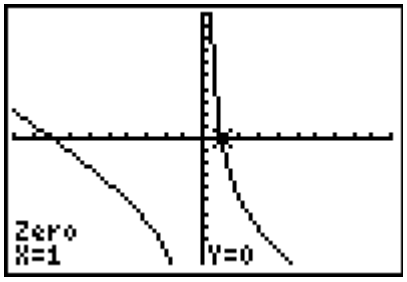
d) The equation has non-permissible values of –3 and 0.

$$1 + \frac{2}{x} = \frac{x}{x+3}$$
$$\frac{x+2}{x} = \frac{x}{x+3}$$
$$x^2 = (x+2)(x+3)$$
$$x^2 = x^2 + 5x + 6$$
$$-5x = 6$$
$$x = -\frac{6}{5}$$

Section 9.3 Page 465 Question 4

a) Graph the single function $y = \frac{8}{x} - x - 7$ and identify the x-intercepts.





The equation has two solutions, $x = -8$ and $x = 1$.
Check:

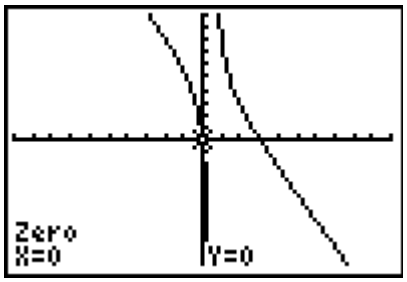
For $x = -8$,

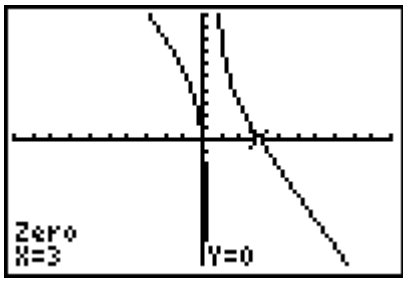
Left Side	Right Side
$\frac{8}{x} - 4$	$x + 3$
$= \frac{8}{-8} - 4$	$= -8 + 3$
$= -5$	$= -5$

For $x = 1$,

Left Side	Right Side
$\frac{8}{x} - 4$	$x + 3$
$= \frac{8}{1} - 4$	$= 1 + 3$
$= 4$	$= 4$

b) Graph the single function $y = \frac{10x}{2x-1} - 2x$ and identify the x-intercepts.





The equation has two solutions, $x = 0$ and $x = 3$.

Check:

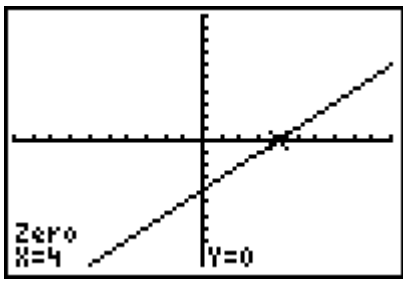
For $x = 0$,

Left Side	Right Side
$2x$	$\frac{10x}{2x-1}$
$= 2(0)$	$= \frac{10(0)}{2(0)-1}$
$= 0$	$= 0$

For $x = 3$,

Left Side	Right Side
$2x$	$\frac{10x}{2x-1}$
$= 2(3)$	$= \frac{10(3)}{2(3)-1}$
$= 6$	$= 6$

c) Graph the single function $y = \frac{3x^2 + 4x - 15}{x+3} - 2x + 1$ and identify the x-intercepts.



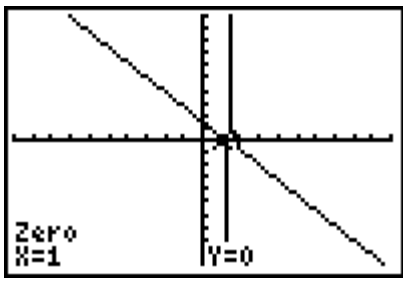
The equation has one solution, $x = 4$.

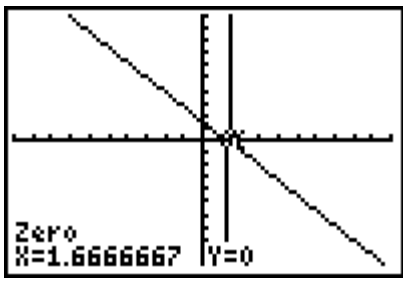
Check:

For $x = 4$,

Left Side	Right Side
$\frac{3x^2 + 4x - 15}{x+3}$	$2x - 1$
$= \frac{3(4)^2 + 4(4) - 15}{4+3}$	$= 2(4) - 1$
$= 7$	$= 7$

d) Graph the single function $y = 1 + \frac{x^2 - 4x}{7 - 5x} - x - \frac{3}{5x - 7}$ and identify the x-intercepts.





The equation has two solutions, $x = 1$ and $x = \frac{5}{3}$.

Check:

For $x = 1$,

Left Side

$$\frac{3}{5x - 7} + x$$

$$= \frac{3}{5(1) - 7} + 1$$

$$= -\frac{1}{2}$$

Right Side

$$1 + \frac{x^2 - 4x}{7 - 5x}$$

$$= 1 + \frac{1^2 - 4(1)}{7 - 5(1)}$$

$$= -\frac{1}{2}$$

For $x = \frac{5}{3}$,

Left Side

$$\frac{3}{5x - 7} + x$$

$$= \frac{3}{5\left(\frac{5}{3}\right) - 7} + \frac{5}{3}$$

$$= \frac{47}{12}$$

Right Side

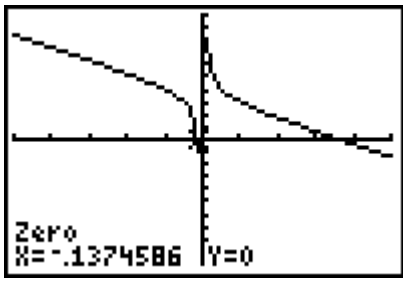
$$1 + \frac{x^2 - 4x}{7 - 5x}$$

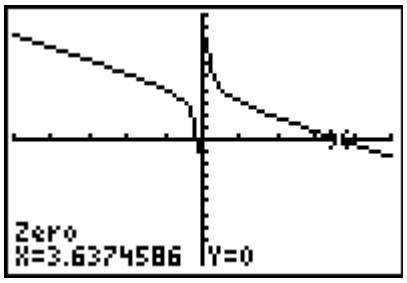
$$= 1 + \frac{\left(\frac{5}{3}\right)^2 - 4\left(\frac{5}{3}\right)}{7 - 5\left(\frac{5}{3}\right)}$$

$$= \frac{47}{12}$$

Section 9.3 Page 466 Question 5

a) Graph the single function $y = \frac{x + 1}{2x} - x + 3$ and identify the x-intercepts.

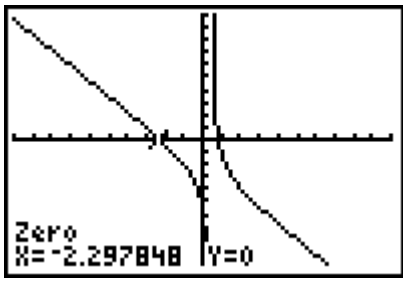


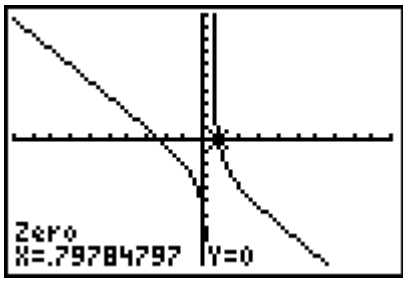


The equation has two solutions, $x \approx -0.14$ and $x \approx 3.64$.

$$\frac{x+1}{2x} = x-3$$
$$x+1 = 2x(x-3)$$
$$x+1 = 2x^2 - 6x$$
$$0 = 2x^2 - 7x - 1$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-7 \pm \sqrt{57}}{4}$$
$x \approx -0.14$ or $x \approx 3.64$

b) Graph the single function $y = \frac{x^2-4x-5}{2-5x} - x - 3$ and identify the x-intercepts.

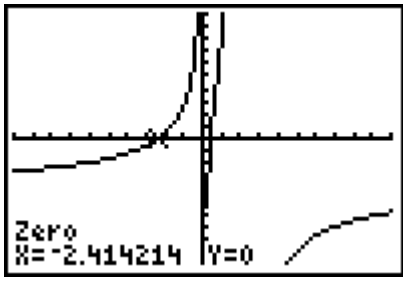


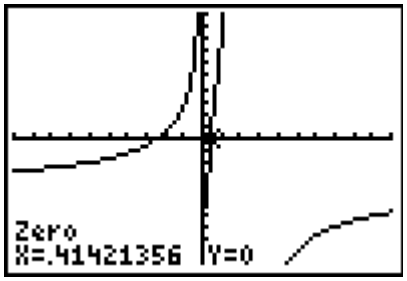


The equation has two solutions, $x \approx -2.30$ and $x \approx 0.80$.

$$\frac{x^2-4x-5}{2-5x} = x+3$$
$$x^2 - 4x - 5 = (x+3)(2-5x)$$
$$x^2 - 4x - 5 = -5x^2 - 13x + 6$$
$$6x^2 + 9x - 11 = 0$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(6)(-11)}}{2(6)}$$
$$= \frac{-9 \pm \sqrt{345}}{12}$$
$x \approx -2.30$ or $x \approx 0.80$

c) Graph the single function $y = 3 - \frac{7x}{x-2} - \frac{2}{x}$ and identify the x-intercepts.



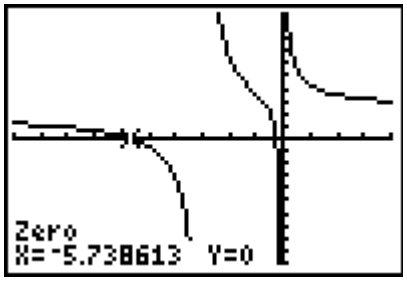


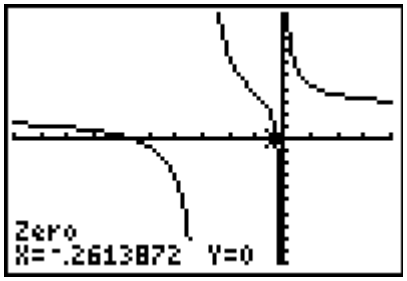
The equation has two solutions, $x \approx -2.41$ and $x \approx 0.41$.

$$\frac{2}{x} = 3 - \frac{7x}{x-2}$$
$$\frac{2}{x} = \frac{-6-4x}{x-2}$$
$$2(x-2) = x(-6-4x)$$
$$2x-4 = -6x-4x^2$$
$$4x^2+8x-4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{8^2-4(4)(-4)}}{2(4)}$$
$$= \frac{-8 \pm \sqrt{128}}{8}$$

$x \approx -2.41$ or $x \approx 0.41$

d) Graph the single function $y = 1 + \frac{5}{x+3} + \frac{x+1}{x}$ and identify the x-intercepts.





The equation has two solutions, $x \approx -5.74$ and $x \approx -0.26$.

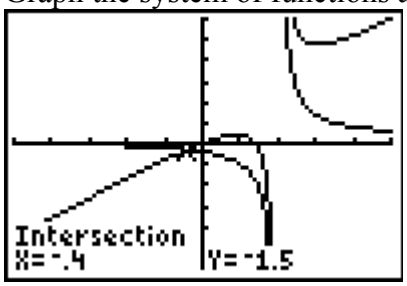
$$2+\frac{5}{x+3}=1-\frac{x+1}{x}$$
$$\frac{2x+11}{x+3}=\frac{-1}{x}$$
$$x(2x+11)=-(x+3)$$
$$2x^2+11x=-x-3$$
$$2x^2+12x+3=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-12\pm\sqrt{12^2-4(2)(3)}}{2(2)}$$
$$=\frac{-12\pm\sqrt{120}}{4}$$
$x\approx -5.74$ or $x=-0.26$

Section 9.3 Page 466 Question 6

a) The equation has a non-permissible value of 2.
$$\frac{3x}{x-2}+5x=\frac{x+4}{x-2}$$
$$\frac{5x^2-7x}{x-2}=\frac{x+4}{x-2}$$
$$5x^2-7x=x+4$$
$$5x^2-8x-4=0$$
$$(5x+2)(x-2)=0$$
$x=-\frac{2}{5}$ or $x=2$

The solution is $x=-\frac{2}{5}$, since $x=2$ is an extraneous root.

Graph the system of functions and find the points of intersection.



The solution is $x=-0.4$, or $-\frac{2}{5}$, which is the same as the answer found algebraically.

b) The equation has a non-permissible value of –4.

$$2x+3=\frac{3x^2+14x+8}{x+4}$$

$$(2x+3)(x+4)=3x^2+14x+8$$

$$2x^2+11x+12=3x^2+14x+8$$

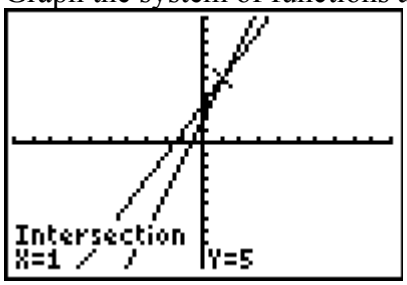
$$0=x^2+3x-4$$

$$0=(x+4)(x-1)$$

$x = –4$ or $x = 1$

The solution is $x = 1$, since $x = –4$ is an extraneous root.

Graph the system of functions and find the points of intersection.



The solution is $x = 1$, which is the same as the answer found algebraically.

c) The equation has a non-permissible value of 3.

$$\frac{6x}{x-3}+3x=\frac{2x^2}{x-3}-5$$

$$\frac{3x^2-3x}{x-3}=\frac{2x^2-5x+15}{x-3}$$

$$3x^2-3x=2x^2-5x+15$$

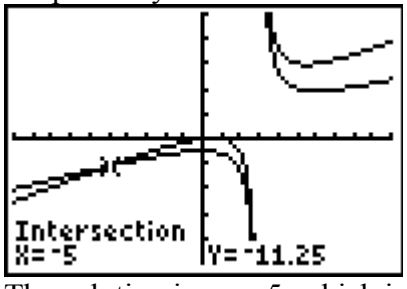
$$x^2+2x-15=0$$

$$(x+5)(x-3)=0$$

$x = –5$ or $x = 3$

The solution is $x = –5$, since $x = 3$ is an extraneous root.

Graph the system of functions and find the points of intersection.



The solution is $x = –5$, which is the same as the answer found algebraically.

d) The equation has non-permissible values of 0 and 1.

$$\frac{2x-1}{x^2-x}+4=\frac{x}{x-1}$$

$$\frac{4x^2-2x-1}{x(x-1)}=\frac{x}{x-1}$$

$$4x^2-2x-1=x^2$$

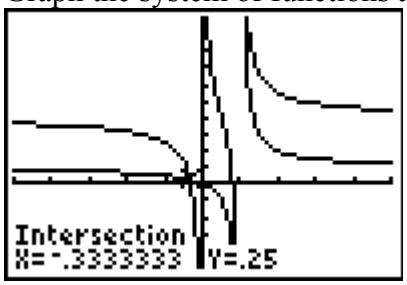
$$3x^2-2x-1=0$$

$$(3x+1)(x-1)=0$$

$x=-\frac{1}{3}$ or $x=1$

The solution is $x=-\frac{1}{3}$, since $x = 1$ is an extraneous root.

Graph the system of functions and find the points of intersection.



The solution is $x=-\frac{1}{3}$, which is the same as the answer found algebraically.

Section 9.3 Page 466 Question 7

Example: Yunah's approach is correct, but she should have indicated that the non-permissible value is 1.

Section 9.3 Page 466 Question 8

Solve algebraically. The equation has non-permissible values of 1 and –2.

$$\frac{2x+1}{x-1}=\frac{2}{x+2}-\frac{3}{2}$$

$$\frac{2x+1}{x-1}=\frac{4-(3x+6)}{2(x+2)}$$

$$2(x+2)(2x+1)=(x-1)(-3x-2)$$

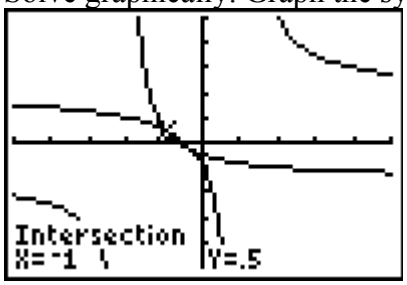
$$4x^2+10x+4=-3x^2+x+2$$

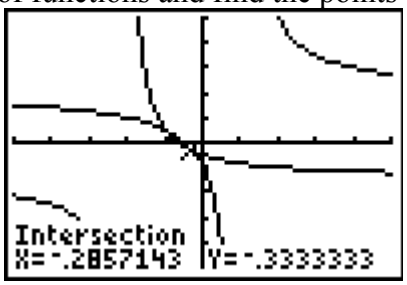
$$7x^2+9x+2=0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(7)(2)}}{2(7)}$$
$$= \frac{-9 \pm \sqrt{25}}{14}$$
$$= \frac{-9 \pm 5}{14}$$

The solution is $x = -\frac{2}{7}$, or about –0.29, and $x = -1$.

Solve graphically. Graph the system of functions and find the points of intersection.





The solution is $x = -1$ and $x \approx -0.29$, which is the same as the solution found algebraically.

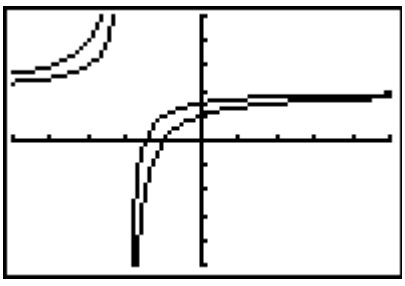
Section 9.3 Page 466 Question 9

Solve algebraically. The equation has non-permissible value is –2.

$$2 - \frac{1}{x+2} = \frac{x}{x+2} + 1$$
$$1 = \frac{x}{x+2} + \frac{1}{x+2}$$
$$1 = \frac{x+1}{x+2}$$

There is no solution.

Solve graphically. Graph the system of functions and find the points of intersection.



The graphs do not intersect, so there is no solution. This is the same as the result found algebraically.

Section 9.3 Page 466 Question 10

Substitute $P = 500$ and $I = 5$ into $I = \frac{P}{4\pi d^2}$.

$$I = \frac{P}{4\pi d^2}$$
$$5 = \frac{500}{4\pi d^2}$$
$$d^2 = \frac{500}{20\pi}$$
$$d^2 = \frac{25}{\pi}$$
$$d = \frac{5}{\sqrt{\pi}}$$
$$d = 2.820...$$

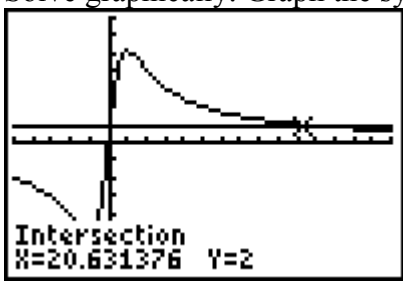
At approximately 2.82 m from a 500-W light source the is intensity 5 W/m^2.

Section 9.3 Page 466 Question 11

Substitute $C = 2$ into $C(t) = \frac{50t}{1.2t^2 + 5}$.

$$C(t) = \frac{50t}{1.2t^2 + 5}$$
$$2 = \frac{50t}{1.2t^2 + 5}$$

Solve graphically. Graph the system of functions and find the points of intersection.



Approximately 20.6 h after drinking coffee the person's level dropped to 2 mg/L.

Section 9.3 Page 467 Question 12

Substitute $T = 10$ and $a = 30$ into $T = \frac{ab}{a+b}$.

$$T = \frac{ab}{a+b}$$
$$10 = \frac{30b}{30+b}$$
$$10(30+b) = 30b$$
$$300 + 10b = 30b$$
$$300 = 20b$$
$$b = 15$$

It would take James 15 min to set it up by himself.

Section 9.3 Page 467 Question 13

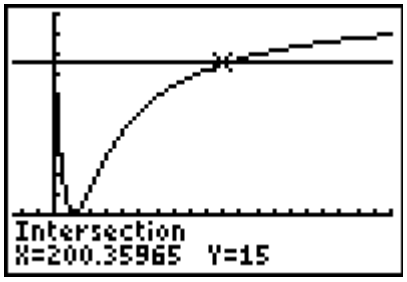
a) Given: 28 shots on net, scored 2 goals
If x represents the number of shots she takes from now on and she scores on half of them, then a function that represents Rachel's shooting percentage is $p = \frac{2+0.5x}{28+x}$.

b) Substitute $p = 0.3$.
$$p = \frac{2+0.5x}{28+x}$$
$$0.3 = \frac{2+0.5x}{28+x}$$
$$0.3(28+x) = 2+0.5x$$
$$6.4 = 0.2x$$
$$x = 32$$

It will take 32 more shots for her to bring her shooting percentage up to her target.

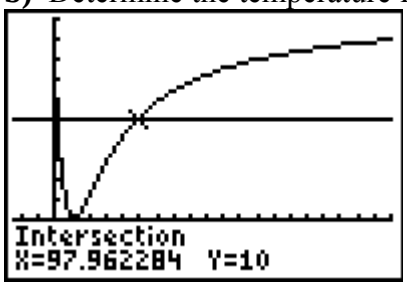
Section 9.3 Page 467 Question 14

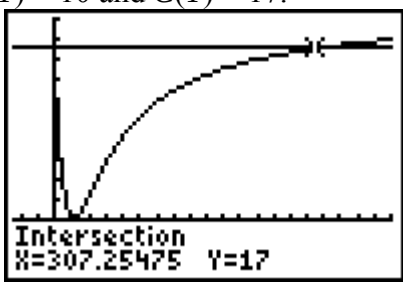
a) Substitute $C(T) = 15$ into $C(T) = \frac{21.2T^2 - 877T + 9150}{T^2 - 23.6T + 760}$. Solve graphically.



According to this model, at a temperature of approximately 200.4 K the coefficient of thermal expansion will be 15.

b) Determine the temperature for $C(T) = 10$ and $C(T) = 17$.





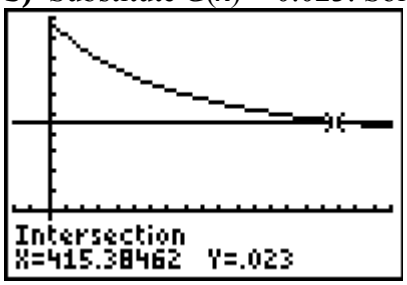
The temperature has to increase about 307.3 – 98.0, or 209.3 K.

Section 9.3 Page 467 Question 15

a) Given: solution A 0.05 g/mL, solution B 0.01 g/mL, start with 200 mL of solution A and pour in x millilitres of solution B

An equation for the concentration, $C(x)$, of the solution after x millilitres have been added is $C(x) = \dfrac{200(0.05) + 0.01x}{200 + x}$ or $C(x) = \dfrac{10 + 0.01x}{200 + x}$.

b) Substitute $C(x) = 0.023$. Solve graphically.

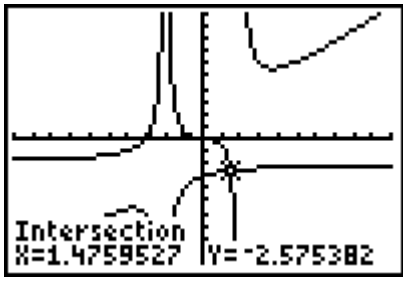


Approximately 415 mL need to be added to make a solution with a concentration of 0.023 g/mL.

Section 9.3 Page 467 Question 16

Solve graphically. Graph the system of functions and find the points of intersection.

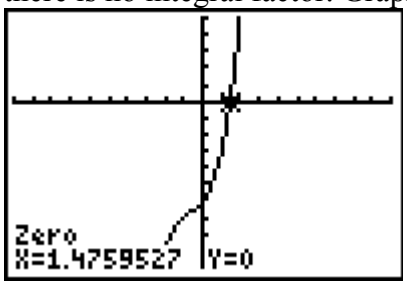
The solution is $x \approx 1.48$.



Solve algebraically. The equation has non-permissible values of –2 and 2.

$$\frac{x}{x+2}-3=\frac{5x}{x^2-4}+x$$
$$x(x-2)-3(x^2-4)=5x+x(x^2-4)$$
$$x^2-2x-3x^2+12=5x+x^3-4x$$
$$0=x^3+2x^2+3x-12$$

Using the factor theorem to test the possible integral factors of ±1, ±2, ±3, ±4, ±6, ±12, there is no integral factor. Graph $y = x^3 + 2x^2 + 3x - 12$ and determine the x-intercept.

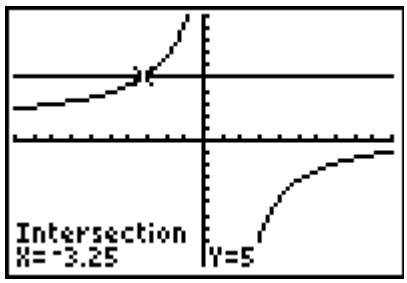


The solution is $x \approx 1.48$.

Section 9.3 Page 467 Question 17

a) $\frac{x-18}{x-1} \le 5$

$$x-18 \le 5(x-1)$$
$$x-18 \le 5x-5$$
$$-4x \le 13$$
$$x \ge -\frac{13}{4}$$

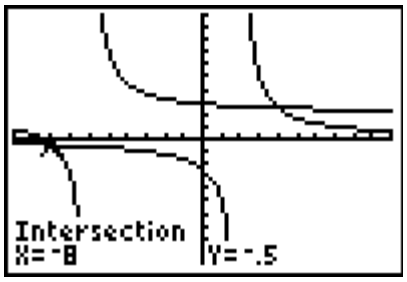


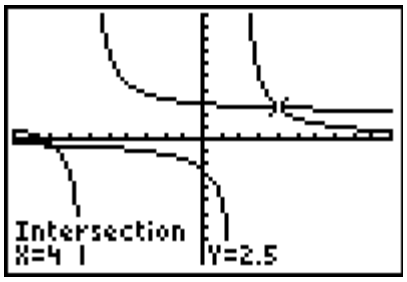
Taking into consideration the non-permissible value of $x = 1$, the solution is $x \le -\frac{13}{4}$ or $x > 1$.

b) $\frac{5}{x-2} \ge \frac{2x+17}{x+6}$

$$5(x+6) \ge (x-2)(2x+17)$$
$$5x+30 \ge 2x^2+13x-34$$
$$0 \ge 2x^2+8x-64$$
$$0 \ge x^2+4x-32$$
$$0 \ge (x+8)(x-4)$$

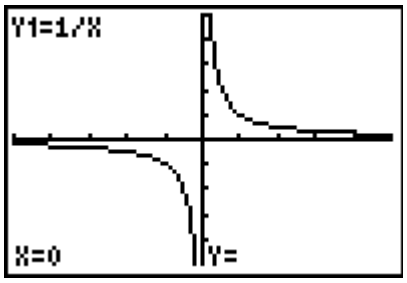
Taking into consideration the non-permissible values of $x = 2$ and $x = -6$, the solution is $-8 \le x < -6$ or $2 < x \le 4$.





Section 9.3 Page 467 Question C1

Example: No, this is incorrect. For example, $\frac{1}{x} = 0$ has no solution.



Section 9.3 Page 467 Question C2

Example: The extraneous root in the radical equation $x = \sqrt{x+6}$ occurs because there is a restriction that the radicand be positive. This same principle of restricted domain is the reason why the rational equation $\frac{3x}{x+2} = x - \frac{6}{x+2}$ has an extraneous root. Extraneous roots can be identified by comparing any solutions with the domain restrictions.

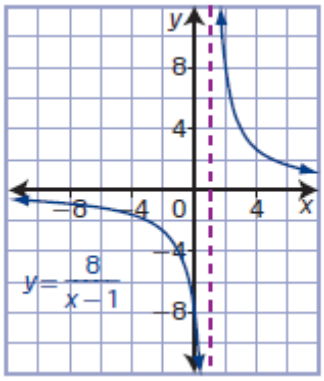
Section 9.3 Page 467 Question C3

Example: I prefer a combination of graphical and algebraic to solve rational equations. The graphical method eliminates any extraneous solutions, but may give only approximate answers. The algebraic method may result in extraneous solutions but can provide exact answers.

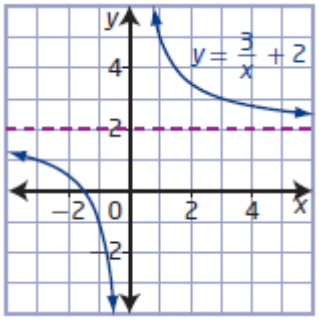
Chapter 9 Review

Chapter 9 Review Page 468 Question 1

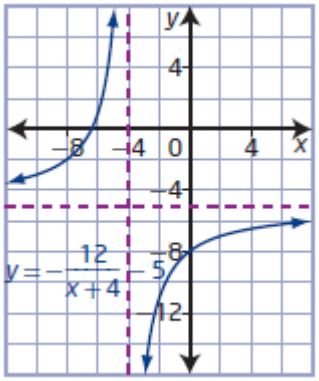
a) For $y = \frac{8}{x-1}$, $a = 8$, $h = 1$, and $k = 0$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 8 and translated 1 unit to the right. The domain is $\{x \mid x \neq 1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$. There is no x-intercept. The y-intercept is –8. The vertical asymptote is at $x = 1$ and the horizontal asymptote is at $y = 0$.



b) For $y = \frac{3}{x} + 2$, $a = 3$, $h = 0$, and $k = 2$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 3 and translated 2 units up. The domain is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 2, y \in \mathrm{R}\}$. The x-intercept is $-\frac{3}{2}$. There is no y-intercept. The vertical asymptote is at $x = 0$ and the horizontal asymptote is at $y = 2$.

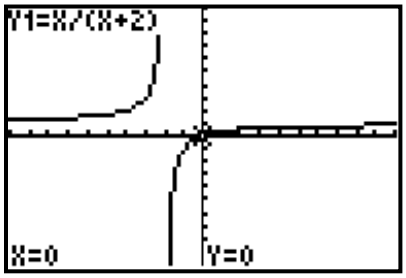


c) For $y = -\frac{12}{x+4} - 5$, $a = -12$, $h = -4$, and $k = -5$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 12, reflected in the x-axis, and translated 4 units to the left and 5 units down. The domain is $\{x \mid x \neq -4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -5, y \in \mathrm{R}\}$. The x-intercept is $-\frac{32}{5}$, or –6.4. The y-intercept is –8. The vertical asymptote is at $x = -4$ and the horizontal asymptote is at $y = -5$.

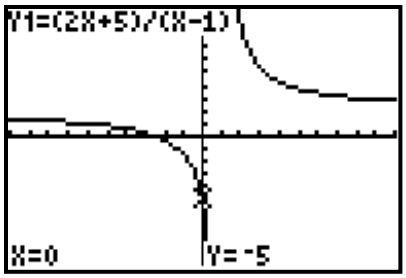


Chapter 9 Review Page 468 Question 2

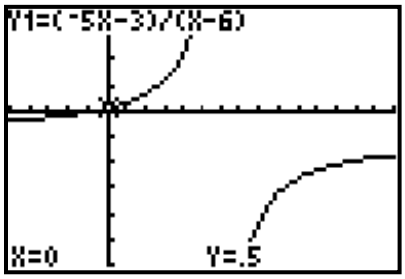
a) For $y = \frac{x}{x+2}$, the vertical asymptote is at $x = -2$, the horizontal asymptote is at $y = 1$, the x-intercept is 0, and the y-intercept is 0.



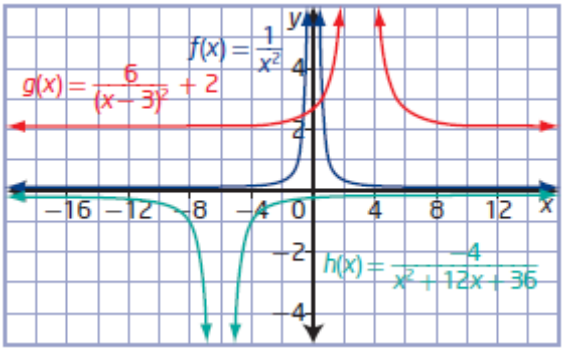
b) For $y = \frac{2x+5}{x-1}$, the vertical asymptote is at $x = 1$, the horizontal asymptote is at $y = 2$, the x-intercept is -2.5, and the y-intercept is -5.



c) For $y = \frac{-5x-3}{x-6}$, the vertical asymptote is at $x = 6$, the horizontal asymptote is at $y = -5$, the x-intercept is -0.6, and the y-intercept is 0.5.



Chapter 9 Review Page 468 Question 3



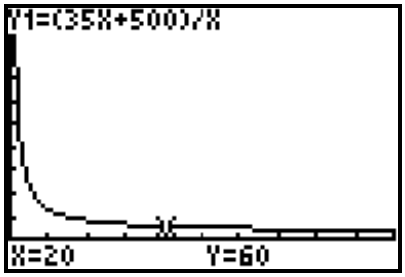
Characteristic	$f(x) = \frac{1}{x^2}$	$g(x) = \frac{6}{(x-3)^2} + 2$	$h(x) = \frac{-4}{x^2+12x+36}$
Non-permissible value	$x = 0$	$x = 3$	$x = -6$

Behaviour near non-permissible value	As x approaches 0, $\|y\|$ becomes very large.	As x approaches 3, $\|y\|$ becomes very large.	As x approaches -6, $\|y\|$ becomes very large.
End behaviour	As $\|x\|$ becomes very large, y approaches 0.	As $\|x\|$ becomes very large, y approaches 2.	As $\|x\|$ becomes very large, y approaches 0.
Domain	$\{x \mid x \neq 0, x \in \mathrm{R}\}$	$\{x \mid x \neq 3, x \in \mathrm{R}\}$	$\{x \mid x \neq -6, x \in \mathrm{R}\}$
Range	$\{y \mid y > 0, y \in \mathrm{R}\}$	$\{y \mid y > 2, y \in \mathrm{R}\}$	$\{y \mid y < 0, y \in \mathrm{R}\}$
Equation of vertical asymptote	$x = 0$	$x = 3$	$x = -6$
Equation of horizontal asymptote	$y = 0$	$y = 2$	$y = 0$

Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consists of all real numbers except for a single value. The range of each function consists of a restricted set of the real numbers. $|y|$ becomes very large for each function when the values of x approach the non-permissible value for the function.

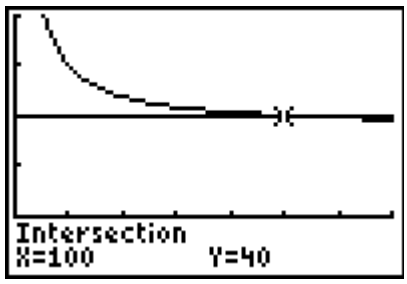
Chapter 9 Review Page 468 Question 4

a) Let x represent the number of uniforms ordered. Let y represent the average cost per uniform. Then, a function to model this situation is $y = \dfrac{500 + 35x}{x}$.



b) As the number of uniforms ordered increases, the average cost per uniform decreases.

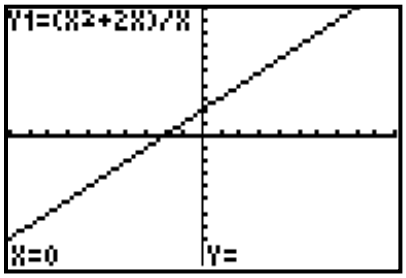
c) Graph $y = \dfrac{500 + 35x}{x}$ and $y = 40$, then determine the point of intersection.



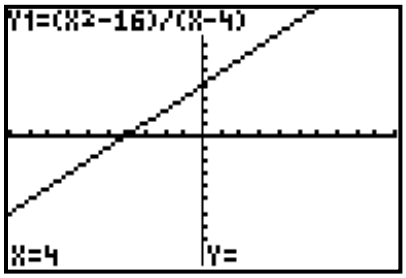
The league needs to order 100 uniforms.

Chapter 9 Review Page 468 Question 5

a) The function $y = \dfrac{x^2 + 2x}{x}$ can be written in factored form as $y = \dfrac{x(x+2)}{x}$. The graph has a point of discontinuity at (0, 2), since x is a factor of both the numerator and the denominator.



b) The function $y = \dfrac{x^2 - 16}{x - 4}$ can be written in factored form as $y = \dfrac{(x-4)(x+4)}{x-4}$. The graph has a point of discontinuity at (4, 8), since $x - 4$ is a factor of both the numerator and the denominator.



c) The function $y = \dfrac{2x^2 - 3x - 5}{2x - 5}$ can be written in factored form as $y = \dfrac{(2x-5)(x+1)}{2x-5}$. The graph has a point of discontinuity at (2.5, 3.5), since $2x - 5$ is a factor of both the numerator and the denominator.

Chapter 9 Review Page 468 Question 6

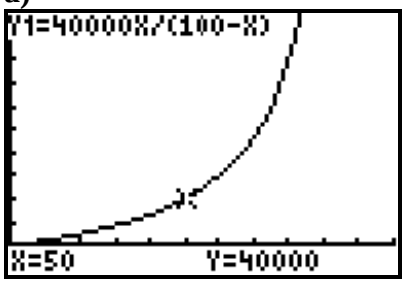
In factored form, $A(x) = \dfrac{x-4}{x^2 - 5x + 4}$ is $A(x) = \dfrac{x-4}{(x-4)(x-1)}$. The graph has a vertical asymptote at $x = 1$, a point of discontinuity at about (4, 0.3), and no x-intercepts: **Graph 3**.

In factored form, $B(x) = \dfrac{x^2 + 5x + 4}{x^2 + 1}$ is $B(x) = \dfrac{(x+4)(x+1)}{x^2+1}$. The graph has no vertical asymptotes, no points of discontinuity, and x-intercepts of –4 and –1: **Graph 1**.

In factored form, $C(x) = \dfrac{x-1}{x^2 - 4}$ is $C(x) = \dfrac{x-1}{(x+2)(x-2)}$. The graph has vertical asymptote at $x = -2$ and $x = 2$, no points of discontinuity, and an x-intercept of 1: **Graph 2**.

Chapter 9 Review Page 469 Question 7

a)



b) As the percent of the spill cleaned up approaches 100, the cost approaches infinity.

c) No, it is not possible to clean up 100% of a spill. The function has a vertical asymptote at $p = 100$.

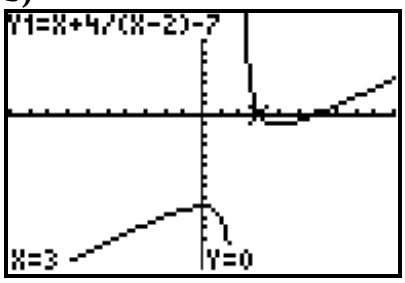
Chapter 9 Review Page 469 Question 8

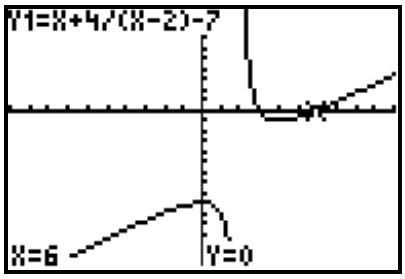
a) The equation has a non-permissible value of 2.

$$x + \frac{4}{x-2} - 7 = 0$$
$$x(x-2) + 4 - 7(x-2) = 0$$
$$x^2 - 9x + 18 = 0$$
$$(x-3)(x-6) = 0$$
$$x = 3 \text{ or } x = 6$$

By inspection, both values check. The equation has two solutions, $x = 3$ and $x = 6$.

b)



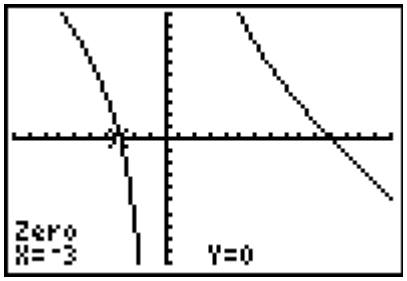


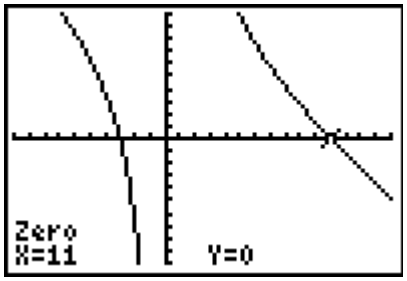
The equation has two solutions, $x = 3$ and $x = 6$.

c) The value of the function is 0 when the value of x is 3 or 6. The x-intercepts of the graph of the corresponding function are the same as the roots of the equation.

Chapter 9 Review Page 469 Question 9

a) Graph the single function $y = \frac{33}{x} - x + 8$ and identify the x-intercepts.

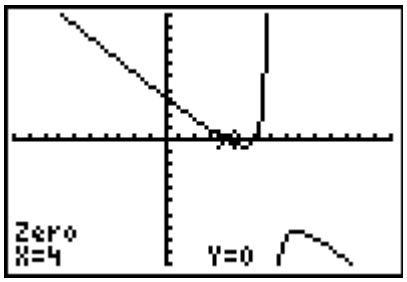


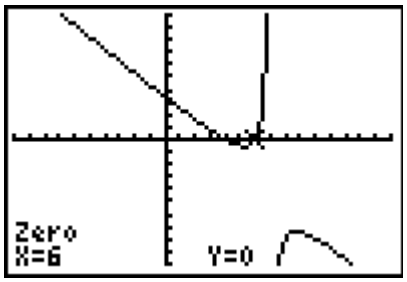


The equation has two solutions, $x = -3$ and $x = 11$.

$$x - 8 = \frac{33}{x}$$
$$x(x-8) = 33$$
$$x^2 - 8x = 33$$
$$x^2 - 8x - 33 = 0$$
$$(x-11)(x+3) = 0$$
$x = -3$ or $x = 11$

b) Graph the single function $y = \frac{x-10}{x-7} - x + 2$ and identify the x-intercepts.

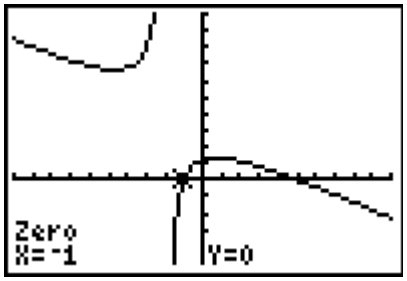


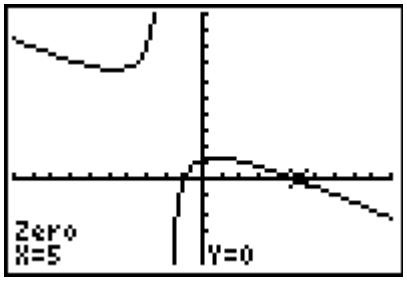


The equation has two solutions, $x = 4$ and $x = 6$.

$$\frac{x-10}{x-7} = x - 2$$
$$x - 10 = (x-2)(x-7)$$
$$x - 10 = x^2 - 9x + 14$$
$$0 = x^2 - 10x + 24$$
$$0 = (x-6)(x-4)$$
$x = 6$ or $x = 4$

c) Graph the single function $y = \frac{3x-1}{x+2} - x + 3$ and identify the x-intercepts.

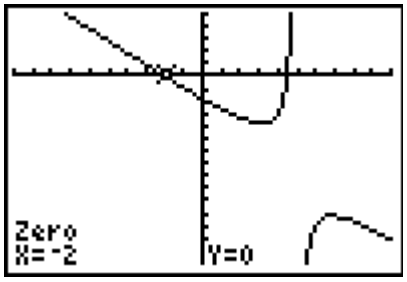


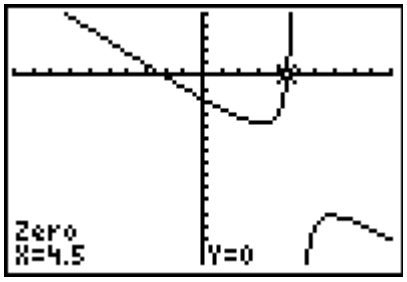


The equation has two solutions, $x = -1$ and $x = 5$.

$$x = \frac{3x-1}{x+2} + 3$$
$$x(x+2) = 3x - 1 + 3(x+2)$$
$$x^2 + 2x = 3x - 1 + 3x + 6$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$x = 5$ or $x = -1$

d) Graph the single function $y = \frac{13-4x}{x-5} - 2x - 1$ and identify the x-intercepts.



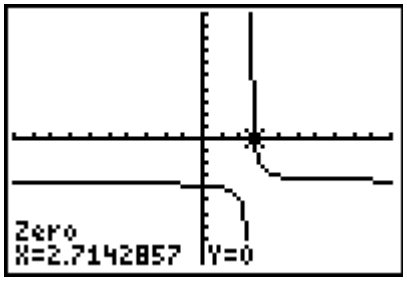


The equation has two solutions, $x = -2$ and $x = 4.5$.

$$2x + 1 = \frac{13-4x}{x-5}$$
$$(2x+1)(x-5) = 13 - 4x$$
$$2x^2 - 9x - 5 = 13 - 4x$$
$$2x^2 - 5x - 18 = 0$$
$$(2x-9)(x+2) = 0$$
$x = 4.5$ or $x = -2$

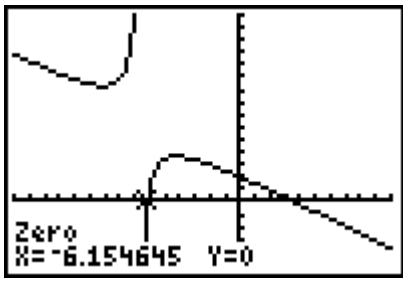
Chapter 9 Review Page 469 Question 10

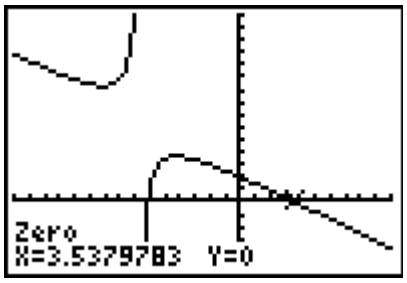
a) Graph the single function $y = \frac{x-4}{5-2x} - 3$ and identify the x-intercepts.



The equation has one solution, $x \approx 2.71$.

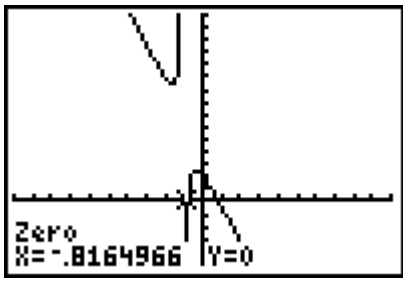
b) Graph the single function $y = \frac{x}{x+6.7} - 1.2x + 3.9$ and identify the x-intercepts.

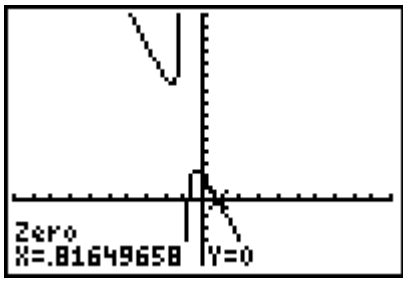




The equation has two solutions, $x \approx -6.15$ and $x \approx 3.54$.

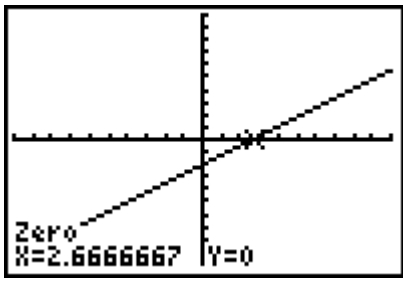
c) Graph the single function $y = \frac{5x+4}{x+1} - 3x - 2$ and identify the x-intercepts.





The equation has two solutions, $x \approx -0.82$ and $x \approx 0.82$.

d) Graph the single function $y = \frac{x^2-2x-8}{x+2} - \frac{1}{4}x + 2$ and identify the x-intercepts.

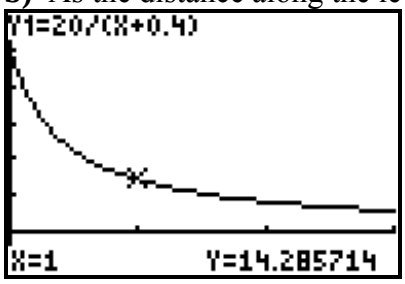


The equation has one solution, $x \approx 2.67$.

Chapter 9 Review Page 469 Question 11

a) The domain is $\{d \mid -0.4 < d \leq 2.6, d \in \mathrm{R}\}$.

b) As the distance along the lever increases, less mass can be lifted.



c) The function has a vertical asymptote at its non-permissible value, $d = -0.4$. Since this is outside the domain, it has no effect on this situation.

The non-permissible value corresponds to the fulcrum point, which does not move when the lever is moved. As the mass gets closer to the fulcrum, it is possible to move a much heavier mass, but when the mass is on the fulcrum, it cannot be moved.

d) Substitute $m = 17.5$.

$$m = \frac{20}{d + 0.4}$$

$$17.5 = \frac{20}{d + 0.4}$$

$$17.5(d + 0.4) = 20$$

$$17.5d = 13$$

$$d = 0.742...$$

The lever can support a maximum possible mass of 17.5 kg at a distance of approximately 0.74 m from the fulcrum.

Chapter 9 Practice Test

Chapter 9 Practice Test Page 470 Question 1

A vertical asymptote at $x = 2$ corresponds to a factor of $x - 2$ in the denominator of the rational function: choice **C**.

Chapter 9 Practice Test Page 470 Question 2

The function $y = \frac{x^2 - 2x}{x^2 - 5x + 6}$ can be written as $y = \frac{x(x-2)}{(x-2)(x-3)}$. The graph will have a vertical asymptote at $x = 3$, a point of discontinuity at $(2, -2)$, and an x-intercept of 0: choice **D**.

Chapter 9 Practice Test Page 470 Question 3

The function $y = -\frac{4}{x-2}$ has a vertical asymptote at $x = 2$. So, as x approaches 2, $|y|$ becomes very large: choice **C**.

Chapter 9 Practice Test Page 470 Question 4

The roots of $5 - x = \frac{x+2}{2x-3}$ can be found by determining the x-intercepts of the graph of $y = \frac{x+2}{2x-3} + x - 5$: choice **B**.

Chapter 9 Practice Test Page 470 Question 5

$$\begin{aligned} y &= \frac{6x-5}{x+7} \\ &= \frac{6x+42-42-5}{x+7} \\ &= \frac{6(x+7)-47}{x+7} \\ &= \frac{6(x+7)}{x+7} - \frac{47}{x+7} \\ &= 6 - \frac{47}{x+7} \\ &= -\frac{47}{x+7} + 6 \end{aligned}$$

Choice **D**.

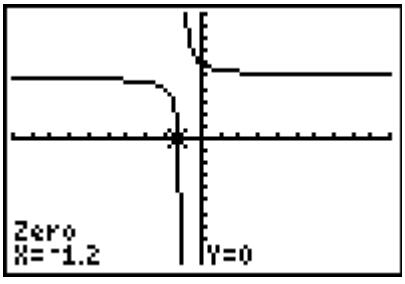
Chapter 9 Practice Test Page 470 Question 6

The graph of the function $y = \frac{x}{x^2 - x}$, or $y = \frac{x}{x(x-1)}$, will have a vertical asymptote at $x = 1$, a point of discontinuity at (0, –1), no intercepts. Choice **C**.

Chapter 9 Practice Test Page 470 Question 7

Graph the single function $y = \frac{x^2}{x+1} - x + 6$ and identify the x-intercepts.

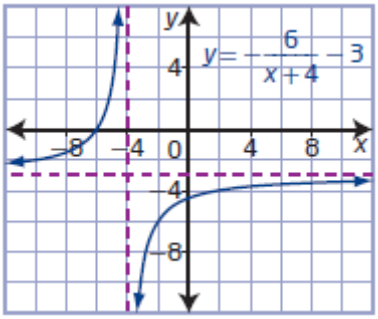
The equation has one solution, $x = -1.2$.



$$x-6=\frac{x^2}{x+1}$$
$$(x-6)(x+1)=x^2$$
$$x^2-5x-6=x^2$$
$$-5x=6$$
$$x=-\frac{6}{5} \quad \text{or } -1.2$$

Chapter 9 Practice Test Page 470 Question 8

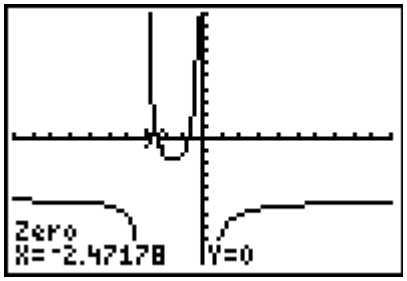
a) For $y=-\frac{6}{x+4}-3$, $a=-6$, $h=-4$, and $k=-3$. The graph of the base function $y=\frac{1}{x}$ is vertically stretched by a factor of 6, reflected in the x-axis, and translated 4 units to the left and 3 units down.

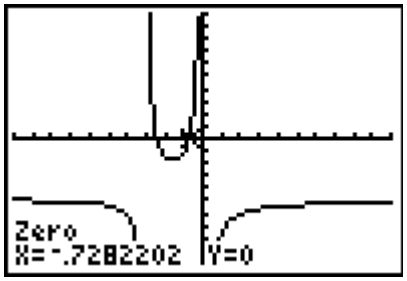


b) The domain is $\{x \mid x \neq -4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -3, y \in \mathrm{R}\}$. The vertical asymptote is at $x=-4$ and the horizontal asymptote is at $y=-3$.The x-intercept is -6. The y-intercept is -4.5.

Chapter 9 Practice Test Page 470 Question 9

Graph the single function $y=\frac{2}{x+3}-\frac{3}{x}-5$ and identify the x-intercepts.





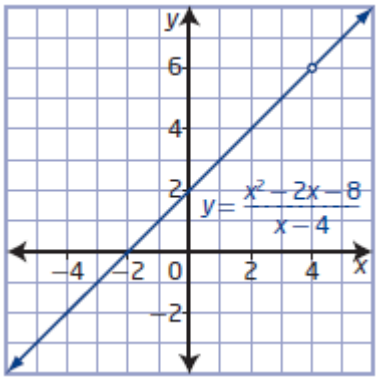
The equation has two solutions, $x \approx -2.47$ and $x \approx -0.73$.

Chapter 9 Practice Test Page 470 Question 10

a) Simplify the function.

$$y = \frac{x^2 - 2x - 8}{x - 4}$$
$$= \frac{(x-4)(x+2)}{x-4}$$
$$= x + 2, x \neq 4$$

The graph has a point of discontinuity at (4, 6).



b) As x approaches 4, y approaches 6.

Chapter 9 Practice Test Page 471 Question 11

$$y = \frac{2x^2 + 7x - 4}{x^2 + x - 12}$$
$$= \frac{(2x-1)(x+4)}{(x+4)(x-3)}$$
$$= \frac{2x-1}{x-3}, x \neq -4, 3$$

The graph will have a vertical asymptote at $x = 3$. Substituting $x = -4$, the graph will have a point of discontinuity at $\left(-4, \frac{9}{7}\right)$. By substituting $y = 0$ and $x = 0$, the graph will have an x-intercept of $\frac{1}{2}$ and a y-intercept of $\frac{1}{3}$, respectively.

Chapter 9 Practice Test Page 471 Question 12

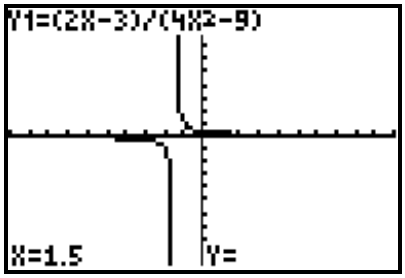
a) In factored form, $A(x) = \frac{x^2 - 9x}{x}$ is $A(x) = \frac{x(x-9)}{x}$. The graph has no vertical asymptotes, a point of discontinuity at (0, –9), and an x-intercept of 9: graph **D**.

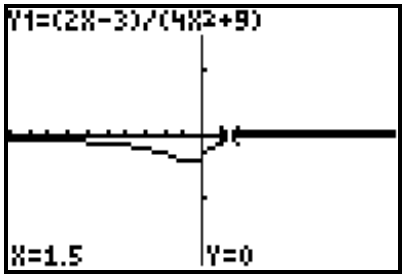
b) In factored form, $B(x) = \frac{x^2}{x^2 - 9}$ is $B(x) = \frac{x^2}{(x+3)(x-3)}$. The graph has a vertical asymptote at $x = \pm 3$, no points of discontinuity, and an x-intercept of 0: graph **A**.

c) In factored form, $C(x) = \frac{x^2 - 9}{x^2}$ is $C(x) = \frac{(x+3)(x-3)}{x^2}$. The graph has a vertical asymptote at $x = 0$, no points of discontinuity, and x-intercepts of –3 and 3: graph **B**.

d) In factored form, $D(x) = \dfrac{x^2}{x^2 - 9x}$ is $D(x) = \dfrac{x^2}{x(x-9)}$. The graph has a vertical asymptote at $x = 9$, a point of discontinuity at (0, 0), and no x-intercepts: graph **C**.

Chapter 9 Practice Test Page 471 Question 13





In factored form, $f(x) = \dfrac{2x-3}{4x^2-9}$ is $f(x) = \dfrac{2x-3}{(2x-3)(2x+3)}$. The function $g(x) = \dfrac{2x-3}{4x^2+9}$ cannot be factored.

The graph of $f(x)$ has a vertical asymptote at $x = -\dfrac{3}{2}$, since $2x + 3$ is a factor of only the denominator. The graph of $f(x)$ has a point of discontinuity at $\left(\dfrac{3}{2}, \dfrac{1}{6}\right)$. This is because $2x - 3$ is a factor of both the numerator and the denominator.
Since the denominator of $g(x)$ will never equal 0, the graph of $g(x)$ has no vertical asymptotes. Since there are no factors common to the numerator and denominator, the graph of $g(x)$ also has no points of discontinuity.

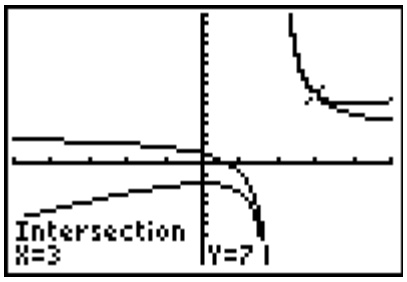
Chapter 9 Practice Test Page 471 Question 14

a) The equation has a non-permissible value of 2.

$$\frac{x^2}{x-2} - 2 = \frac{3x-2}{x-2}$$
$$x^2 - 2(x-2) = 3x-2$$
$$x^2 - 2x + 4 = 3x - 2$$
$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$

$x = 2$ or $x = 3$
The solution is $x = 3$, since $x = 2$ is an extraneous root.
Alex should have excluded the extraneous root.

b) Graph the system of functions and find the points of intersection.

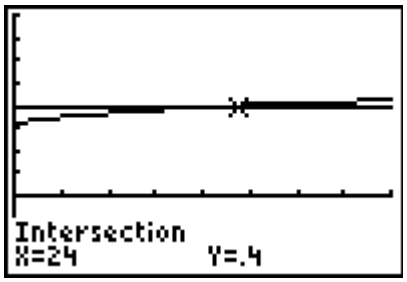


The solution is $x = 3$. The graphical method eliminates extraneous roots.

Chapter 9 Practice Test Page 471 Question 15

a) Given: 31 putts, 10 successes
If x represents the number of putts Jennifer takes from now on and she is successful on half of them, then a function that represents her average putting success rate is $A = \frac{10 + 0.5x}{31 + x}$.

b) Graph the functions $A = \frac{10 + 0.5x}{31 + x}$ and $A = 0.40$ and find the point of intersection.

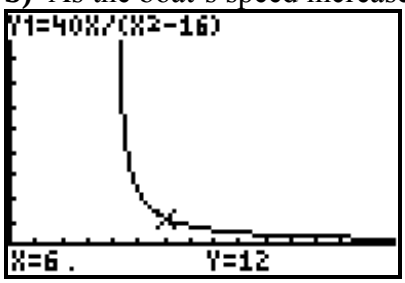


It will take 24 more putts for Jennifer to bring her average putting success rate up to her target.

Chapter 9 Practice Test Page 471 Question 16

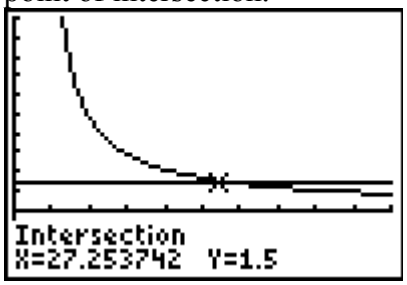
a) The domain is $\{v \mid v > 4, v \in \mathrm{R}\}$, since speed (and time) must be positive and $v = 4$ is a non-permissible value.

b) As the boat's speed increases, the total time for the round trip decreases.



c) As the boat's speed approaches 4 km/h, the time it takes for a round trip approaches infinity. The water flows at 4 km/h. If the boat's speed is less, the boat will never make the return trip, which is why there is an asymptote at $x = 4$.

d) The round trip will take 90 min, or 1.5 h. Graph $t = \frac{40v}{v^2 - 16}$ and $t = 1.5$ and find the point of intersection.



The boat speed needed is approximately 27.25 km/h.

Chapter 10 Function Operations

Section 10.1 Sums and Differences of Functions

Section 10.1 Page 483 Question 1

a) For $f(x) = |x + 3|$ and $g(x) = 4$,
$h(x) = f(x) + g(x)$
$= |x + 3| + 4$

b) For $f(x) = 3x - 5$ and $g(x) = -x + 2$,
$h(x) = f(x) + g(x)$
$= 3x - 5 + (-x + 2)$
$= 3x - 5 - x + 2$
$= 2x - 3$

c) For $f(x) = x^2 + 2x$ and $g(x) = x^2 + x + 2$,
$h(x) = f(x) + g(x)$
$= x^2 + 2x + x^2 + x + 2$
$= 2x^2 + 3x + 2$

d) For $f(x) = -x - 5$ and $g(x) = (x + 3)^2$,
$h(x) = f(x) + g(x)$
$= -x - 5 + (x + 3)^2$
$= -x - 5 + x^2 + 6x + 9$
$= x^2 + 5x + 4$

Section 10.1 Page 483 Question 2

a) For $f(x) = 6x$ and $g(x) = x - 2$,
$h(x) = f(x) - g(x)$
$= 6x - (x - 2)$
$= 6x - x + 2$
$= 5x + 2$

b) For $f(x) = -3x + 7$ and $g(x) = 3x^2 + x - 2$,
$h(x) = f(x) - g(x)$
$= -3x + 7 - (3x^2 + x - 2)$
$= -3x + 7 - 3x^2 - x + 2$
$= -3x^2 - 4x + 9$

c) For $f(x) = 6 - x$ and $g(x) = (x + 1)^2 - 7$,
$h(x) = f(x) - g(x)$
$= 6 - x - ((x + 1)^2 - 7)$
$= 6 - x - (x^2 + 2x + 1 - 7)$
$= 6 - x - x^2 - 2x + 6$
$= -x^2 - 3x + 12$

d) For $f(x) = \cos x$ and $g(x) = 4$,
$h(x) = f(x) - g(x)$
$= \cos x - 4$

Section 10.1 Page 483 Question 3

For $f(x) = -6x + 1$ and $g(x) = x^2$,

a) $h(x) = f(x) + g(x)$
$= -6x + 1 + x^2$
$= x^2 - 6x + 1$
$h(2) = 2^2 - 6(2) + 1$
$= 4 - 12 + 1$
$= -7$

b) $m(x) = f(x) - g(x)$
$= -6x + 1 - x^2$
$= -x^2 - 6x + 1$
$m(1) = -1^2 - 6(1) + 1$
$= -1 - 6 + 1$
$= -6$

c) $p(x) = g(x) - f(x)$
$= x^2 - (-6x + 1)$
$= x^2 + 6x - 1$
$p(1) = 1^2 + 6(1) - 1$
$= 1 + 6 - 1$
$= 6$

Section 10.1 Page 483 Question 4

For $f(x) = 3x^2 + 2$, $g(x) = \sqrt{x+4}$, and $h(x) = 4x - 2$,

a) $y = (f + g)(x)$
$= f(x) + g(x)$
$= 3x^2 + 2 + \sqrt{x+4}$
The domain of $f(x) = 3x^2 + 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = \sqrt{x+4}$ is $\{x \mid x \geq -4, x \in \mathrm{R}\}$.
The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

b) $y = (h - g)(x)$
$= h(x) - g(x)$
$= 4x - 2 - \sqrt{x+4}$
The domain of $h(x) = 4x - 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = \sqrt{x+4}$ is $\{x \mid x \geq -4, x \in \mathrm{R}\}$.
The domain of $y = (h - g)(x)$ consists of all values that are in both the domain of $h(x)$ and the domain of $g(x)$: $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

c) $y = (g - h)(x)$
$= g(x) - h(x)$
$= \sqrt{x+4} - (4x - 2)$
$= \sqrt{x+4} - 4x + 2$

The domain of $g(x) = \sqrt{x+4}$ is $\{x \mid x \geq -4, x \in \mathrm{R}\}$.
The domain of $h(x) = 4x - 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (g - h)(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

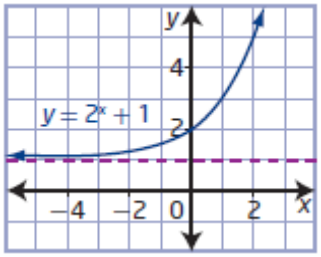
d) $y = (f + h)(x)$
$= f(x) + h(x)$
$= 3x^2 + 2 + 4x - 2$
$= 3x^2 + 4x$

The domain of $f(x) = 3x^2 + 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = 4x - 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (f + h)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $h(x)$: $\{x \mid x \in \mathrm{R}\}$.

Section 10.1 Page 483 Question 5

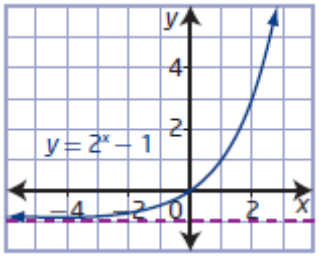
For $f(x) = 2^x$ and $g(x) = 1$,

a) $y = (f + g)(x)$
$= f(x) + g(x)$
$= 2^x + 1$



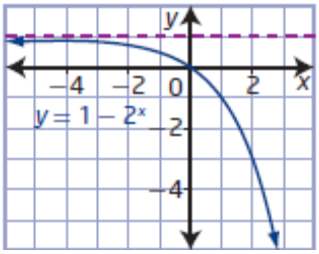
The domain of $f(x) = 2^x$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = 1$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f + g)(x)$ is $\{y \mid y > 1, y \in \mathrm{R}\}$.

b) $y = (f - g)(x)$
$= f(x) - g(x)$
$= 2^x - 1$



The domain of $f(x) = 2^x$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = 1$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid y > -1, y \in \mathrm{R}\}$.

c) $y = (g - f)(x)$
$= g(x) - f(x)$
$= 1 - 2^x$



The domain of $g(x) = 1$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $f(x) = 2^x$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (g - f)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (g - f)(x)$ is $\{y \mid y < 1, y \in \mathrm{R}\}$.

Section 10.1 Page 483 Question 6

a) From the graph, $f(4) = 5$ and $g(4) = 3$ and. So, $(f + g)(4) = 5 + 3 = 8$.

b) From the graph, $f(-4) = 5$ and $g(-4) = 1$. So, $(f + g)(-4) = 5 + 1 = 6$.

c) From the graph, $f(-5) = 7$ and $g(-5) = 0$. So, $(f + g)(-5) = 7 + 0 = 7$.

d) From the graph, $f(-6) = 9$ and $g(-6)$ does not exist. So, $(f + g)(-6)$ is not in the domain.

Section 10.1 Page 484 Question 7

First, determine the equations of the functions in the graph as $f(x) = (x - 2)^2 - 5$ and $g(x) = -0.5x^2 + 3$.

a) $y = (f + g)(x)$
$= f(x) + g(x)$
$= (x - 2)^2 - 5 + (-0.5x^2 + 3)$
$= x^2 - 4x + 4 - 5 - 0.5x^2 + 3$
$= 0.5x^2 - 4x + 2$

By completing the square, the function in vertex form is $y = 0.5(x - 4)^2 - 6$, where $a = 0.5$, $h = 4$, and $k = -6$. The graph of this parabola has vertex at $(4, -6)$ and has been vertically stretched by a factor of 0.5: graph **B**.

b) $y = (f - g)(x)$
$= f(x) + g(x)$
$= (x - 2)^2 - 5 - (-0.5x^2 + 3)$
$= x^2 - 4x + 4 - 5 + 0.5x^2 - 3$
$= 1.5x^2 - 4x - 4$

By completing the square, the function in vertex form is $y = \frac{3}{2}\left(x - \frac{4}{3}\right)^2 - \frac{20}{3}$, where $a = \frac{3}{2}$, $h = \frac{4}{3}$, and $k = -\frac{20}{3}$. The graph of this parabola has vertex at $\left(\frac{4}{3}, -\frac{20}{3}\right)$ and has been vertically stretched by a factor of $\frac{3}{2}$: graph **C**.

c) $y = (g - f)(x)$
$= g(x) + f(x)$
$= -0.5x^2 + 3 - ((x - 2)^2 - 5)$
$= -0.5x^2 + 3 - (x^2 - 4x + 4 - 5)$
$= -0.5x^2 + 3 - x^2 + 4x + 1$
$= -1.5x^2 + 4x + 4$

By completing the square, the function in vertex form is $y = -\frac{3}{2}\left(x - \frac{4}{3}\right)^2 + \frac{20}{3}$, where $a = -\frac{3}{2}$, $h = \frac{4}{3}$, and $k = \frac{20}{3}$. The graph of this parabola has vertex at $\left(\frac{4}{3}, \frac{20}{3}\right)$ and has been vertically stretched by a factor of $\frac{3}{2}$ and reflected in the x-axis: graph **A**.

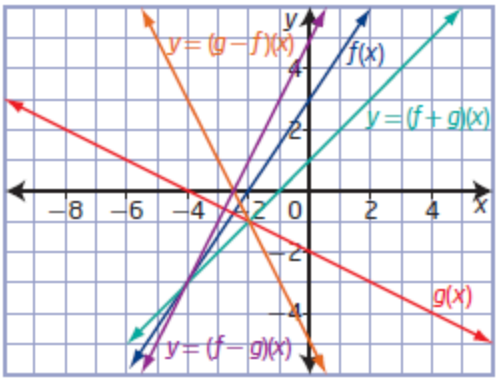
Section 10.1 Page 484 Question 8

a) First, determine the equations of the functions in the graph as $f(x) = 1.5x + 3$ and $g(x) = -0.5x - 2$.

i) $y = (f + g)(x)$
$= f(x) + g(x)$
$= 1.5x + 3 + (-0.5x - 2)$
$= x + 1$

ii) $y = (f - g)(x)$
$= f(x) - g(x)$
$= 1.5x + 3 - (-0.5x - 2)$
$= 2x + 5$

iii) $y = (g - f)(x)$
$= g(x) - f(x)$
$= -0.5x - 2 - (1.5x + 3)$
$= -2x - 5$

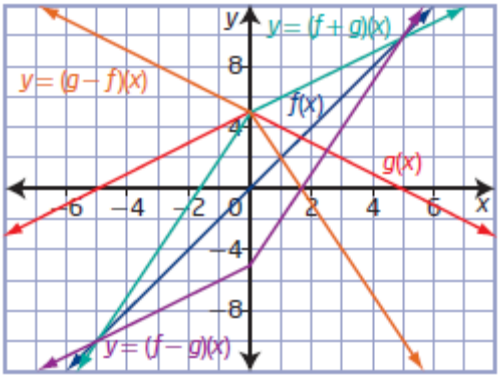


b) First, determine the equations of the functions in the graph as $f(x) = 2x$ and $g(x) = -|x| + 5$.

i) $y = (f + g)(x)$
$= f(x) + g(x)$
$= 2x + (-|x| + 5)$
$= 2x - |x| + 5$

ii) $y = (f - g)(x)$
$= f(x) - g(x)$
$= 2x - (-|x| + 5)$
$= 2x + |x| - 5$

iii) $y = (g - f)(x)$
$= g(x) - f(x)$
$= -|x| + 5 - 2x$



Section 10.1 Page 484 Question 9

For $f(x) = 3x^2 + 2$, $g(x) = 4x$, and $h(x) = 7x - 1$,

a) $y = f(x) + g(x) + h(x)$
$= 3x^2 + 2 + 4x + 7x - 1$
$= 3x^2 + 11x + 1$

b) $y = f(x) + g(x) - h(x)$
$= 3x^2 + 2 + 4x - (7x - 1)$
$= 3x^2 - 3x + 3$

c) $y = f(x) - g(x) + h(x)$
$= 3x^2 + 2 - 4x + 7x - 1$
$= 3x^2 + 3x + 1$

d) $y = f(x) - g(x) - h(x)$
$= 3x^2 + 2 - 4x - (7x - 1)$
$= 3x^2 - 11x + 3$

Section 10.1 Page 484 Question 10

$h(x) = (f + g)(x)$
$h(x) = f(x) + g(x)$
$g(x) = h(x) - f(x)$

a) Substitute $h(x) = x^2 + 5x + 2$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= x^2 + 5x + 2 - (5x + 2)$
$= x^2$

b) Substitute $h(x) = \sqrt{x+7} + 5x + 2$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= \sqrt{x+7} + 5x + 2 - (5x + 2)$
$= \sqrt{x+7}$

c) Substitute $h(x) = 2x + 3$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= 2x + 3 - (5x + 2)$
$= -3x + 1$

d) Substitute $h(x) = 3x^2 + 4x - 2$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= 3x^2 + 4x - 2 - (5x + 2)$
$= 3x^2 - x - 4$

Section 10.1 Page 484 Question 11

$h(x) = (f - g)(x)$
$h(x) = f(x) - g(x)$
$g(x) = f(x) - h(x)$

a) Substitute $h(x) = -x^2 + 5x + 3$ and $f(x) = 5x + 2$.

$g(x) = f(x) - h(x)$
$= 5x + 2 - (-x^2 + 5x + 3)$
$= x^2 - 1$

b) Substitute $h(x) = \sqrt{x-4} + 5x + 2$ and $f(x) = 5x + 2$.

$g(x) = f(x) - h(x)$
$= 5x + 2 - (\sqrt{x-4} + 5x + 2)$
$= -\sqrt{x-4}$

c) Substitute $h(x) = -3x + 11$ and $f(x) = 5x + 2$.

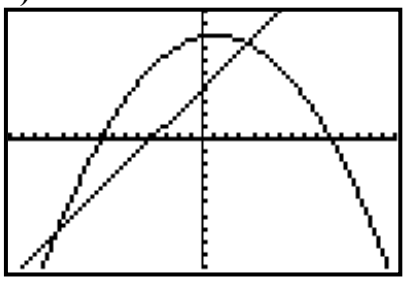
$g(x) = f(x) - h(x)$
$= 5x + 2 - (-3x + 11)$
$= 8x - 9$

d) Substitute $h(x) = -2x^2 + 16x + 8$ and $f(x) = 5x + 2$.

$g(x) = f(x) - h(x)$
$= 5x + 2 - (-2x^2 + 16x + 8)$
$= 2x^2 - 11x - 6$

Section 10.1 Page 485 Question 12

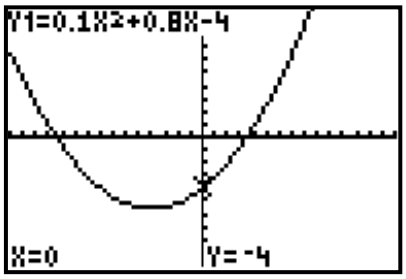
a)



The points of intersection represent where the supply equals the demand. The intersection point in quadrant III should not be considered since the price cannot be negative.

b) Substitute $S(p) = p + 4$ and $D(p) = -0.1(p + 8)(p - 10)$.

$y = S(p) - D(p)$
$= p + 4 - (-0.1(p + 8)(p - 10))$
$= p + 4 + 0.1(p^2 - 2p - 80)$
$= p + 4 + 0.1p^2 - 0.2p - 8$
$= 0.1p^2 + 0.8p - 4$

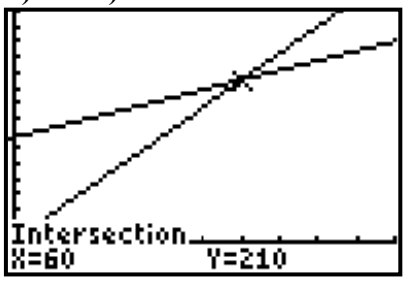


This function models the excess supply as a function of cost.

Section 10.1 Page 485 Question 13

a) The total cost is represented by $C(n) = 135 + 1.25n$.
The total revenue is represented by $R(n) = 3.5n$.

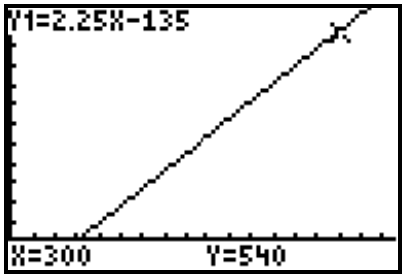
b) and **c)**



The break-even point is (60, 210). When 60 hamburgers are sold, the cost equals the revenue, $210.

d) Profit can be represented by a difference function:

$$P(n) = R(n) - C(n)$$
$$= 3.5n - (135 + 1.25n)$$
$$= 2.25n - 135$$



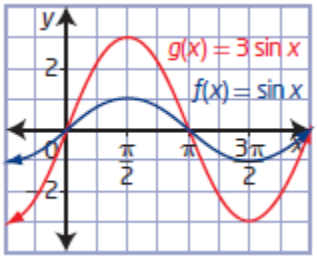
e) Substitute $n = 300$, the maximum number of hamburgers that can be sold in a day.

$$P(n) = 2.25n - 135$$
$$P(300) = 2.25(300) - 135$$
$$P(300) = 675 - 135$$
$$P(300) = 540$$

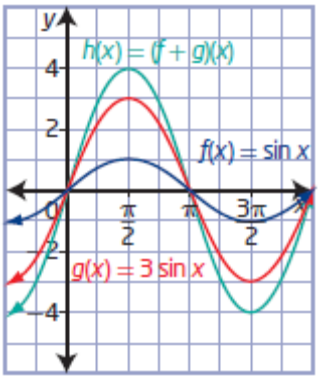
The maximum daily profit the vendor can earn is $540.

Section 10.1 Page 485 Question 14

a)



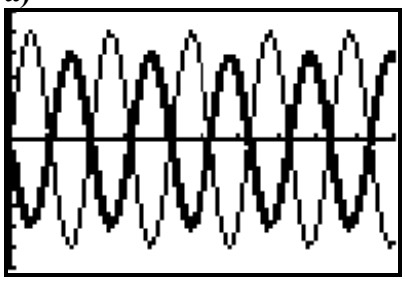
b) Add the y-values at each x-value and draw a smooth curve through the points.



c) The maximum height of the resultant wave is 4 cm.

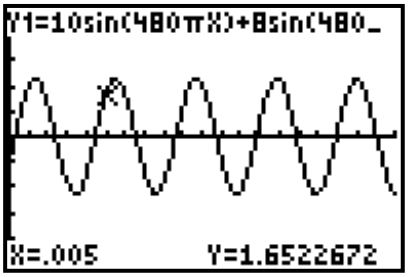
Section 10.1 Page 485 Question 15

a)



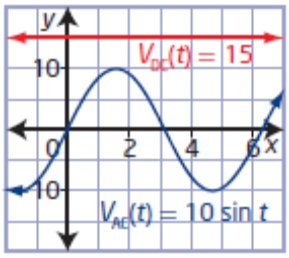
b) The maximum values of one function are located at the same x-coordinates as the minimum values of the other function, and vice versa. This will result in destructive interference.

c) $y = E(t) + R(t)$
$= 10 \sin 480\pi t + 8 \sin 480\pi(t - 0.002)$

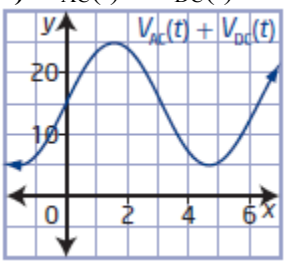


Section 10.1 Page 486 Question 16

a)



b) $V_{AC}(t) + V_{DC}(t) = 10 \sin t + 15$



c) The domain of $V_{AC}(t) + V_{DC}(t)$ is $\{t \mid t \in \mathrm{R}\}$ and the range is $\{V \mid 5 \leq V \leq 25, V \in \mathrm{R}\}$.

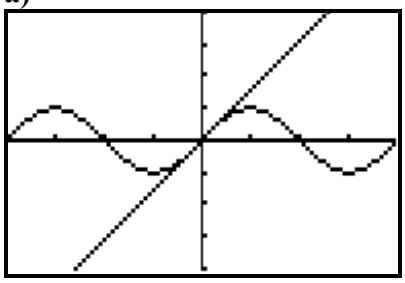
d) i) The minimum value of the voltage signal is 5 V.
ii) The maximum value of the voltage signal is 25 V.

Section 10.1 Page 486 Question 17

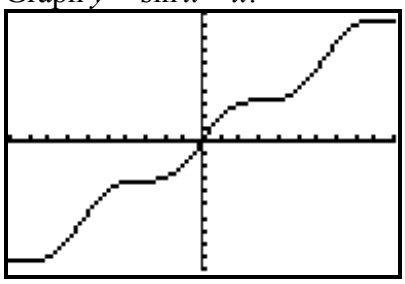
$$\begin{aligned} h(t) &= d_1(t) - d_2(t) \\ &= 10t^2 - 5(t + 2)^2 \\ &= 10t^2 - 5(t^2 + 4t + 4) \\ &= 10t^2 - 5t^2 - 20t - 20 \\ &= 10t^2 - 20t - 20 \end{aligned}$$

Section 10.1 Page 486 Question 18

a)



b) My prediction is that the shape of $h(x) = f(x) + g(x)$ will be a sinusoidal function on a diagonal according to $y = x$.
Graph $y = \sin x + x$:

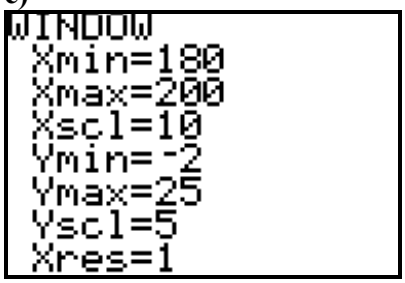


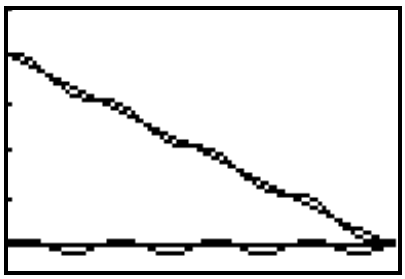
Section 10.1 Page 486 Question 19

a) A function representing the skier's distance, d, from the base of the hill versus time, t, in seconds, is $d = 200 - t$.

b) If the height, m, of the skier through the moguls is $m(t) = 0.75 \sin 1.26t$, then a function that represents the skier's actual path of height versus time is $h(t) = 200 - t + 0.75 \sin 1.26t$.

c)





Section 10.1 Page 487 Question 20

Example: Replace all x with $-x$ and then simplify. If the new function is equal to the original, then it is even. If it is equal to -1 times the original, then it is odd. If it is neither equal to the original nor equal to -1 times the original, then it is neither.
Let $f(x) = |x + 1|$, $g(x) = x^2 + 2x + 1$, and $h(x) = 2^x - 1$.
Check $k(x) = f(x) + g(x)$.
$k(x) = f(x) + g(x)$
$= |x + 1| + x^2 + 2x + 1$
$k(-x) = |-x + 1| + (-x)^2 + 2(-x) + 1$
$= |-x + 1| + x^2 - 2x + 1$
Since $k(-x) \neq k(x)$ and $k(-x) \neq -k(x)$, it is neither even nor odd.
Check $k(x) = f(x) + h(x)$.
$k(x) = f(x) + h(x)$
$= |x + 1| + 2^x - 1$
$k(-x) = |-x + 1| + 2^{-x} - 1$
Since $k(-x) \neq k(x)$ and $k(-x) \neq -k(x)$, it is neither even nor odd.

Check $k(x) = g(x) + h(x)$.

$k(x) = g(x) + h(x)$

$= x^2 + 2x + 1 + 2^x - 1$

$= x^2 + 2x + 2^x$

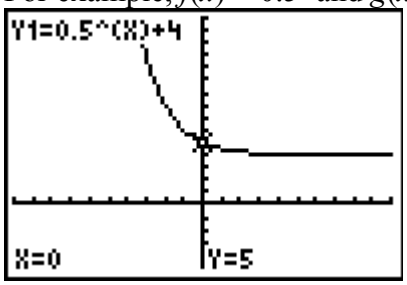
$k(-x) = (-x)^2 + 2(-x) + 2^{-x}$

$= x^2 - 2x + 2^{-x}$

Since $k(-x) \neq k(x)$ and $k(-x) \neq -k(x)$, it is neither even nor odd.

Section 10.1 Page 487 Question 21

The graph shows the sum of an exponential function and a constant function.
For example, $f(x) = 0.5^x$ and $g(x) = 4$.



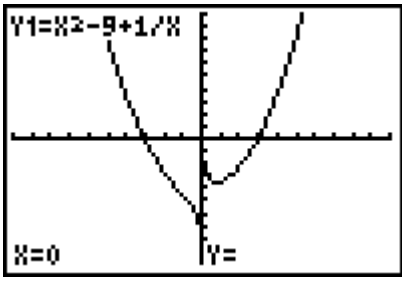
Section 10.1 Page 487 Question 22

a) The domain of $f(x) = x^2 - 9$ is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$.
The domain of $g(x) = \dfrac{1}{x}$ is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$.

b) $h(x) = f(x) + g(x)$

$= x^2 - 9 + \dfrac{1}{x}$

c) The domain of $h(x)$ is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$.
The domain and range of $f(x)$ are different from the domain and range of $h(x)$.
The domain and range of $g(x)$ are the same as that of $h(x)$.



Section 10.1 Page 487 Question C1

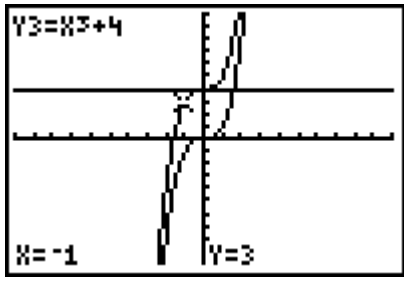
a) $f(x) + g(x) = g(x) + f(x)$ is true for all functions because addition is commutative.

b) $(f - g)(x) = (g - f)(x)$ is not true for all functions because subtraction is not commutative.

Section 10.1 Page 487 Question C2

a) For $y_1 = x^3$ and $y_2 = 4$, y_3 is the sum of the functions given for y_1 and y_2.

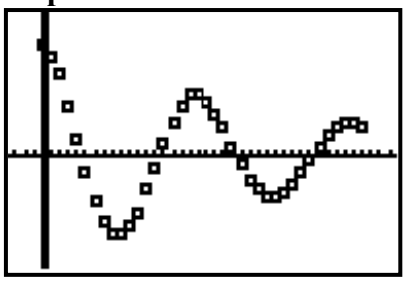
$y_3 = y_1 + y_2$
$= x^3 + 4$



b) The domain of y_3 is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

Section 10.1 Page 487 Question C3

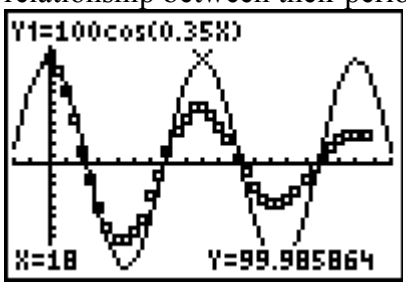
Step 1:



The graph exhibits sinusoidal features in its shape and the fact that it is periodic.

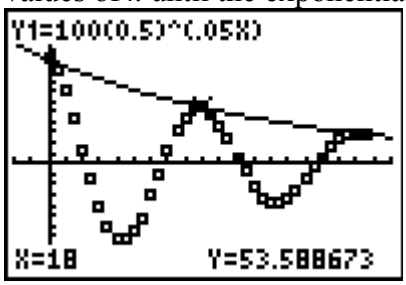
Step 2: The graph exhibits exponential features in that it is decreasing and approaching $y = 0$.

Step 3: Use technology to graph the function $h = 100 \cos(kt)$, where k is a constant. Test values of k until the crests and troughs of the function occur at the same times as for the scatter plot in step 1. Using $a = 100$ allows you to see both graphs on the screen the relationship between their periods.



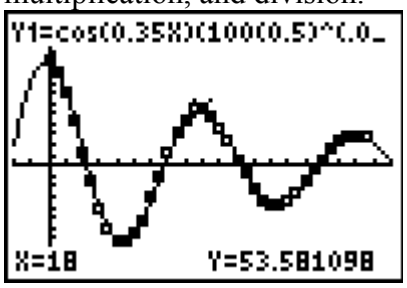
So, a cosine function that has the same period as the scatter plot in step 1 is $h = \cos 0.35t$.

Step 4: Use technology to graph the function $h = 100(0.5)^{kt}$, where k is a constant. Test values of k until the exponential curve just touches each crest of the scatter plot in step 1.



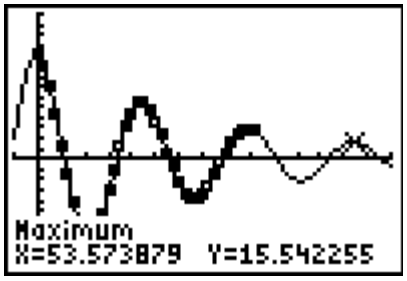
So, an exponential function that models the decay in the amplitude of the scatter plot in step 1 is $h = 100(0.5)^{0.05t}$.

Step 5: Try various operations with the two functions: addition, subtraction, multiplication, and division.



So, a combined function that models the height-time relationship of the bungee jumper is $h = (\cos 0.35t)(100(0.5)^{0.05t})$ or $h = (100 \cos 0.35t)((0.5)^{0.05t})$.

Step 6: Change the window settings to extend the horizontal axis and determine the maximum value of the fourth crest.



The bungee jumper will be approximately 15.5 m above the rest position at his fourth crest.

Section 10.2 Products and Quotients of Functions

Section 10.2 Page 496 Question 1

a) For $f(x) = x + 7$ and $g(x) = x - 7$,

$h(x) = f(x)g(x)$
$= (x + 7)(x - 7)$
$= x^2 - 49$

$$k(x) = \frac{f(x)}{g(x)} = \frac{x+7}{x-7}, x \neq 7$$

b) For $f(x) = 2x - 1$ and $g(x) = 3x + 4$,

$h(x) = f(x)g(x)$
$= (2x - 1)(3x + 4)$
$= 6x^2 + 5x - 4$

$$k(x) = \frac{f(x)}{g(x)} = \frac{2x-1}{3x+4}, x \neq -\frac{4}{3}$$

c) For $f(x) = \sqrt{x+5}$ and $g(x) = x + 2$,

$h(x) = f(x)g(x)$
$= (\sqrt{x+5})(x + 2)$
$= x\sqrt{x+5} + 2\sqrt{x+5}$

$$k(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+5}}{x+2}, x \geq -5, x \neq -2$$

d) For $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{6-x}$,

$h(x) = f(x)g(x)$
$= (\sqrt{x-1})(\sqrt{6-x})$
$= \sqrt{x^2+7x-6}$

$k(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{\sqrt{x-1}}{\sqrt{6-x}}, 1 \le x < 6$

Section 10.2 Page 496 Question 2

a) From the graph, $f(-2) = 3$ and $g(-2) = -1$. So, $(f \bullet g)(-2) = 3(-1) = -3$.

b) From the graph, $f(1) = 0$ and $g(1) = 2$. So, $(f \bullet g)(1) = 0(2) = 0$.

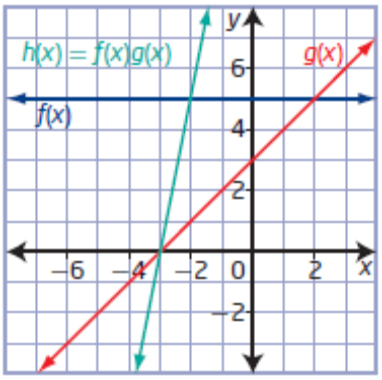
c) From the graph, $f(0) = -1$ and $g(0) = 1$. So, $\left(\dfrac{f}{g}\right)(0) = \dfrac{-1}{1} = -1$.

d) From the graph, $f(1) = 0$ and $g(1) = 2$. So, $\left(\dfrac{f}{g}\right)(1) = \dfrac{0}{2} = 0$.

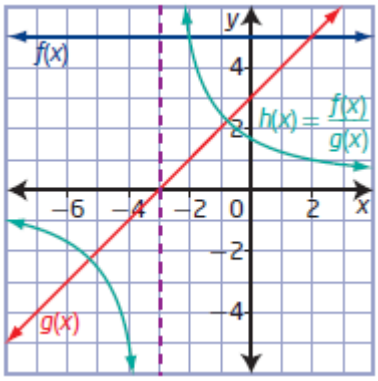
Section 10.2 Page 496 Question 3

First, determine the equations of the functions in the graph as $f(x) = 5$ and $g(x) = x + 3$.

a) $h(x) = f(x)g(x)$
$= 5(x + 3)$
$= 5x + 15$



b) $k(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{5}{x+3}, x \ne -3$



Section 10.2 Page 496 Question 4

a) For $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$,

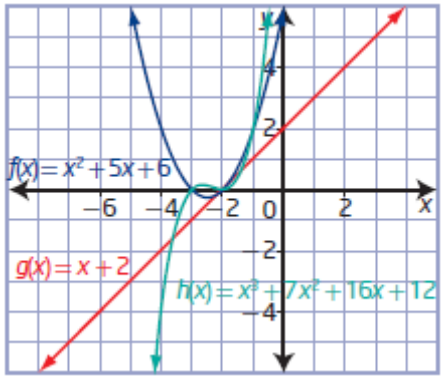
$h(x) = (f \bullet g)(x)$

$= f(x)g(x)$

$= (x^2 + 5x + 6)(x + 2)$

$= x^3 + 2x^2 + 5x^2 + 10x + 6x + 12$

$= x^3 + 7x^2 + 16x + 12$



The function $f(x) = x^2 + 5x + 6$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.

The function $g(x) = x + 2$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.

The domain of $h(x) = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \in \mathrm{R}\}$.

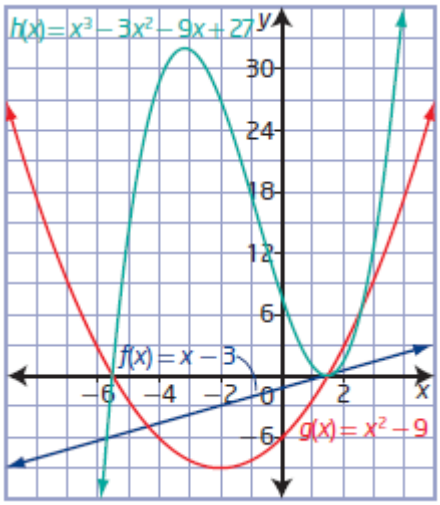
b) For $f(x) = x - 3$ and $g(x) = x^2 - 9$,

$h(x) = (f \bullet g)(x)$

$= f(x)g(x)$

$= (x - 3)(x^2 - 9)$

$= x^3 - 3x^2 - 9x + 27$



The function $f(x) = x - 3$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.

The function $g(x) = x^2 - 9$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.

The domain of $h(x) = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \in \mathrm{R}\}$.

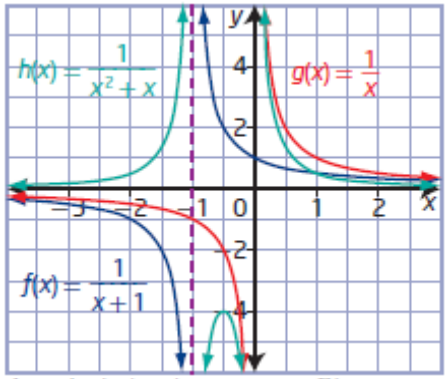
c) For $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{1}{x}$,

$h(x) = (f \bullet g)(x)$

$= f(x)g(x)$

$= \dfrac{1}{x+1}\left(\dfrac{1}{x}\right)$

$= \dfrac{1}{x^2 + x}$



The function $f(x) = \dfrac{1}{x+1}$ has domain $\{x \mid x \neq -1, x \in \mathrm{R}\}$.

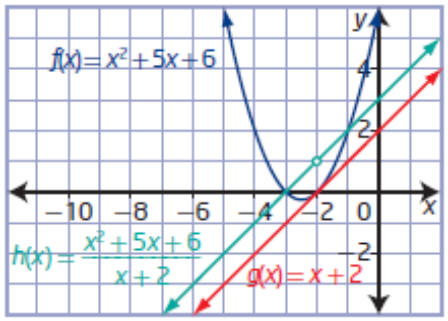
The function $g(x) = \dfrac{1}{x}$ has domain $x \mid x \neq 0, x \in \mathrm{R}\}$.

The domain of $h(x) = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \neq -1, 0, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathrm{R}\}$.

Section 10.2 Page 496 Question 5

a) For $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$,

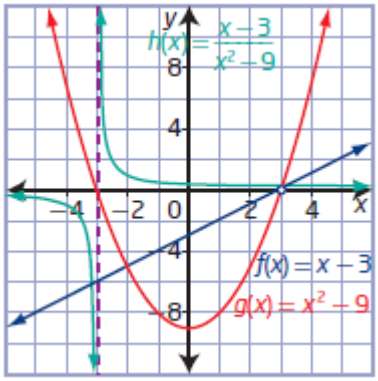
$$h(x) = \left(\frac{f}{g}\right)(x)$$
$$= \frac{f(x)}{g(x)}$$
$$= \frac{x^2 + 5x + 6}{x + 2}$$
$$= \frac{(x+2)(x+3)}{x+2}$$
$$= x + 3, x \neq -2$$



The function $f(x) = x^2 + 5x + 6$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = x + 2$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -2, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \neq 1, y \in \mathrm{R}\}$.

b) For $f(x) = x - 3$ and $g(x) = x^2 - 9$,

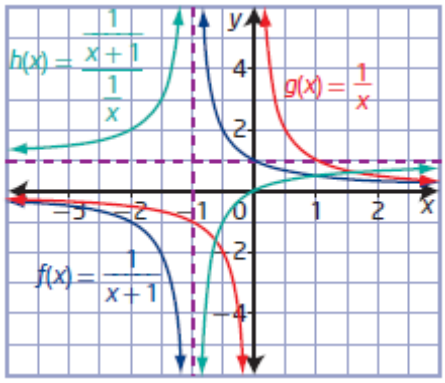
$$h(x) = \left(\frac{f}{g}\right)(x)$$
$$= \frac{f(x)}{g(x)}$$
$$= \frac{x-3}{x^2-9}$$
$$= \frac{x-3}{(x-3)(x+3)}$$
$$= \frac{1}{x+3}, x \neq -3, 3$$



The function $f(x) = x - 3$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = x^2 - 9$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -3, 3, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \neq 0, \frac{1}{6}, y \in \mathrm{R}\}$.

c) For $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{1}{x}$,

$$h(x) = \left(\frac{f}{g}\right)(x)$$
$$= \frac{f(x)}{g(x)}$$
$$= \frac{\dfrac{1}{x+1}}{\dfrac{1}{x}}$$
$$= \frac{x}{x+1}, x \neq -1, 0$$



The function $f(x) = \dfrac{1}{x+1}$ has domain $\{x \mid x \neq -1, x \in \mathrm{R}\}$.

The function $g(x) = \dfrac{1}{x}$ has domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$.

The domain of $h(x) = \left(\dfrac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -1, 0, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \neq 0, 1, y \in \mathrm{R}\}$.

Section 10.2 Page 496 Question 6

For $f(x) = x + 2$, $g(x) = x - 3$, and $h(x) = x + 4$,

a) $y = f(x)g(x)h(x)$
$= (x + 2)(x - 3)(x + 4)$
$= (x^2 - x - 6)(x + 4)$
$= x^3 + 4x^2 - x^2 - 4x - 6x - 24$
$= x^3 + 3x^2 - 10x - 24$

b) $y = \dfrac{f(x)g(x)}{h(x)}$
$= \dfrac{(x+2)(x-3)}{x+4}$
$= \dfrac{x^2 - x - 6}{x+4}, x \neq -4$

c) $y = \dfrac{f(x) + g(x)}{h(x)}$
$= \dfrac{x+2+x-3}{x+4}$
$= \dfrac{2x-1}{x+4}, x \neq -4$

d) $y = \dfrac{f(x)}{h(x)} \times \dfrac{g(x)}{h(x)}$
$= \dfrac{x+2}{x+4}\left(\dfrac{x-3}{x+4}\right)$
$= \dfrac{x^2 - x - 6}{x^2 + 8x + 16}, x \neq -4$

Section 10.2 Page 496 Question 7

$h(x) = f(x)g(x)$

$g(x) = \dfrac{h(x)}{f(x)}$

a) Substitute $h(x) = 6x + 15$ and $f(x) = 2x + 5$.

$$g(x) = \frac{h(x)}{f(x)}$$
$$= \frac{6x+15}{2x+5}$$
$$= \frac{3(2x+5)}{2x+5}$$
$$= 3$$

b) Substitute $h(x) = -2x^2 - 5x$ and $f(x) = 2x + 5$.

$$g(x) = \frac{h(x)}{f(x)}$$
$$= \frac{-2x^2-5x}{2x+5}$$
$$= \frac{-x(2x+5)}{2x+5}$$
$$= -x$$

c) Substitute $h(x) = 2x\sqrt{x} + 5\sqrt{x}$ and $f(x) = 2x + 5$.

$$g(x) = \frac{h(x)}{f(x)}$$
$$= \frac{2x\sqrt{x}+5\sqrt{x}}{2x+5}$$
$$= \frac{\sqrt{x}(2x+5)}{2x+5}$$
$$= \sqrt{x}$$

d) Substitute $h(x) = 10x^2 + 13x - 30$ and $f(x) = 2x + 5$.

$$g(x) = \frac{h(x)}{f(x)}$$
$$= \frac{10x^2+13x-30}{2x+5}$$
$$= \frac{(5x-6)(2x+5)}{2x+5}$$
$$= 5x - 6$$

Section 10.2 Page 496 Question 8

$h(x) = \dfrac{f(x)}{g(x)}$ and, therefore, $g(x) = \dfrac{f(x)}{h(x)}$

a) Substitute $h(x) = \dfrac{3x-1}{x+7}$ and $f(x) = 3x - 1$.

$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{\dfrac{3x-1}{x+7}} \\ &= x+7 \end{aligned}$$

b) Substitute $h(x) = \dfrac{3x-1}{\sqrt{x+6}}$ and $f(x) = 3x - 1$.

$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{\dfrac{3x-1}{\sqrt{x+6}}} \\ &= \sqrt{x+6} \end{aligned}$$

c) Substitute $h(x) = 1.5x - 0.5$ and $f(x) = 3x - 1$.

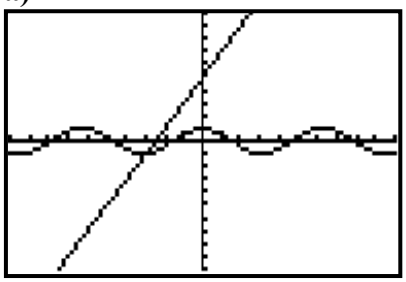
$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{1.5x-0.5} \\ &= \frac{3x-1}{0.5(3x-1)} \\ &= 2 \end{aligned}$$

d) Substitute $h(x) = \dfrac{1}{x+9}$ and $f(x) = 3x - 1$.

$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{\dfrac{1}{x+9}} \\ &= (3x-1)(x+9) \\ &= 3x^2 + 26x - 9 \end{aligned}$$

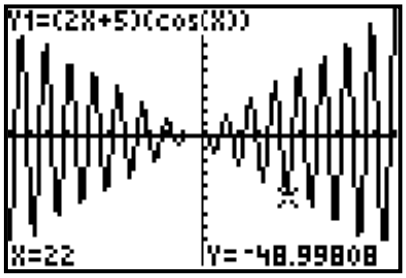
Section 10.2 Page 496 Question 9

a)



For $f(x) = 2x + 5$, the domain is $\{x \mid x \in \mathrm{R}\}$ and range is $\{y \mid y \in \mathrm{R}\}$.
For $g(x) = \cos x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and range is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$.

b) $y = f(x)g(x)$
$= (2x + 5)(\cos x)$

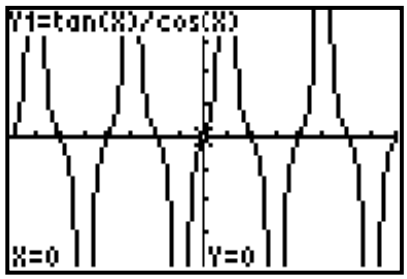


For $y = f(x)g(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and range is $\{y \mid y \in \mathrm{R}\}$.

Section 10.2 Page 496 Question 10

a) Substitute $f(x) = \tan x$ and $g(x) = \cos x$.

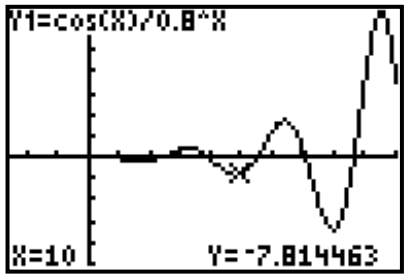
$$y = \left(\frac{f}{g}\right)(x)$$
$$= \frac{\tan x}{\cos x}$$



The domain is $\left\{x \mid x \ne (2n-1)\frac{\pi}{2}, n \in \mathrm{I}, x \in \mathrm{R}\right\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

b) Substitute $f(x) = \cos x$ and $g(x) = 0.8^x$.

$$y = \left(\frac{f}{g}\right)(x)$$
$$= \frac{\cos x}{0.8^x}$$



The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

Section 10.2 Page 497 Question 11

a) $\tan x = \dfrac{\sin x}{\cos x}$

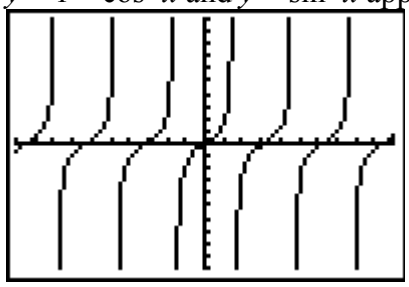
$= \dfrac{f(x)}{g(x)}$

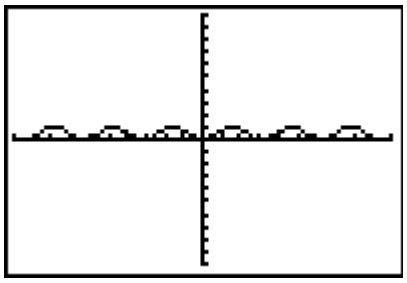
b) $1 - \cos^2 x = \sin^2 x$

$= (\sin x)(\sin x)$

$= f(x)f(x)$

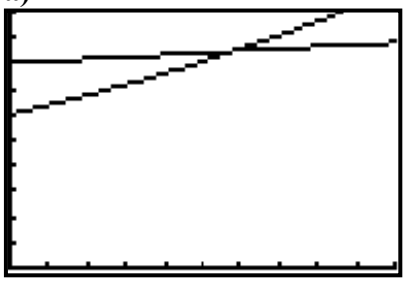
c) The graphs of $y = \dfrac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.





Section 10.2 Page 497 Question 12

a)

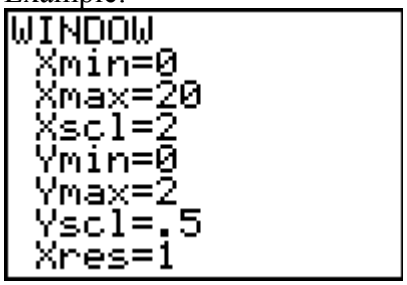


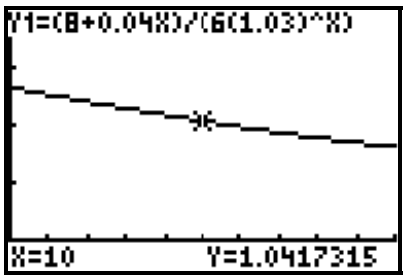
Both graphs are increasing over time. However, the graph of $P(t)$ increases more rapidly and overtakes the graph of $F(t)$.

b) $y = \dfrac{F(t)}{P(t)}$

$= \dfrac{8 + 0.04t}{6(1.03)^t}$

Example:



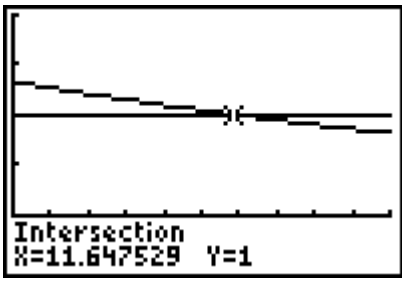


Values of t that are less than 0 should not be considered. Time cannot be negative.

c) From the graph, the amount of food per fish is a maximum at $t = 0$.

d) The fish farm will no longer be viable when $\frac{F(t)}{P(t)} < 1$.

Graph $y = \frac{8 + 0.04t}{6(1.03)^t}$ and $y = 1$ and find the point of intersection.



This occurs in about 11.6 years.

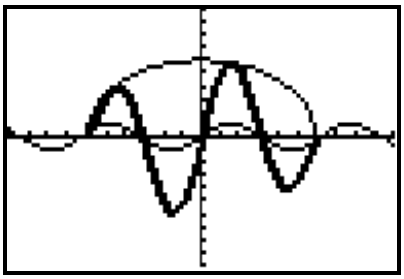
Section 10.2 Page 497 Question 13

a) Substitute $f(x) = \sqrt{36 - x^2}$ and $g(x) = \sin x$.

$y = (f \bullet g)(x)$

$= f(x)g(x)$

$= \sqrt{36 - x^2}\ (\sin x)$



b) The domain of $f(x) = \sqrt{36 - x^2}$ is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$.

The domain for $g(x) = \sin x$ is $\{x \mid x \in \mathrm{R}\}$.

Then, for $y = \sqrt{36 - x^2}\ (\sin x)$, the domain is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$.

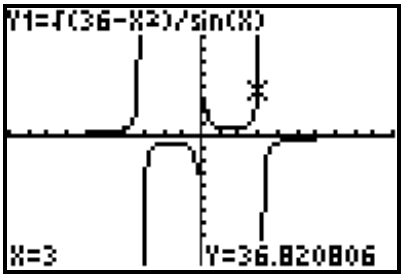
From the graph, the range is about $\{y \mid -5.8 \le y \le 5.8, y \in \mathrm{R}\}$.

c) Substitute $f(x) = \sqrt{36 - x^2}$ and $g(x) = \sin x$.

$y = \left(\frac{f}{g}\right)(x)$

$= \frac{f(x)}{g(x)}$

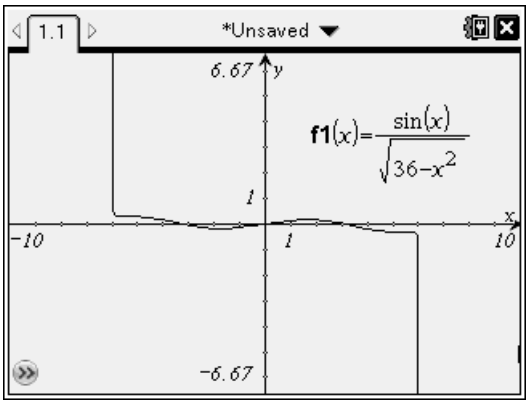
$= \frac{\sqrt{36 - x^2}}{\sin x}$



For $y = \frac{\sqrt{36 - x^2}}{\sin x}$, the domain is $\{x \mid -6 \le x \le 6, x \ne n\pi, n \in \mathrm{I}, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

d) Substitute $f(x) = \sqrt{36 - x^2}$ and $g(x) = \sin x$.

$$y = \left(\frac{g}{f}\right)(x)$$
$$= \frac{g(x)}{f(x)}$$
$$= \frac{\sin x}{\sqrt{36 - x^2}}$$



For $y = \frac{\sin x}{\sqrt{36 - x^2}}$, the domain is $\{x \mid -6 < x < 6, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

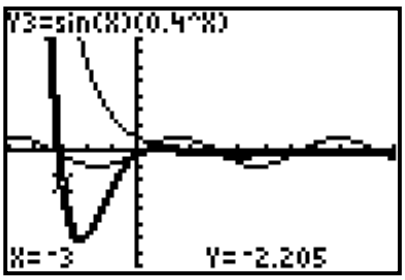
The domain in part d) is restricted to $-6 < x < 6$ but has no non-permissible values. In part c), the domain is restricted to $-6 \leq x \leq 6$ *with* non-permissible values. The ranges in parts c) and d) are the same.

Section 10.2 Page 497 Question 14

a) and **b)**

$d(t) = (A \sin kt) \times 0.4^{ct}$
$= f(t)g(t)$

Then, $f(t) = A \sin kt$ and $g(t) = 0.4^{ct}$.

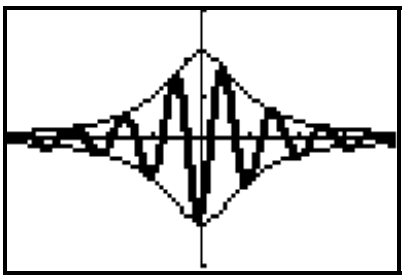


Section 10.2 Page 498 Question 15

a) Substitute $f(x) = \frac{2}{x^2 + 1}$ and $g(x) = \sin(6x - 1)$.

$y = f(x)g(x)$
$= \frac{2}{x^2 + 1} \sin(6x - 1)$

Graph $f(x) = \frac{2}{x^2 + 1}$, $-f(x) = -\frac{2}{x^2 + 1}$, and $y = \frac{2}{x^2 + 1} \sin(6x - 1)$.



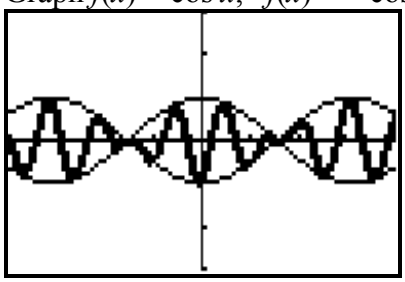
The graph of $y = f(x)g(x)$ oscillates between the graphs of $f(x)$ and $-f(x)$.

b) Substitute $f(x) = \cos x$ and $g(x) = \sin(6x - 1)$.

$y = f(x)g(x)$

$= \cos x \sin(6x - 1)$

Graph $f(x) = \cos x$, $-f(x) = -\cos x$, and $y = \cos x \sin(6x - 1)$.



Yes. The graph of $y = f(x)g(x)$ oscillates between the graphs of $f(x)$ and $-f(x)$.

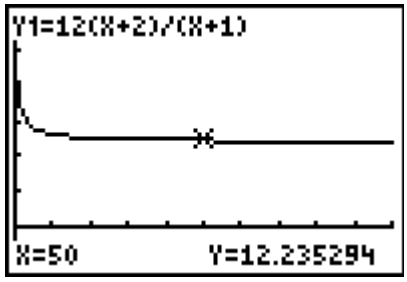
Section 10.2 Page 498 Question 16

The price per tonne can be modelled by

$$y = \frac{p(x)}{x}$$

$$= \frac{12x\left(\frac{x+2}{x+1}\right)}{x}$$

$$= \frac{12(x+2)}{x+1}$$



The price per tonne decreases as the number of tonnes increases.

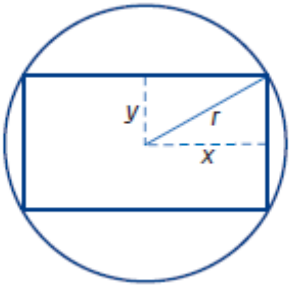
Section 10.2 Page 498 Question 17

Given that the length of the rectangle is $2x$, first determine an expression for the width of the rectangle.

Use the Pythagorean theorem to find that $y = \sqrt{r^2 - x^2}$.

Then, the area of the rectangle can be modelled by

$A = 2x(2\sqrt{r^2 - x^2})$ or $A = 4x\sqrt{r^2 - x^2}$.



Section 10.2 Page 498 Question C1

Yes; multiplication is commutative. For example, let $f(x) = 6x$ and $g(x) = x - 2$.

$f(x)g(x) = 6x(x - 2)$ $\qquad$ $g(x)f(x) = (x - 2)6x$

$= 6x^2 - 12x$ $\qquad$ $= 6x^2 - 12x$

$f(x)g(x) = g(x)f(x)$

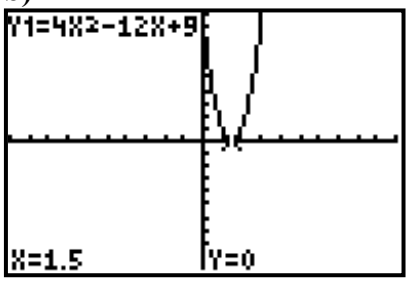
Section 10.2 Page 498 Question C2

Example: Multiplication of functions generally increases the domain, although this is not always true. Quotients of functions generally produce asymptotes and points of discontinuity, which restricts the domain, although this is not always true.

Section 10.2 Page 498 Question C3

a) $A(x) = (2x - 3)(2x - 3)$
$= 4x^2 - 12x + 9$

b)



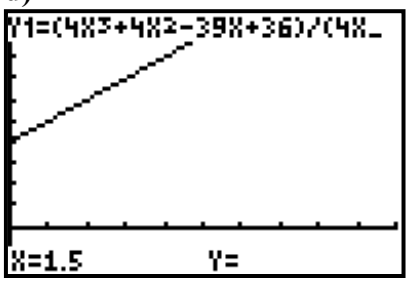
In this context, the domain is $\{x \mid x \geq 1.5, x \in \mathrm{R}\}$ and the range is $\{A \mid A \geq 0, A \in \mathrm{R}\}$.

c) Substitute $V(x) = 4x^3 + 4x^2 - 39x + 36$ and $A(x) = 4x^2 - 12x + 9$.

$$h(x) = \frac{V(x)}{A(x)}$$
$$= \frac{4x^3 + 4x^2 - 39x + 36}{4x^2 - 12x + 9}$$
$$= \frac{(2x-3)^2(x+4)}{(2x-3)^2}$$
$$= x + 4, \; x \neq \frac{3}{2}$$

This represents the height of the box.

d)



In this context, the domain is $\{x \mid x > 1.5, x \in \mathrm{R}\}$ and the range is $\{h \mid h > 5.5, h \in \mathrm{R}\}$.

Section 10.3 Composite Functions

Section 10.3 Page 507 Question 1

Given: $f(2) = 3, f(3) = 4, f(5) = 0, g(2) = 5, g(3) = 2$, and $g(4) = -1$

a) Substitute $g(3) = 2$.

$f(g(3)) = f(2)$
$= 3$

b) Substitute $g(2) = 5$.

$f(g(2)) = f(5)$
$= 0$

c) Substitute $f(2) = 3$.

$g(f(2)) = g(3)$
$= 2$

d) Substitute $f(3) = 4$.

$g(f(3)) = g(4)$
$= -1$

Section 10.3 Page 507 Question 2

a) From the graph, $g(-4) = 0$ and $f(0) = 2$. So, $f(g(-4)) = 2$.

b) From the graph, $g(0) = -4$ and $f(-4) = 2$. So, $f(g(0)) = 2$.

c) From the graph, $f(-2) = 0$ and $g(0) = -4$. So, $g(f(-2)) = -4$.

d) From the graph, $f(-3) = 1$ and $g(1) = -5$. So, $g(f(-3)) = -5$.

Section 10.3 Page 507 Question 3

Given: $f(x) = 2x + 8$ and $g(x) = 3x - 2$

a) Determine $g(1)$.

$g(x) = 3x - 2$
$g(1) = 3(1) - 2$
$g(1) = 1$
Substitute $g(1) = 1$ into $f(x)$.
$f(g(1)) = f(1)$
$= 2(1) + 8$
$= 10$

b) Determine $g(-2)$.

$g(x) = 3x - 2$
$g(-2) = 3(-2) - 2$
$g(-2) = -8$
Substitute $g(-2) = -8$ into $f(x)$.
$f(g(-2)) = f(-8)$
$= 2(-8) + 8$
$= -8$

c) Determine $f(-4)$.

$f(x) = 2x + 8$
$f(-4) = 2(-4) + 8$
$f(-4) = 0$
Substitute $f(-4) = 0$ into $g(x)$.
$g(f(-4)) = g(0)$
$= 3(0) - 2$
$= -2$

d) Determine $f(1)$.

$f(x) = 2x + 8$
$f(1) = 2(1) + 8$
$f(1) = 10$
Substitute $f(1) = 10$ into $g(x)$.
$g(f(1)) = g(10)$
$= 3(10) - 2$
$= 28$

Section 10.3 Page 507 Question 4

Given: $f(x) = 3x + 4$ and $g(x) = x^2 - 1$

a) Determine $g(a)$.

$g(x) = x^2 - 1$

$g(a) = a^2 - 1$

Substitute $g(a) = a^2 - 1$ into $f(x)$.

$f(g(a)) = f(a^2 - 1)$

$= 3(a^2 - 1) + 4$

$= 3a^2 + 1$

b) Determine $f(a)$.

$f(x) = 3x + 4$

$f(a) = 3(a) + 4$

$f(a) = 3a + 4$

Substitute $f(a) = 3a + 4$ into $g(x)$.

$g(f(a)) = g(3a + 4)$

$= (3a + 4)^2 - 1$

$= 9a^2 + 24a + 15$

c) $f(g(x)) = f(x^2 - 1)$

$= 3(x^2 - 1) + 4$

$= 3x^2 + 1$

d) $g(f(x)) = g(3x + 4)$

$= (3x + 4)^2 - 1$

$= 9x^2 + 24x + 15$

e) $f(f(x)) = f(3x + 4)$

$= 3(3x + 4) + 4$

$= 9x + 16$

f) $g(g(x)) = g(x^2 - 1)$

$= (x^2 - 1)^2 - 1$

$= x^4 - 2x^2$

Section 10.3 Page 507 Question 5

a) For $f(x) = x^2 + x$ and $g(x) = x^2 + x$,

$f(g(x)) = f(x^2 + x)$

$= (x^2 + x)^2 + x^2 + x$

$= x^4 + 2x^3 + x^2 + x^2 + x$

$= x^4 + 2x^3 + 2x^2 + x$

$g(f(x)) = g(x^2 + x)$

$= (x^2 + x)^2 + x^2 + x$

$= x^4 + 2x^3 + x^2 + x^2 + x$

$= x^4 + 2x^3 + 2x^2 + x$

b) For $f(x) = \sqrt{x^2 + 2}$ and $g(x) = x^2$,

$f(g(x)) = f(x^2)$

$= \sqrt{(x^2)^2 + 2}$

$= \sqrt{x^4 + 2}$

$g(f(x)) = g(\sqrt{x^2 + 2})$

$= (\sqrt{x^2 + 2})^2$

$= x^2 + 2$

c) For $f(x) = |x|$ and $g(x) = x^2$,

$f(g(x)) = f(x^2)$

$= |x^2|$

$= x^2$

$g(f(x)) = g(|x|)$

$= |x|^2$

$= x^2$

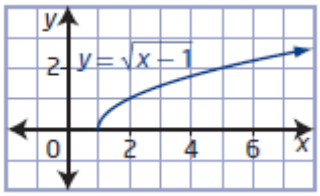
Section 10.3 Page 507 Question 6

a) For $f(x) = \sqrt{x}$ and $g(x) = x - 1$,

$y = f(g(x))$

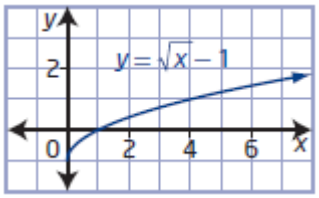
$= f(x - 1)$

$= \sqrt{x - 1}$



The domain is $\{x \mid x \geq 1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) For $f(x) = \sqrt{x}$ and $g(x) = x - 1$,



$y = g(f(x))$
$= g(\sqrt{x})$
$= \sqrt{x} - 1$

The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$.

Section 10.3 Page 507 Question 7

$h(x) = (f \circ g)(x)$
$= f(g(x))$

a) For $h(x) = (2x - 5)^2$ and $f(x) = x^2$,
$h(x) = f(g(x))$
$= g(x)^2$
$= (2x - 5)^2$

Therefore, $g(x) = 2x - 5$.

b) For $h(x) = (5x + 1)^2 - (5x + 1)$ and $f(x) = x^2 - x$,
$h(x) = f(g(x))$
$= g(x)^2 - g(x)$
$= (5x + 1)^2 - (5x + 1)$

Therefore, $g(x) = 5x + 1$.

Section 10.3 Page 507 Question 8

Christine's work is correct. Ron forgot to substitute $x^2 + 2$ for all xs in the first step.

Section 10.3 Page 507 Question 9

For $j(x) = x^2$ and $k(x) = x^3$,

$k(j(x)) = k(x^3)$
$= (x^3)^2$
$= x^6$

$j(k(x)) = k(x^2)$
$= (x^2)^3$
$= x^6$

So, $k(j(x)) = j(k(x))$ for all values of x.

Section 10.3 Page 507 Question 10

For $s(x) = x^2 + 1$ and $t(x) = x - 3$,

$s(t(x)) = s(x - 3)$
$= (x - 3)^2 + 1$
$= x^2 - 6x + 10$

$t(s(x)) = t(x^2 + 1)$
$= x^2 + 1 - 3$
$= x^2 - 2$

So, $s(t(x)) \neq t(s(x))$ for all values of x.

Section 10.3 Page 507 Question 11

a) Substitute $C(t) = 100 + 35t$.

$W(C) = 3\sqrt{C}$

$W(C(t)) = 3\sqrt{100 + 35t}$

b) In this context, the domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and the range is $\{W \mid W \geq 30, W \in \mathrm{W}\}$.

Section 10.3 Page 508 Question 12

a) The function, $s(p)$, that relates the regular price, p, to the sale price, s, is $s(p) = 0.75p$.

b) The function, $t(s)$, that relates the sale price, s, to the total cost including taxes, t, is $t(s) = 1.05s$.

c) A composite function that expresses the total cost in terms of the regular price is

$t(s(p)) = t(0.75p)$

$= 1.05(0.75p)$

$= 0.7875p$

Substitute $p = 89.99$.

$t(s(p)) = 0.7875p$

$t(s(89.99)) = 0.7875(89.99)$

$t(s(89.99)) = 70.867\ldots$

Tobias paid \$70.87 for the jacket.

Section 10.3 Page 508 Question 13

a) The function, $g(d)$, that relates the distance, d, in kilometres, driven to the quantity, g, in litres, of gasoline used is $g(d) = 0.06d$.

b) The function, $c(g)$, that relates the quantity, g, in litres, of gasoline used to the average cost, c, in dollars, of a litre of gasoline is $c(g) = 1.23g$.

c) The composite function that expresses the cost of gasoline in terms of the distance driven is

$c(g(d)) = c(0.06d)$

$= 1.23(0.06d)$

$= 0.0738d$

Substitute $d = 200$.

$c(g(d)) = 0.0738d$

$c(g(200)) = 0.0738(200)$

$c(g(200)) = 14.76$

It would cost Jordan \$14.76 to drive her car 200 km.

d) From part a), $d(g) = \frac{g}{0.06}$. From part b), $g(c) = \frac{c}{1.23}$.
The composite function that expresses the distance driven in terms of the cost of gasoline is

$$d(g(c)) = d\left(\frac{c}{1.23}\right)$$

$$= \frac{\frac{c}{1.23}}{0.06}$$

$$= \frac{c}{0.0738} \approx 13.55c$$

Substitute $c = 40$.

$$d(g(c)) = \frac{c}{0.0738}$$

$$d(g(40)) = \frac{40}{0.0738}$$

$d(g(40)) = 542.005\ldots$
Jordan could drive about 542 km on $40.

Section 10.3 Page 508 Question 14

For $f(x) = 3x$, $g(x) = x - 7$, and $h(x) = x^2$,
a) $(f \circ g \circ h)(x) = (f \circ (g \circ h))(x)$
First, determine $(g \circ h)(x)$.
$(g \circ h)(x) = g(x^2)$
$= x^2 - 7$
Then,
$(f \circ g \circ h)(x) = (f \circ (g \circ h))(x)$
$= f(x^2 - 7)$
$= 3(x^2 - 7)$
$= 3x^2 - 21$

b) For $g(f(h(x))$, first, determine $f(h(x))$.
$f(h(x)) = f(x^2)$
$= 3x^2$
Then,
$g(f(h(x)) = g(3x^2)$
$= 3x^2 - 7$

c) For $f(h(g(x))$, first, determine $h(g(x))$.
$h(g(x)) = h(x - 7)$
$= (x - 7)^2$
$= x^2 - 14x + 49$
Then,
$f(h(g(x)) = f(x^2 - 14x + 49)$

$= 3(x^2 - 14x + 49)$
$= 3x^2 - 42x + 147$

d) $(h \circ g \circ f)(x) = (h \circ (g \circ f))(x)$
First, determine $(g \circ f)(x)$.
$(g \circ f)(x) = g()$
$= 3x - 7$
Then,
$(h \circ g \circ f)(x) = (h \circ (g \circ f))(x)$
$= h(3x - 7)$
$= (3x - 7)^2$
$= 9x^2 - 42x + 49$

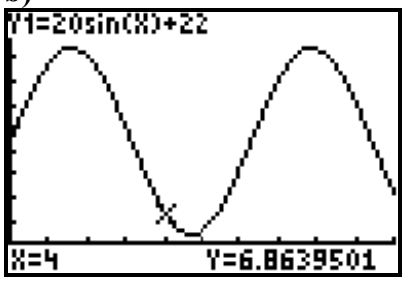
Section 10.3 Page 508 Question 15

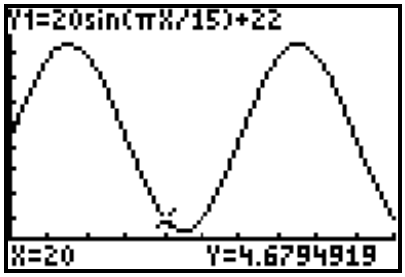
a) The equation of the rider's height in terms of time is

$h(\theta(t)) = h\left(\frac{\pi t}{15}\right)$

$= 20 \sin \frac{\pi t}{15} + 22$

b)





For $h(\theta) = 20 \sin \theta + 22$, $b = 1$. So, the period is 2π.

For $h(\theta(t)) = 20 \sin \frac{\pi t}{15} + 22$, $b = \frac{\pi}{15}$. So, the period is $\frac{2\pi}{\frac{\pi}{15}}$, or 30.

The period of the composite function is much greater.

Section 10.3 Page 508 Question 16

a) The equation of the concentration of pollutant as a function of time is

$C(P(t)) = C\left(12.5(2)^{\frac{t}{10}}\right)$

$= 1.15\left(12.5(2)^{\frac{t}{10}}\right) + 53.12$

$= 14.375\,(2)^{\frac{t}{10}} + 53.12$

b) Substitute $C = 100$.

$$C(P(t)) = 14.375\,(2)^{\frac{t}{10}} + 53.12$$

$$100 = 14.375\,(2)^{\frac{t}{10}} + 53.12$$

$$46.88 = 14.375\,(2)^{\frac{t}{10}}$$

$$\frac{46.88}{14.375} = (2)^{\frac{t}{10}}$$

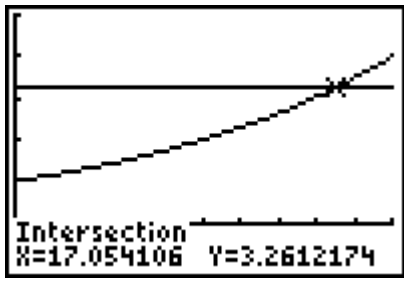
This can be solved algebraically or graphically.

$$\log\frac{46.88}{14.375} = \log 2^{\frac{t}{10}}$$

$$\log\frac{46.88}{14.375} = \frac{t}{10}\log 2$$

$$\frac{10}{\log 2}\log\frac{46.88}{14.375} = t$$

$$t = 17.054...$$



It will take approximately 17.1 years for the concentration to be over 100 ppm.

Section 10.3 Page 509 Question 17

a) Example: Let $g(x) = x^2$. Then, work backward.

$h(x) = 2x^2 - 1$

$f(g(x)) = 2g(x) - 1$

$f(x) = 2x - 1$

b) Example: Let $g(x) = x^2$. Then, work backward.

$$h(x) = \frac{2}{3 - x^2}$$

$$f(g(x)) = \frac{2}{3 - g(x)}$$

$$f(x) = \frac{2}{3 - x}$$

c) Let $g(x) = x^2 - 4x + 5$. Then, work backward.

$h(x) = |x^2 - 4x + 5|$

$f(g(x)) = |g(x)|$

$f(x) = |x|$

Section 10.3 Page 509 Question 18

For $f(x) = 1 - x$ and $g(x) = \dfrac{x}{1-x}$, $x \neq 1$,

a) $g(f(x)) = g(1-x)$

$$= \frac{1-x}{1-(1-x)}$$

$$= \frac{1-x}{x}$$

$$= \frac{1}{g(x)}$$

b) $f(g(x)) = f\left(\dfrac{x}{1-x}\right)$

$$= 1 - \frac{x}{1-x}$$

$$= \frac{1-2x}{1-x}$$

$$\neq \frac{1}{f(x)}$$

Section 10.3 Page 509 Question 19

a) $m(v(t)) = m(t^3)$

$$= \frac{m_0}{\sqrt{1-\dfrac{(t^3)^2}{c^2}}}$$

$$= \frac{m_0}{\sqrt{1-\dfrac{t^6}{c^2}}}$$

b) Substitute $t = \sqrt[3]{\dfrac{c}{2}}$.

$$m(v(t)) = \frac{m_0}{\sqrt{1-\dfrac{t^6}{c^2}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{1-\dfrac{\left(\sqrt[3]{\frac{c}{2}}\right)^6}{c^2}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{1-\dfrac{\frac{c^2}{4}}{c^2}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{1-\frac{1}{4}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{\frac{3}{4}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{2m_0}{\sqrt{3}}$$

The particle's mass is $\dfrac{2m_0}{\sqrt{3}}$.

Section 10.3 Page 509 Question 20

a) For $f(x) = 5x + 10$ and $g(x) = \frac{1}{5}x - 2$,

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{5}x - 2\right) \\ &= 5\left(\frac{1}{5}x - 2\right) + 10 \\ &= x - 10 + 10 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(5x + 10) \\ &= \frac{1}{5}(5x + 10) - 2 \\ &= x + 2 - 2 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

b) For $f(x) = \frac{x-1}{2}$ and $g(x) = 2x + 1$,

$$\begin{aligned} f(g(x)) &= f(2x + 1) \\ &= \frac{2x+1-1}{2} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g\left(\frac{x-1}{2}\right) \\ &= 2\left(\frac{x-1}{2}\right) + 1 \\ &= x - 1 + 1 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

c) For $f(x) = \sqrt[3]{x+1}$ and $g(x) = x^3 - 1$,

$$\begin{aligned} f(g(x)) &= f(x^3 - 1) \\ &= \sqrt[3]{x^3 - 1 + 1} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(\sqrt[3]{x+1}) \\ &= (\sqrt[3]{x+1})^3 - 1 \\ &= x + 1 - 1 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

d) For $f(x) = 5^x$ and $g(x) = \log_5 x$,

$$\begin{aligned} f(g(x)) &= f(\log_5 x) \\ &= 5^{\log_5 x} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(5^x) \\ &= \log_5 5^x \\ &= x \log_5 5 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

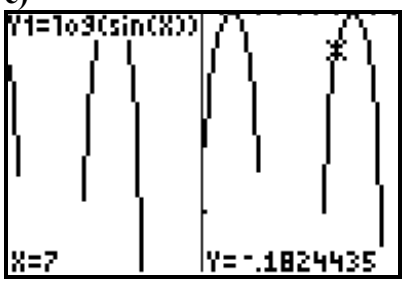
Section 10.3 Page 509 Question 21

a) The domain of $f(x) = \log x$ is $\{x \mid x > 0, x \in \mathrm{R}\}$.

b) For $f(x) = \log x$ and $g(x) = \sin x$,

$$\begin{aligned} f(g(x)) &= f(\sin x) \\ &= \log (\sin x) \end{aligned}$$

c)



d) For $f(g(x)) = \log(\sin x)$, the domain is $\{x \mid 2\pi n < x < (2n + 1)\pi n, n \in \mathrm{I}, x \in \mathrm{R}\}$, since $\sin x$ is positive in quadrants I and II of each rotation on a unit circle. The range is $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

Section 10.3 Page 509 Question 22

For $f(x) = \dfrac{1}{1+x}$ and $g(x) = \dfrac{1}{2+x}$,

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{2+x}\right) \\ &= \frac{1}{1+\frac{1}{2+x}} \\ &= \frac{1}{\frac{x+3}{2+x}} \\ &= \frac{x+2}{x+3}, x \neq -1, -2, -3 \end{aligned}$$

Section 10.3 Page 509 Question 23

a) For $f_1(x) = x, f_2(x) = \dfrac{1}{x}, f_3(x) = 1 - x, f_4(x) = \dfrac{x}{x-1}, f_5(x) = \dfrac{1}{1-x}$, and $f_6(x) = \dfrac{x-1}{x}$,

i) $f_2(f_3(x)) = f_2(1 - x)$

$$= \frac{1}{1-x}, x \neq 1$$

ii) $(f_3 \circ f_5)(x) = f_3\left(\dfrac{1}{1-x}\right)$

$$\begin{aligned} &= 1 - \frac{1}{1-x} \\ &= \frac{1-x-1}{1-x} \\ &= \frac{-x}{1-x}, x \neq 1 \end{aligned}$$

iii) $f_1(f_2(x)) = f_1\left(\frac{1}{x}\right)$

$= \frac{1}{x}, x \neq 0$

iv) $f_2(f_1(x)) = f_2(x)$

$= \frac{1}{x}, x \neq 0$

b) $f_6(x) = \frac{x-1}{x}$

$y = \frac{x-1}{x}$

$x = \frac{y-1}{y}$

$xy = y - 1$

$xy - y = -1$

$y(x - 1) = -1$

$y = \frac{-1}{x-1}$

$y = \frac{1}{1-x}$

This is the same as $f_2(f_3(x))$.

Section 10.3 Page 509 Question C1

No, $f(g(x))$ does not mean the same as $(f \bullet g)(x)$. One is a composite function, $f(g(x))$, and the other is the product of functions, $(f \bullet g)(x)$. For example, let $f(x) = x + 4$ and $g(x) = x^2$.

$f(g(x)) = f(x^2)$

$= x^2 + 4$

$(f \bullet g)(x) = (x + 4)(x^2)$

$= x^3 + 4x^2$

Section 10.3 Page 509 Question C2

a) From the list, $f(1) = 5$ and $g(5) = 10$.

So,

$g(f(1)) = g(5)$

$= 10$

b) From the list, $f(3) = 7$ and $g(7) = 0$.

So,

$g(f(3)) = g(7)$

$= 0$

Section 10.3 Page 509 Question C3

For $f(x) = 4 - 3x$ and $g(x) = \frac{4-x}{3}$,

$g(f(x)) = g(4 - 3x)$

$= \frac{4-(4-3x)}{3}$

$= \frac{3x}{3}$

$= x$

$f(g(x)) = f\left(\frac{4-x}{3}\right)$

$= 4 - 3\left(\frac{4-x}{3}\right)$

$= 4 - 4 + x$

$= x$

Yes, $g(f(x)) = f(g(x))$ for all x. The pair of functions are inverses of each other.

Section 10.3 Page 509 Question C4

Step 1:

a) $f(x) = 2x + 3$

$f(x + h) = 2(x + h) + 3$

$f(x + h) = 2x + 2h + 3$

b)
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{2x+2h+3-(2x+3)}{h}\\ &= \frac{2x+2h+3-2x-3}{h}\\ &= \frac{2h}{h}\\ &= 2\end{aligned}$$

Step 2:

a) $f(x) = -3x - 5$

$f(x + h) = -3(x + h) - 5$

$f(x + h) = -3x - 3h - 5$

b)
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{-3x-3h-5-(-3x-5)}{h}\\ &= \frac{-3x-3h-5+3x+5}{h}\\ &= \frac{-3h}{h}\\ &= -3\end{aligned}$$

Step 3: I predict that $\frac{f(x+h)-f(x)}{h}$ for $f(x) = \frac{3}{4}x - 5$ will equal $\frac{3}{4}$.

Each value is the slope of the related linear function.

Chapter 10 Review

Chapter 10 Review Page 510 Question 1

For $f(x) = 3x - 1$ and $g(x) = 2x + 7$,

a) $(f + g)(x) = f(x) + g(x)$

$= 3x - 1 + 2x + 7$

$= 5x + 6$

$(f + g)(4) = 5(4) + 6$

$= 26$

b) From part a), $(f + g)(x) = 5x + 6$.

$(f + g)(-1) = 5(-1) + 6$

$= 1$

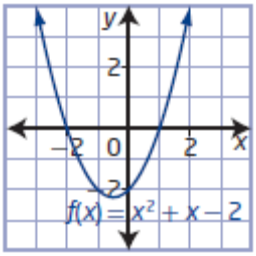
c) $(f-g)(x) = f(x) - g(x)$
$= 3x - 1 - (2x + 7)$
$= x - 8$
$(f-g)(3) = 3 - 8$
$= -5$

d) $(g-f)(x) = g(x) - f(x)$
$= 2x + 7 - (3x - 1)$
$= -x + 8$
$(g-f)(-5) = -(-5) + 8$
$= 13$

Chapter 10 Review Page 510 Question 2

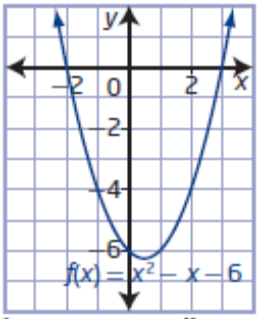
a) For $g(x) = x + 2$ and $h(x) = x^2 - 4$,

i) $f(x) = g(x) + h(x)$
$= x + 2 + x^2 - 4$
$= x^2 + x - 2$



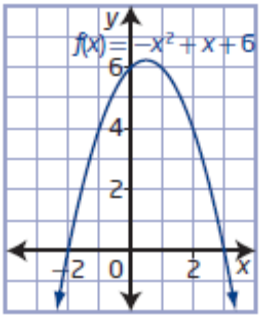
domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \geq -2.25, y \in \mathrm{R}\}$

ii) $f(x) = h(x) - g(x)$
$= x^2 - 4 - (x + 2)$
$= x^2 - x - 6$



domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \geq -6.25, y \in \mathrm{R}\}$

iii) $f(x) = g(x) - h(x)$
$= x + 2 - (x^2 - 4)$
$= -x^2 + x + 6$



domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \leq 6.25, y \in \mathrm{R}\}$

b)

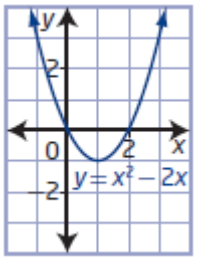
i) $f(x) = x^2 + x - 2$
$f(2) = 2^2 + 2 - 2$
$f(2) = 4$

ii) $f(x) = x^2 - x - 6$
$f(2) = 2^2 - 2 - 6$
$f(2) = -4$

iii) $f(x) = -x^2 + x + 6$
$f(2) = -2^2 + 2 + 6$
$f(2) = 4$

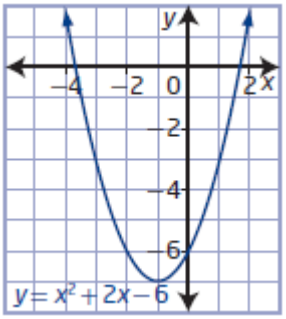
Chapter 10 Review Page 510 Question 3

a) For $f(x) = x^2 - 3$ and $g(x) = -2x + 3$,
$y = (f + g)(x)$
$= f(x) + g(x)$
$= x^2 - 3 + (-2x + 3)$
$= x^2 - 2x$



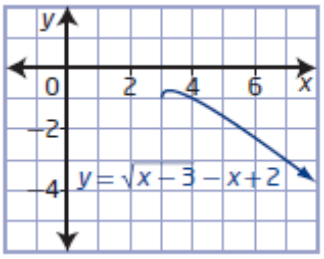
The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f + g)(x)$ is $\{y \mid y \geq -1, y \in \mathrm{R}\}$.

$y = (f - g)(x)$
$= f(x) - g(x)$
$= x^2 - 3 - (-2x + 3)$
$= x^2 + 2x - 6$



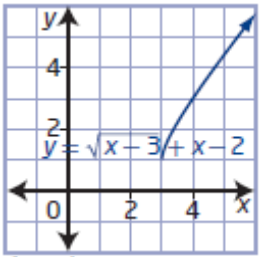
The domain of $y = (f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid y \geq -7, y \in \mathrm{R}\}$.

b) For $f(x) = \sqrt{x-3}$ and $g(x) = -x + 2$,
$y = (f + g)(x)$
$= f(x) + g(x)$
$= \sqrt{x-3} + (-x + 2)$
$= \sqrt{x-3} - x + 2$



The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$:
$\{x \mid x \geq 3, x \in \mathrm{R}\}$.
From the graph, the range of $y = (f + g)(x)$ is approximately $\{y \mid y \geq -0.75, y \in \mathrm{R}\}$.

$y = (f - g)(x)$
$= f(x) - g(x)$
$= \sqrt{x-3} - (-x + 2)$
$= \sqrt{x-3} + x - 2$

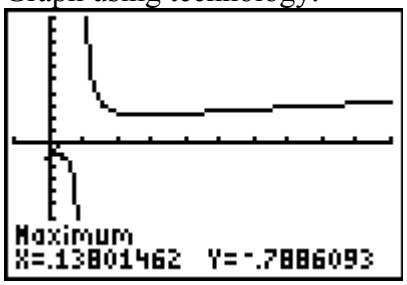


The domain of $y = (f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 3, x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid y \geq 1, y \in \mathrm{R}\}$.

Chapter 10 Review Page 510 Question 4

a) For $f(x) = \dfrac{1}{x-1}$ and $g(x) = \sqrt{x}$,
$(f + g)(x) = f(x) + g(x)$
$= \dfrac{1}{x-1} + \sqrt{x}$

The domain of $(f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 0, x \neq 1, x \in \mathrm{R}\}$.
Graph using technology.



The range of $(f + g)(x)$ is approximately $\{y \mid y \leq -0.79 \text{ or } y \geq 2.2, y \in \mathrm{R}\}$.

b) For $f(x) = \dfrac{1}{x-1}$ and $g(x) = \sqrt{x}$,

$(f-g)(x) = f(x) - g(x)$

$= \dfrac{1}{x-1} - \sqrt{x}$

The domain of $(f-g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 0, x \neq 1, x \in \mathrm{R}\}$.

Graph using technology.

The range of $(f+g)(x)$ is $\{y \mid y \in \mathrm{R}\}$.

Chapter 10 Review Page 510 Question 5

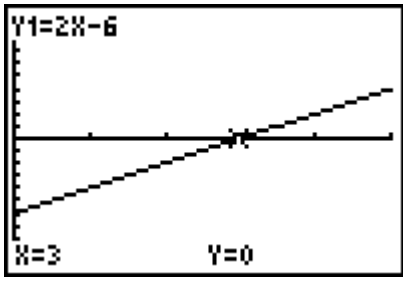
a) An expression that reflects the net change in population at any given time is

$P = b(x) - d(x)$

$= -4x + 78 - (-6x + 84)$

$= 2x - 6$

b) The net change in population will continue to increase, going from negative to positive at $x = 3$.



c) The population begins to increase after year 3, since the net change is then positive.

Chapter 10 Review Page 510 Question 6

For $g(x) = x + 2$ and $h(x) = x^2 - 4$,

a) $f(x) = g(x)h(x)$

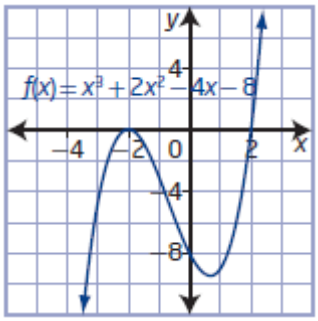
$= (x + 2)(x^2 - 4)$

$= x^3 + 2x^2 - 4x - 8$

The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \in \mathrm{R}\}$.

The range of $f(x)$ is $\{y \mid y \in \mathrm{R}\}$.

There are no asymptotes.

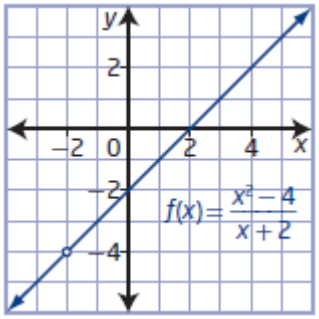


b) $f(x) = \dfrac{h(x)}{g(x)}$

$= \dfrac{x^2 - 4}{x + 2}$

$= \dfrac{(x + 2)(x - 2)}{x + 2}$

$= x - 2, x \neq -2$



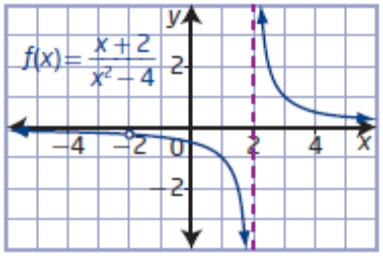
The domain of $f(x)$ consists of all values that are in both the domain of $h(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -2, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -4, y \in \mathrm{R}\}$.
There are no asymptotes.

c) $f(x) = \dfrac{g(x)}{h(x)}$

$= \dfrac{x + 2}{x^2 - 4}$

$= \dfrac{x + 2}{(x + 2)(x - 2)}$

$= \dfrac{1}{x - 2}, x \neq -2, 2$



The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$, excluding values of x where $h(x) = 0$: $\{x \mid x \neq -2, 2, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -0.25, 0, y \in \mathrm{R}\}$.
The equation of the asymptotes are $x = 2$ and $y = 0$.

Chapter 10 Review Page 510 Question 7

a) $f(x) = x^3 + 2x^2 - 4x - 8$
$f(-2) = (-2)^3 + 2(-2)^2 - 4(-2) - 8$
$f(-2) = -8 + 8 + 8 - 8$
$f(-2) = 0$

b) Since $f(x) = x - 2, x \neq -2, f(-2)$ does not exist.

c) Since $f(x) = \dfrac{1}{x - 2}, x \neq -2, 2, f(-2)$ does not exist.

Chapter 10 Review Page 511 Question 8

For $g(x) = \frac{1}{x+4}$ and $h(x) = \frac{1}{x^2-16}$,

a) $f(x) = g(x)h(x)$

$$= \frac{1}{x+4}\left(\frac{1}{x^2-16}\right)$$

$$= \frac{1}{x^3+4x^2-16x-64}, x \neq -4, 4$$

The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq 0, y \in \mathrm{R}\}$.

b) $f(x) = \frac{g(x)}{h(x)}$

$$= \frac{\frac{1}{x+4}}{\frac{1}{x^2-16}}$$

$$= \frac{x^2-16}{x+4}$$

$$= \frac{(x+4)(x-4)}{x+4}$$

$$= x-4, x \neq -4, 4$$

The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$, excluding values of x where $h(x) = 0$: $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -8, 0, y \in \mathrm{R}\}$.

c) $f(x) = \frac{h(x)}{g(x)}$

$$= \frac{\frac{1}{x^2-16}}{\frac{1}{x+4}}$$

$$= \frac{x+4}{x^2-16}$$

$$= \frac{x+4}{(x+4)(x-4)}$$

$$= \frac{1}{x-4}, x \neq -4, 4$$

The domain of $f(x)$ consists of all values that are in both the domain of $h(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -0.125, 0, y \in \mathrm{R}\}$.

Chapter 10 Review Page 511 Question 9

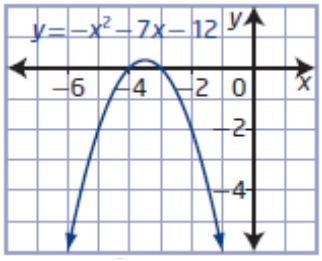
a) For $f(x) = x + 3$ and $g(x) = -x - 4$,

$y = (f \cdot g)(x)$

$= f(x)g(x)$

$= (x + 3)(-x - 4)$

$= -x^2 - 7x - 12$



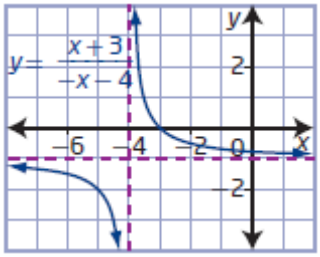
The domain of $y = (f \cdot g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.

The range of $y = (f \cdot g)(x)$ is $\{y \mid y \leq 0.25, y \in \mathrm{R}\}$.

$$y = \left(\frac{f}{g}\right)(x)$$

$$= \frac{f(x)}{g(x)}$$

$$= \frac{x+3}{-x-4}, x \neq -4$$



The domain of $y = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -4, x \in \mathrm{R}\}$.

The range of $y = (f - g)(x)$ is $\{y \mid \neq -1, y \in \mathrm{R}\}$.

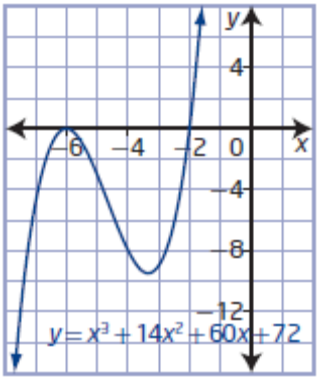
b) For $f(x) = x^2 + 8x + 12$ and $g(x) = x + 6$,

$y = (f \cdot g)(x)$

$= f(x)g(x)$

$= (x^2 + 8x + 12)(x + 6)$

$= x^3 + 14x^2 + 60x + 72$



The domain of $y = (f \cdot g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.

The range of $y = (f \cdot g)(x)$ is $\{y \mid y \in \mathrm{R}\}$.

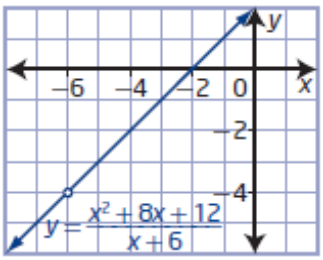
$$y = \left(\frac{f}{g}\right)(x)$$

$$= \frac{f(x)}{g(x)}$$

$$= \frac{x^2+8x+12}{x+6}$$

$$= \frac{(x+2)(x+6)}{x+6}$$

$= x + 2, x \neq -6$



The domain of $y = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $g(x) = 0$: $\{x \mid x \neq -6, x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid \neq -4, y \in \mathrm{R}\}$.

Chapter 10 Review Page 511 Question 10

Given: $f(x) = x^2$ and $g(x) = x + 1$.

a) Determine $g(-2)$.

$g(x) = x + 1$
$g(-2) = -2 + 1$
$g(-2) = -1$
Substitute $g(-2) = -1$ into $f(x)$.
$f(g(-2)) = f(-1)$
$= (-1)^2$
$= 1$

b) Determine $f(-2)$.

$f(x) = x^2$
$f(-2) = (-2)^2$
$f(-2) = 4$
Substitute $f(-2) = 4$ into $g(x)$.
$g(f(-2)) = g(4)$
$= 4 + 1$
$= 5$

Chapter 10 Review Page 511 Question 11

Given: $f(x) = 2x^2$ and $g(x) = \frac{4}{x}$,

a) $f(g(x)) = f\left(\frac{4}{x}\right)$

$= 2\left(\frac{4}{x}\right)^2$

$= 2\left(\frac{16}{x^2}\right)$

$= \frac{32}{x^2}, x \neq 0$

b) $g(f(x)) = g(2x^2)$

$= \frac{4}{2x^2}$

$= \frac{2}{x^2}, x \neq 0$

c) $g(f(x)) = \frac{2}{x^2}$

$g(f(-2)) = \frac{2}{(-2)^2}$

$g(f(-2)) = \frac{1}{2}$

Chapter 10 Review Page 511 Question 12

a) For $f(x) = -\frac{2}{x}$ and $g(x) = \sqrt{x}$,

$y = f(g(x))$

$= f\left(\sqrt{x}\right)$

$= -\frac{2}{\sqrt{x}}, x > 0$

b) The domain of $y = f(g(x))$ is the set of all values of x in the domain of g for which $g(x)$ is in the domain of f: $\{x \mid x > 0, x \in \mathrm{R}\}$. The range of $y = f(g(x))$ is $\{y \mid y < 0, y \in \mathrm{R}\}$.

Chapter 10 Review Page 511 Question 13

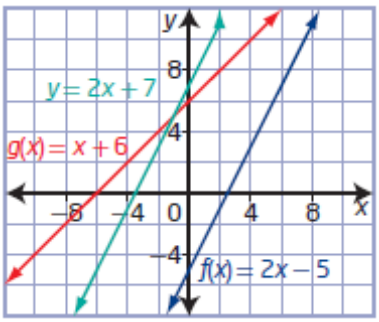
For $f(x) = 2x - 5$ and $g(x) = x + 6$,

$y = (f \circ g)(x)$

$= f(x + 6)$

$= 2(x + 6) - 5$

$= 2x + 7$



Chapter 10 Review Page 511 Question 14

First, determine a model for travel down the shaft: $d = 5t$. The temperature as a function of time can be modelled by

$T(d(t)) = T(5t)$

$= 0.01(5t) + 20$

$= 0.05t + 20$

Chapter 10 Review Page 511 Question 15

a) Let x represent the current price of the tablet. Then, the price, d, of the tablet after the discount is $d(x) = 0.75x$.

The price, c, of the tablet after the coupon as a function of the current price is $c(x) = x - 10$.

b) $c(d(x)) = c(0.75x)$

$= 0.75x - 10$

This represents using the coupon after the discount.

c) $d(c(x)) = d(x - 10)$

$= 0.75(x - 10)$

$= 0.75x - 7.5$

This represents applying the discount after the coupon.

d) Comparing the functions from part b) and c), $c(d(x)) = 0.75x - 10$ using the coupon after the discount, will result in the lower sale price of $290.

Chapter 10 Practice Test

Chapter 10 Practice Test Page 512 Question 1

For $f(x) = (x + 3)^2$ and $g(x) = x + 4$,

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= (x + 3)^2 + x + 4 \\ &= x^2 + 6x + 9 + x + 4 \\ &= x^2 + 7x + 13 \end{aligned}$$

Choice **B**.

Chapter 10 Practice Test Page 512 Question 2

For $f(x) = x + 8$ and $g(x) = 2x^2 - 128$,

$$\begin{aligned} y &= \frac{g(x)}{f(x)} \\ &= \frac{2x^2 - 128}{x + 8} \\ &= \frac{2(x^2 - 64)}{x + 8} \\ &= \frac{2(x - 8)(x + 8)}{x + 8} \\ &= 2(x - 8) \\ &= 2x - 16, \; x \neq -8 \end{aligned}$$

The domain of $y = \frac{g(x)}{f(x)}$ consists of all values that are in both the domain of $g(x)$ and the domain of $f(x)$, excluding values of x where $f(x) = 0$: $\{x \mid x \neq -8, x \in \mathrm{R}\}$.
Choice **D**.

Chapter 10 Practice Test Page 512 Question 3

From the graph, $f(x) = x^2 + 2$ and $g(x) = x$.

$$\begin{aligned} g(x) - f(x) &= x - (x^2 + 2) \\ &= -x^2 + x - 2 \end{aligned}$$

The equation of the parabola in vertex form is $y = \left(x - \frac{1}{2}\right)^2 - \frac{7}{4}$. Since this parabola opens downward and has vertex $\left(\frac{1}{2}, -\frac{7}{4}\right)$, the entire parabola is below the x-axis.

Therefore, $g(x) - f(x) < 0$ is true for $x \in \mathrm{R}$: choice **A**.

The other options can be shown to be false.
For choice B, consider $x = -1$. From the graph $f(-1) = 3$ and $g(-1) = -1$, so $\frac{f(-1)}{g(-1)} = \frac{3}{-1}$ or -3, which is less than 1. Therefore choice B is not true.
Choice C is not true by observing the graph. For all value of x, $f(x) > g(x)$.
For choice D, consider $x = -1$. $g(-1) + f(-1) = 3 + (-1)$ or 2. So $g(x) + f(x) > 0$ at $x = -1$.
Choice D is not true.

Chapter 10 Practice Test Page 512 Question 4

For $f(x) = 5 - x$ and $g(x) = 2\sqrt{3x}$,
$f(g(x)) = f(2\sqrt{3x})$
$= 5 - 2\sqrt{3x}$
$f(g(3)) = 5 - 2\sqrt{3(3)}$
$= 5 - 2(3)$
$= -1$
Choice **C**.

Chapter 10 Practice Test Page 512 Question 5

For $f(x) = x + 5$ and $g(x) = x^2$,
$y = f(g(x))$
$= f(x^2)$
$= x^2 + 5$
Choice **A**.

Chapter 10 Practice Test Page 512 Question 6

For $f(x) = \sin x$ and $g(x) = 2x^2$,

a) $h(x) = (f + g)(x)$
$= f(x) + g(x)$
$= \sin x + 2x^2$

b) $h(x) = (f - g)(x)$
$= f(x) - g(x)$
$= \sin x - 2x^2$

c) $h(x) = (f \cdot g)(x)$
$= f(x)g(x)$
$= (\sin x)2x^2$

d) $h(x) = \left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{\sin x}{2x^2}, x \neq 0$

Chapter 10 Practice Test Page 512 Question 7

Use mental math.

	$g(x)$	$f(x)$	$(f+g)(x)$	$(f \circ g)(x)$
a)	$x-8$	$\sqrt{x}$	$\sqrt{x}+x-8$	$\sqrt{x-8}$
b)	$x+3$	$4x$	$5x+3$	$4x+12$
c)	x^2	$\sqrt{x-4}$	$\sqrt{x-4}+x^2$	$\sqrt{x^2-4}$
d)	$\frac{1}{x}$	$\frac{1}{x}$	$\frac{2}{x}$	x

Chapter 10 Practice Test Page 512 Question 8

For $g(x) = \frac{1}{1+x}$ and $h(x) = \frac{1}{3+2x}$,

$y = g(x)h(x)$

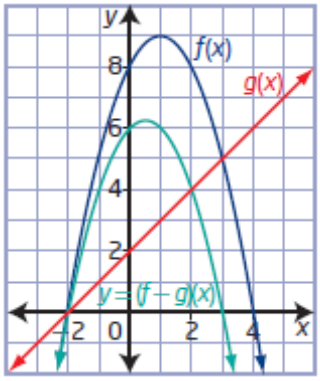
$= \frac{1}{1+x}\left(\frac{1}{3+2x}\right)$

$= \frac{1}{2x^2+5x+3}, x \neq -1.5, -1$

The domain of $y = g(x)h(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \neq -1.5, -1, x \in \mathrm{R}\}$.

Chapter 10 Practice Test Page 513 Question 9

From the graph, $f(x) = -(x-1)^2 + 9$ and $g(x) = x + 2$.



a) $y = (f-g)(x)$

$= f(x) - g(x)$

$= -(x-1)^2 + 9 - (x+2)$

$= -x^2 + 2x - 1 + 9 - x - 2$

$= -x^2 + x + 6$

b) $y = \left(\frac{f}{g}\right)(x)$

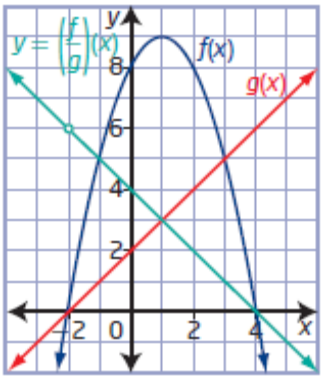
$= \frac{f(x)}{g(x)}$

$= \frac{-(x-1)^2+9}{x+2}$

$= \frac{-x^2+2x+8}{x+2}$

$= \frac{-(x-4)(x+2)}{x+2}$

$= -x + 4, x \neq -2$



Chapter 10 Practice Test Page 513 Question 10

a) For $f(x) = 3 - x$ and $g(x) = |x + 3|$,
$g(f(x)) = g(3 - x)$
$= |3 - x + 3|$
$= |6 - x|$
The domain of $f(x)$ is $\{x \mid x \in \mathrm{R}\}$. The domain of $g(x)$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(f(x))$ is the set of all values of x in the domain of f for which $f(x)$ is in the domain of g: $\{x \mid x \in \mathrm{R}\}$. The range of $g(f(x))$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) For $f(x) = 4^x$ and $g(x) = x + 1$,
$g(f(x)) = g(4^x)$
$= 4^x + 1$
The domain of $f(x)$ is $\{x \mid x \in \mathrm{R}\}$. The domain of $g(x)$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(f(x))$ is the set of all values of x in the domain of f for which $f(x)$ is in the domain of g: $\{x \mid x \in \mathrm{R}\}$. The range of $g(f(x))$ is $\{y \mid y \geq 1, y \in \mathrm{R}\}$.

c) For $f(x) = x^4$ and $g(x) = \sqrt{x}$,
$g(f(x)) = g(x^4)$
$= \sqrt{x^4}$
$= x^2$
The domain of $f(x)$ is $\{x \mid x \in \mathrm{R}\}$. The domain of $g(x)$ is $\{x \mid x \geq 0, x \in \mathrm{R}\}$.
The domain of $g(f(x))$ is the set of all values of x in the domain of f for which $f(x)$ is in the domain of g: $\{x \mid x \in \mathrm{R}\}$. The range of $g(f(x))$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

Chapter 10 Practice Test Page 513 Question 11

a) Let x represent Becky's earnings per pay period.
Then, her income, r, after the retirement deduction as a function of her earnings per pay period can be modelled by $r(x) = x - 200$.
Her income, t, after federal taxes as a function of her earnings per pay period can be modelled by $t(x) = 0.72x$.

b) $t(r(x)) = t(x - 200)$
$= 0.72(x - 200)$
$= 0.72x - 144$
This represents determining federal income taxes after Becky's retirement deduction.

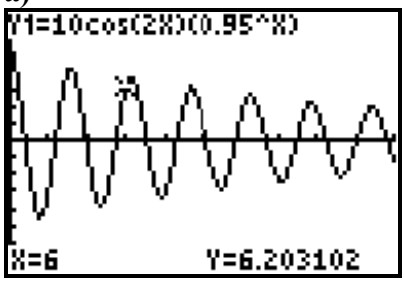
c) Substitute $x = 2700$.
$t(r(x)) = 0.72x - 144$
$t(r(2700)) = 0.72(2700) - 144$
$t(r(2700)) = 1800$
Becky's net income is $1800.

d) $r(t(x)) = r(0.72x)$
$= 0.72x - 200$
Substitute $x = 2700$.
$r(t(x)) = 0.72x - 200$
$r(t(2700)) = 0.72(2700) - 200$
$r(t(2700)) = 1744$
Becky's net income is \$1744.

e) The calculation order changes the net income. If federal income tax is calculated using the income after the retirement deduction, Becky receives more money.

Chapter 10 Practice Test Page 513 Question 12

a)

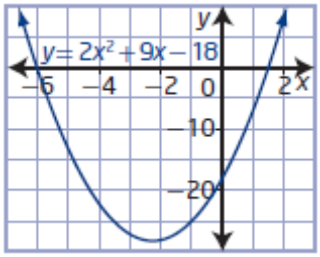


b) $x(t) = (10 \cos 2t)(0.95^t)$
$= f(t)g(t)$
Then, $f(t) = 10 \cos 2t$ and $g(t) = 0.95^t$.
The function $f(t)$ is responsible for the periodic motion, while the function $g(t)$ is responsible for the exponential decay of the amplitude.

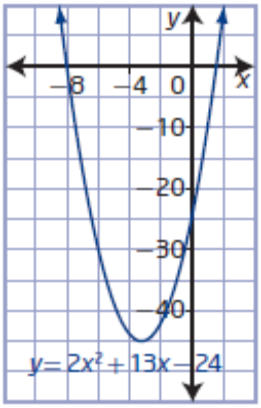
Chapter 10 Practice Test Page 513 Question 13

For $f(x) = 2x^2 + 11x - 21$ and $g(x) = 2x - 3$,

a) $y = f(x) - g(x)$
$= 2x^2 + 11x - 21 - (2x - 3)$
$= 2x^2 + 9x - 18$



b) $y = f(x) + g(x)$
$= 2x^2 + 11x - 21 + 2x - 3$
$= 2x^2 + 13x - 24$

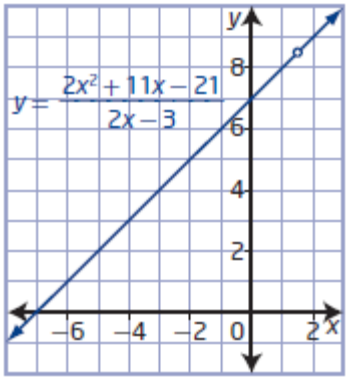


c) $y = \dfrac{f(x)}{g(x)}$

$= \dfrac{2x^2 + 11x - 21}{2x - 3}$

$= \dfrac{(2x - 3)(x + 7)}{2x - 3}$

$= x + 7, x \neq 1.5$



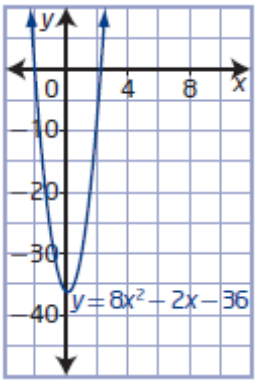
d) $y = f(g(x))$

$= f(2x - 3)$

$= 2(2x - 3)^2 + 11(2x - 3) - 21$

$= 2(4x^2 - 12x + 9) + 22x - 33 - 21$

$= 8x^2 - 2x - 36$



Chapter 10 Practice Test Page 513 Question 14

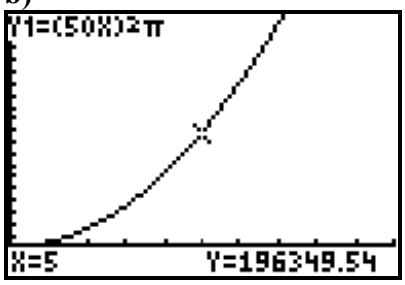
a) Let r represent the radius of the circular ripple. Then, the area of the circular ripple is given by $A(r) = \pi r^2$. The radius changes with time according to $r(t) = 50t$.
An equation that represents the area of the circle as a function of time is the composite function $A(r(t))$.

$A(r(t)) = A(50t)$

$= \pi(50t)^2$

$= 2500\pi t^2$

b)



c) Substitute $t = 5$.

$A(r(t)) = 2500\pi t^2$

$A(r(5)) = 2500\pi(5)^2$

$A(r(5)) = 196\ 349.540\ldots$

The area of the circle after 5 s is approximately 196 350 cm^2.

d) Example: No. In 30 s, the radius would be 1500 cm. Most likely the circular ripples would no longer be visible on the surface of the water due to turbulence.

Chapter 11 Permutations, Combinations, and the Binomial Theorem

Section 11.1 Permutations

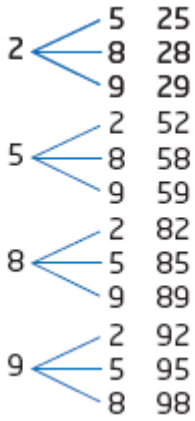
Section 11.1 Page 524 Question 1

a)

Position 1	Position 2	Position 3
Jo	Amy	Mike
Jo	Mike	Amy
Amy	Jo	Mike
Amy	Mike	Jo
Mike	Jo	Amy
Mike	Amy	Jo

There are six different arrangements.

b)



There are 12 different two-digit numbers.

c)

Starter	Main	Dessert
soup	chili	ice cream
soup	chili	fruit salad
soup	hamburger	ice cream
soup	hamburger	fruit salad
soup	chicken	ice cream
soup	chicken	fruit salad
soup	fish	ice cream
soup	fish	fruit salad
salad	chili	ice cream
salad	chili	fruit salad
salad	hamburger	ice cream
salad	hamburger	fruit salad
salad	chicken	ice cream
salad	chicken	fruit salad
salad	fish	ice cream
salad	fish	fruit salad

There are 16 different meals.

Section 11.1 Page 524 Question 2

a)

$$
\begin{aligned}
{}_8P_2 &= \frac{8!}{(8-2)!} \\
&= \frac{8!}{6!} \\
&= \frac{8(7)\,\cancel{(6!)}^{1}}{\cancel{6!}_{1}} \\
&= 56
\end{aligned}
$$

b)

$$
\begin{aligned}
{}_7P_5 &= \frac{7!}{(7-5)!} \\
&= \frac{7!}{2!} \\
&= \frac{7(6)(5)(4)(3)\,\cancel{(2!)}^{1}}{\cancel{2!}_{1}} \\
&= 2520
\end{aligned}
$$

c)

$$
\begin{aligned}
{}_6P_6 &= \frac{6!}{(6-6)!} \\
&= \frac{6!}{0!} \\
&= \frac{6(5)(4)(3)(2)(1)}{1} \\
&= 720
\end{aligned}
$$

d)

$$
\begin{aligned}
{}_4P_1 &= \frac{4!}{(4-1)!} \\
&= \frac{4!}{3!} \\
&= \frac{4\,\cancel{(3!)}^{1}}{\cancel{3!}_{1}} \\
&= 4
\end{aligned}
$$

Section 11.1 Page 524 Question 3

Left Side = 4! + 3!
= 4(3!) + 3!
= (4 + 1)(3!)
= 5(3!)

Right Side = (4 + 3)!
= 7!

Left Side ≠ Right Side

Section 11.1 Page 524 Question 4

a) 9! = 9(8)(7)(6)(5)(4)(3)(2)(1)
= 362 880

b)

$$
\begin{aligned}
\frac{9!}{5!4!} &= \frac{9(8)(7)(6)\,\cancel{(5!)}^{1}}{\cancel{5!}_{1}(4)(3)(2)(1)} \\
&= 126
\end{aligned}
$$

c) (5!)(3!) = 5(4)(3)(2)(1)(3)(2)(1)
= 720

d) 6(4!) = 6(4)(3)(2)(1)
= 144

e) $\frac{102!}{100!2!} = \frac{102(101)\cancel{(100!)}^{1}}{\cancel{100!}_{1}(2)(1)}$

$= 5151$

f) $7! - 5!$

$= 7(6)(5)(4)(3)(2)(1) - 5(4)(3)(2)(1)$

$= 5040 - 120$

$= 4920$

Section 11.1 Page 524 Question 5

a) There are six letters in *hoodie*. There are 2! ways to arrange the two *o*'s.

$\frac{6!}{2!} = \frac{6(5)(4)(3)\cancel{(2!)}^{1}}{\cancel{2!}_{1}}$

$= 360$

The number of different six-letter arrangements is 360.

b) There are seven letters in *decided*. There are 3! ways to arrange the three *d*'s and 2! ways to arrange the two *e*'s

$\frac{7!}{3!2!} = \frac{7(6)(5)(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}(2)(1)}$

$= 420$

The number of different seven-letter arrangements is 420.

c) There are 11 letters in *aqilluqqaaq*. There are 3! ways to arrange the three *a*'s, 4! ways to arrange the four *q*'s, and 2! ways to arrange the two *l*'s.

$\frac{11!}{3!4!2!} = \frac{11(10)(9)(8)(7)(6)(5)\cancel{(4!)}^{1}}{3(2)(1)\cancel{4!}_{1}(2)(1)}$

$= 138\ 600$

The number of different 11-letter arrangements is 138 600.

d) There are six letters in *deeded*. There are 3! ways to arrange the three *d*'s and 3! ways to arrange the three *e*'s

$\frac{6!}{3!3!} = \frac{6(5)(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}(3)(2)(1)}$

$= 20$

The number of different six-letter arrangements is 20.

e) There are five letters in *puppy*. There are 3! ways to arrange the three *p*'s.

$$\frac{5!}{3!} = \frac{5(4)\,\cancel{(3!)}^{1}}{\cancel{3!}_{1}}$$

$$= 20$$

The number of different five-letter arrangements is 20.

f) There are eight letters in *baguette*. There are 2! ways to arrange the two *e*'s and 2! ways to arrange the two *t*'s.

$$\frac{8!}{2!2!} = \frac{8(7)(6)(5)(4)(3)\,\cancel{(2!)}^{1}}{\cancel{2!}_{1}(2)(1)}$$

$$= 10\ 080$$

The number of different six-letter arrangements is 10 080.

Section 11.1 Page 525 Question 6

$$4! = 4(3)(2)(1)$$
$$= 24$$

The four names can be listed in 24 ways.

Section 11.1 Page 525 Question 7

a)

$${}_nP_2 = 30$$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)\,\cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$$

$$n(n-1) = 6(5)$$

Using reasoning, the solution to ${}_nP_2$ is $n = 6$.

b)

$${}_nP_3 = 990$$

$$\frac{n!}{(n-3)!} = 990$$

$$\frac{n(n-1)(n-2)\,\cancel{(n-3)!}}{\cancel{(n-3)!}} = 990$$

$$n(n-1)(n-2) = 11(10)(9)$$

Using reasoning, the solution to ${}_nP_3$ is $n = 11$.

c) $${}_6P_r = 30$$

$$\frac{6!}{(6-r)!} = 30$$

$$\frac{6!}{30} = (6-r)!$$

$$\frac{\cancel{6}\ (5)(4!)}{\cancel{30}} = (6-r)!$$

$$4! = (6-r)!$$

$$4 = 6-r$$

$$r = 2$$

Using reasoning, the solution to ${}_6P_r$ is $r = 2$.

d) $$2({}_nP_2) = 60$$

$${}_nP_2 = \frac{60}{2}$$

$${}_nP_2 = 30$$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)\ \cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$$

$$n(n-1) = 6(5)$$

Using reasoning, the solution to $2({}_nP_2)$ is $n = 6$.

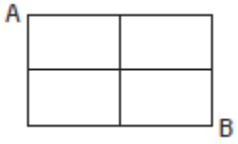
Section 11.1 Page 525 Question 8

a) Determine the number of arrangements of moving down 2 units and right 2 units.

$$\frac{4!}{2!2!} = \frac{4(3)(2!)}{2(1)(2!)}$$

$$= 6$$

There are 6 pathways from A to B.

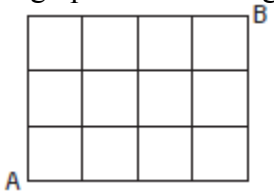


b) Determine the number of arrangements of moving up 3 units and right 4 units.

$$\frac{7!}{3!4!} = \frac{7(6)(5)(4!)}{3(2)(1)(4!)}$$

$$= 35$$

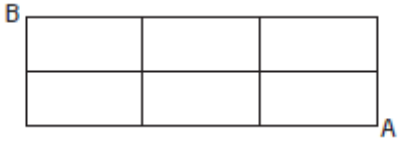
There are 35 pathways from A to B.



c) Determine the number of arrangements of moving up 2 units and left 3 units.

$$\frac{5!}{2!3!} = \frac{5(4)(3!)}{2(1)(3!)}$$
$$= 10$$

There are 10 pathways from A to B.



Section 11.1 Page 525 Question 9

a) Case 1: first digit is 3 or 5; Case 2: first digit is 2 or 4

b) Case 1: first letter is B; Case 2: first letter is E

Section 11.1 Page 525 Question 10

a) The boys can be arranged on the ends in 2! = 2 ways. The girls can be arranged in the middle in 4! = 24 ways.

2!4! = 2(24)
= 48

The two boys and four girls can be arranged in 48 ways.

b) There are 2! = 2 ways to arrange the boys together. Considering the boys as one object means there are five objects to arrange in 5! = 120 ways. Since the boys must be seated together, there are 5!2! = 240 ways to arrange the two boys and four girls.

c) There are 2! = 2 ways to arrange the boys in the middle of the row. There are 4! = 24 ways to arrange the girls. Since the boys must be seated together, there are 4!2! = 48 ways to arrange the two boys and four girls.

Section 11.1 Page 525 Question 11

a) 7! = 7(6)(5)(4)(3)(2)(1)
= 5040

Seven books can be arranged in 7! = 5040 ways.

b) $\frac{7!}{2!} = \frac{7(6)(5)(4)(3)(2!)}{(2!)}$
$= 2520$

If two of the books are identical, the seven books can be arranged in 2520 ways.

c) Case 1: mathematics book is on the left end

There are 6! = 720 ways to arrange the other six books.

Case 2: mathematics book is not on the left end, so it must be on the right end

There are 6! = 720 ways to arrange the other six books.

The books can be arranged in 720 + 720 = 1440 ways.

d) There are 4! = 24 ways to arrange the four particular books together. Considering those four books as one object means there are 3 + 1 = 4 objects to arrange in 4! = 24 ways. Since four of the books must be together, there are 4!4! = 576 ways to arrange the books.

Section 11.1 Page 525 Question 12

The letters can be arranged in 6! = 720 ways.

First letter	Middle letters	Last letter
4 choices	4! choices	4 – 1 = 3 choices

Total number of arrangements that begin and end with a consonant: 4(4!)(3) = 288

Section 11.1 Page 525 Question 13

First character	Last 3 characters
26 – 1 = 25 choices	10(9)(8) = 720 choices

total number of different codes: 25(720) = 18 000
Since 18 000 < 25 300, the organization will not have enough different ID codes for its members.

Section 11.1 Page 525 Question 14

Iblauk has five colour choices for the body of the mitt and four choices for the wrist. She can make 5(4) = 20 different colour combinations of mitts.

Section 11.1 Page 525 Question 15

There are 40 choices for each of the three numbers in the sequence.
40(40)(40) = 64 000
Total time to test all combinations: 64 000(15 s) = 960 000 s
Convert 960 000 s to hours:

$$960\ 000\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right) = 266\frac{2}{3}\ \text{h}$$

It would take $266\frac{2}{3}$ h to test all possible combinations.

Section 11.1 Page 526 Question 16

a) Jodi can park the vehicles in 7! = 5040 ways.

b) There are 2! = 2 ways to arrange the truck and hybrid together. Considering those two vehicles as one object means there are six objects to arrange in 6! = 720 ways. Since the truck and hybrid must be beside each other, there are 6!2! = 1440 ways to arrange the seven vehicles.

c) Number of ways the vehicles can be parked so the convertible is not next to the subcompact
= total arrangements of the vehicles – number of ways two of the vehicles can be parked beside each other
= 5040 – 1440 (from parts a) and b))
= 3600
There are 3600 ways for the vehicles to be parked without the convertible next to the subcompact.

Section 11.1 Page 526 Question 17

a) There are eight letters in *parallel*. There are 3! ways to arrange the three *l*'s and 2! ways to arrange the two *a*'s

$$\frac{8!}{3!2!} = \frac{8(7)(6)(5)(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}(2)(1)}$$

$$= 3360$$

The number of eight-letter arrangements possible is 3360.

b) Considering the *l*'s as one object means there are 5 + 1 = 6 objects to arrange in 6! = 720 ways. Since there are 2! ways to arrange the two *a*'s, there are

$$\frac{6!}{2!} = \frac{6(5)(4)(3)\cancel{(2!)}}{\cancel{2!}}$$

$$= 360$$

ways to arrange the letters.

Section 11.1 Page 526 Question 18

a) The word has five letters, with two pairs of identical letters. Example: AABBS

b) Examples: teeth, radar, sells

Section 11.1 Page 526 Question 19

First digit	Second digit	Third digit	Last digit
5 possibilities (3, 4, 5, 6, or 8)	9 possibilities	9 possibilities	9 possibilities

Number of integers from 3000 to 8999 that do not contain *7*s: 5(9)(9)(9) = 3645

Section 11.1 Page 526 Question 20

a) 26(10)(26)(10)(26)(10)
There are 17 576 000 different postal codes.

b) Example: Yes, Canada will eventually exceed 15.5 million postal communities.

Section 11.1 Page 526 Question 21

a) For each of the 14 lines, there are 10 pages of lines that could be substituted.
There are 10^{14} arrangements of the lines

b) Yes, the title is reasonable because 10^{14} = 100 000 000 000 000, which is 100 million million.

Section 11.1 Page 526 Question 22

a)

$$_3P_r = 3!$$
$$\frac{3!}{(3-r)!} = 3!$$
$$1 = (3-r)!$$
$$0! = (3-r)!$$
$$0 = 3-r$$
$$r = 3$$

b)

$$_7P_r = 7!$$
$$\frac{7!}{(7-r)!} = 7!$$
$$1 = (7-r)!$$
$$0! = (7-r)!$$
$$0 = 7-r$$
$$r = 7$$

c)

$$_nP_3 = 4(_{n-1}P_2)$$
$$\frac{n!}{(n-3)!} = 4\left(\frac{(n-1)!}{(n-1-2)!}\right)$$
$$\frac{n!}{(n-3)!} = \frac{4(n-1)!}{(n-3)!}$$
$$n! = 4(n-1)!$$
$$n(n-1)! = 4(n-1)!$$
$$n = 4$$

d)

$$n(_5P_3) = {}_7P_5$$
$$n\left(\frac{5!}{(5-3)!}\right) = \frac{7!}{(7-5)!}$$
$$\frac{n(5!)}{2!} = \frac{7!}{2!}$$
$$n = \frac{7!}{5!}$$
$$n = \frac{7(6)(5!)}{5!}$$
$$n = 42$$

Section 11.1 Page 526 Question 23

$$_nP_n = \frac{n!}{(n-n)!}$$
$$= \frac{n!}{0!}$$

And $_nP_n = n!$; so, 0! = 1.

Section 11.1 Page 526 Question 24

${}_3P_5$ gives an error message on a calculator because the number of items taken from the set is greater than the number of items in the set.

Section 11.1 Page 526 Question 25

Case 1: One-digit number
3 possibilities for a single digit odd number

Case 2: Two-digit number
4 possibilities for the first digit (1, 2, 3, 4, 5 minus one digit, since the last digit must be odd)
3 possibilities for the last digit (1, 3, 5)
4(3) = 12 ways two-digit numbers to be formed

Case 3: Three-digit number
4 possibilities for the first digit (1, 2, 3, 4, 5 minus one digit, since the last digit must be odd)
4 possibilities for the middle digit (0, 1, 2, 3, 4, 5 minus two, for the first and last digits)
3 possibilities for the last digit (1, 3, 5)
4(4)(3) = 48 ways for three-digit numbers to be formed

The odd numbers can be formed in 3 + 12 + 48 = 63 ways.

Section 11.1 Page 526 Question 26

Case 1a: Four-digit number starting with 2

First digit	Second digit	Third digit	Last digit
1 choice	3 choices	2 choices	1 choice

Four digit numbers can be formed in 1(3)(2)(1) = 6 ways.

Case 1b: Four-digit number starting with 1, 3, or 5

First digit	Second digit	Third digit	Last digit
3 choices	3 choices	2 choices	2 choices

Four digit numbers can be formed in 3(3)(2)(2) = 36 ways.

Case 2a: Five-digit number starting with 2

First digit	Second digit	Third digit	Fourth Digit	Last digit
1 choice	3 choices	2 choices	1 choice	1 choice

Five digit numbers can be formed in 1(3)(2)(1)(1) = 6 ways.

Case 2b: Five-digit number starting with 1, 3, or 5

First digit	Second digit	Third digit	Fourth Digit	Last digit
3 choices	3 choices	2 choices	1 choice	2 choices

Five digit numbers can be formed in 3(3)(2)(1)(2) = 36 ways.

Total number of even numbers: 6 + 36 + 6 + 36 = 84

Section 11.1 Page 526 Question 27

Case 1: One-digit number
8 ways

Case 2: Two-digit number

First digit	Second digit
9 choices	9 choices

Two-digit numbers can be formed in 9(9) = 81 ways

Case 3: Three-digit number

First digit	Second digit	Last digit
9 choices	9 choices	8 choices

Three-digit numbers can be formed in 9(9)(8) = 648 ways

Total number of integers: 8 + 81 + 648 = 737

Section 11.1 Page 527 Question 28

There are six possible locations for the first yellow cube and five possible locations for the other yellow cube.

Total number of arrangements: $\frac{6(5)}{2} = 15$ (Divide by 2 because the cubes are identical.)

Total number of permutations with two objects the same and four objects the same:

$$\frac{6!}{2!4!} = \frac{6(5)(4!)}{2(1)4!}$$
$$= \frac{6(5)}{2}$$
$$= 15$$

Section 11.1 Page 527 Question 29

Consider the cases for each number of sides painted green and other sides painted grey.

Zero sides painted green: 1 way
One side painted green: 1 way
Two sides painted green: 2 ways
Three sides painted green: 2 ways
Four sides painted green: 2 ways
Five sides painted green: 1 way
Six sides painted green: 1 way

Total ways of painting the cube using two colours: 1 + 1 + 2 + 2 + 2 + 1 + 1 = 10

Section 11.1 Page 527 Question 30

Example: Use the numbers 1 to 9 to represent the different students.

Day 1	Day 2	Day 3	Day 4
1 2 3	1 4 7	1 4 9	1 6 8
4 5 6	2 5 8	2 6 7	2 4 9
7 8 9	3 6 9	3 5 8	3 5 7

Section 11.1 Page 527 Question 31

Determine how many factors of 5 there are in 100!. Each multiple of 5 has one factor of 5 except 25, 50, 75, and 100, which have two factors of 5. So, there are 20 + 4 = 24 factors of 5 in 100!. There are more than enough factors of 2 to match up with the 5s to make factors of 10, so there are 24 zeros.

Section 11.1 Page 527 Question 32

a) EDACB or BCADE

b) 2

c) None. Since F only knows A, then F must stand next to A. However, in both arrangements from part a), A must stand between C and D. Therefore, no possible arrangement satisfies the conditions.

Section 11.1 Page 527 Question C1

a) ${}_aP_b$ represents the number of permutations or arrangements in order of b objects taken from a group of a objects. For example, ${}_{20}P_3$ represents the number of arrangements of three council positions taken from a team of 20 members.

b) $b \leq a$ because the number of objects selected must be less than or equal to the number of objects in the set

Section 11.1 Page 527 Question C2

If all of the objects are different, they can be arranged in $n!$ ways. If some of the objects are identical, you need to divide by the number of ways of arranging the objects in each group of identical objects.

Section 11.1 Page 527 Question C3

a) $\dfrac{3!(n+2)!}{4!(n-1)!} = \dfrac{3!(n+2)(n+1)(n)(n-1)!}{4!(n-1)!}$

$= \dfrac{3!(n+2)(n+1)n}{4!}$

$= \dfrac{3!(n+2)(n+1)n}{4(3!)}$

$= \dfrac{(n+2)(n+1)n}{4}$

b) $\dfrac{7!(r-1)!}{6!(r+1)!} + \dfrac{5!r!}{3!(r+1)!} = \dfrac{7(6)(5)(4)\,\cancel{(3!)}^{1}(r-1)!}{6(5)(4)\,\cancel{3!}_{1}(r+1)!} + \dfrac{5(4)\,\cancel{(3!)}^{1}r!}{\cancel{3!}_{1}(r+1)!}$

$= \dfrac{7(6)(5)(4)(r-1)!}{6(5)(4)(r+1)!} + \dfrac{5(4)r!}{(r+1)!} \times \dfrac{6(5)(4)}{6(5)(4)}$

$= \dfrac{7(6)(5)(4)(r-1)! \ + \ 5(4)r!(6)(5)(4)}{6(5)(4)(r+1)!}$

$= \dfrac{7(r-1)! \ + \ 5(4)r!}{(r+1)!}$

$= \dfrac{7(r-1)! \ + \ 5(4)(r)(r-1)!}{(r+1)(r)(r-1)!}$

$= \dfrac{7 \ + \ 5(4)(r)}{(r+1)(r)}$

$= \dfrac{7+20r}{r(r+1)}$

Section 11.1 Page 527 Question C5

a) 9! = 9(8)(7)(6)(5)(4)(3)(2)(1)
= 362 880

b) log (9!) = 5.559 763 …

c) log (10!) = 6.559 763 …

d) The answer to part c) is 1 greater than the answer to part b).
This is because 10! = 10(9!) and log (10!) = log 10 + log (9!) = 1 + log (9!).

Section 11.2 Combinations

Section 11.2 Page 534 Question 1

a) Combination, because the order that members shake hands is not important

b) Permutation, because the order of the digits is important

c) Combination, because the order that the cars are purchased is not important

d) Combination, because the order that players are selected to ride in the van is not important

Section 11.2 Page 534 Question 2

${}_5P_3$ is a permutation representing the number of ways of arranging 3 objects taken from a group of 5 objects. ${}_5C_3$ is a combination representing the number of ways of choosing any 3 objects from a group of 5 objects.

$$
\begin{aligned}
{}_5P_3 &= \frac{5!}{2!} \\
&= \frac{5(4)(3)(2!)}{2!} \\
&= 60
\end{aligned}
\qquad
\begin{aligned}
{}_5C_3 &= \frac{5!}{2!3!} \\
&= \frac{5(4)(3!)}{2(1)3!} \\
&= 10
\end{aligned}
$$

Section 11.2 Page 534 Question 3

a)
$$
\begin{aligned}
{}_6P_4 &= \frac{6!}{2!} \\
&= \frac{6(5)(4)(3)(2!)}{2!} \\
&= 360
\end{aligned}
$$

b)
$$
\begin{aligned}
{}_7C_3 &= \frac{7!}{4!3!} \\
&= \frac{7(6)(5)\cancel{(4!)}^{1}}{\cancel{4!}^{1}(3)(2)(1)} \\
&= 35
\end{aligned}
$$

c)
$$
\begin{aligned}
{}_5C_2 &= \frac{5!}{3!2!} \\
&= \frac{5(4)\cancel{(3!)}^{1}}{\cancel{3!}^{1}(2)(1)} \\
&= 10
\end{aligned}
$$

d)
$$
\begin{aligned}
{}_{10}C_7 &= \frac{10!}{3!7!} \\
&= \frac{10(9)(8)\cancel{(7!)}^{1}}{3(2)(1)\cancel{(7!)}^{1}} \\
&= 120
\end{aligned}
$$

Section 11.2 Page 534 Question 4

a) $${}_{10}C_4 = \frac{10!}{6!4!}$$

$$= \frac{10(9)(8)(7)\ \cancel{(6!)}^{1}}{\cancel{6!}_{1}(4)(3)(2)(1)}$$

$$= 210$$

b) $${}_{10}P_4 = \frac{10!}{6!}$$

$$= \frac{10(9)(8)(7)\ \cancel{(6!)}^{1}}{\cancel{6!}_{1}}$$

$$= 5040$$

Section 11.2 Page 534 Question 5

a) AB, AC, AD, BC, BD, CD

b) AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, DC

c) The number of permutations is 2! times the number of combinations.

Section 11.2 Page 534 Question 6

a) $${}_nC_1 = 10$$

$$\frac{n!}{(n-1)!1!} = 10$$

$$\frac{n!}{(n-1)!} = 10$$

$$\frac{n\ \cancel{(n-1)!}^{1}}{\cancel{(n-1)!}_{1}} = 10$$

$$n = 10$$

b) $${}_nC_2 = 21$$

$$\frac{n!}{(n-2)!2!} = 21$$

$$\frac{n!}{(n-2)!} = 21(2!)$$

$$\frac{n(n-1)\ \cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}} = 21(2)(1)$$

$$n(n-1) = 42$$

$$n^2 - n - 42 = 0$$

$$(n-7)(n+6) = 0$$

$$n - 7 = 0 \quad \text{or} \quad n + 6 = 0$$

$$n = 7 \qquad\qquad n = -6$$

Since $n \geq 2$, the solution is $n = 7$.

c) $${}_nC_{n-2} = 6$$

$$\frac{n!}{(n-(n-2))!(n-2)!} = 6$$

$$\frac{n!}{(n-n+2)!(n-2)!} = 6$$

$$\frac{n!}{2!(n-2)!} = 6$$

$$\frac{n(n-1)\overset{1}{\cancel{(n-2)!}}}{2!\underset{1}{\cancel{(n-2)!}}} = 6$$

$$\frac{n(n-1)}{2!} = 6$$

$$n(n-1) = 12$$

$$n(n-1) = 4(3)$$

$$n = 4$$

d) $${}_{n+1}C_{n-1} = 15$$

$$\frac{(n+1)!}{(n+1-(n-1))!(n-1)!} = 15$$

$$\frac{(n+1)!}{(n+1-n+1)!(n-1)!} = 15$$

$$\frac{(n+1)!}{2!(n-1)!} = 15$$

$$\frac{(n+1)(n)\overset{1}{\cancel{(n-1)!}}}{2!\underset{1}{\cancel{(n-1)!}}} = 15$$

$$\frac{(n+1)(n)}{2!} = 15$$

$$n(n+1) = 15(2)(1)$$

$$n(n+1) = 30$$

$$n(n+1) = 5(6)$$

$$n = 5$$

Section 11.2 Page 534 Question 7

a) Case 1: one-digit numbers, Case 2: two-digit numbers, Case 3: three-digit numbers
b) Cases involve grouping the 4 members from either grade.
Case 1: four grade 11s, Case 2: three grade 11s and one grade 12, Case 3: two grade 11s and two grade 12s, Case 4: one grade 11 and three grade 12s, Case 5: four grade 12s

Section 11.2 Page 534 Question 8

Left Side = ${}_{11}C_3$

$$= \frac{11!}{(11-3)!3!}$$

$$= \frac{11!}{8!3!}$$

Right Side = ${}_{11}C_8$

$$= \frac{11!}{(11-8)!8!}$$

$$= \frac{11!}{3!8!}$$

Left Side = Right Side

Section 11.2 Page 534 Question 9

a) $${}_5C_5 = \frac{5!}{(5-5)!5!}$$

$$= \frac{5!}{0!5!}$$

$$= 1$$

b) ${}_5C_0 = \dfrac{5!}{(5-0)!0!}$

$= \dfrac{5!}{5!0!}$

$= 1$

There is only one way to choose 5 objects from a group of 5 objects, and only one way to choose 0 objects from a group of 5 objects.

Section 11.2 Page 534 Question 10

a) ${}_4C_3 = \dfrac{4!}{(4-3)!3!}$

$= \dfrac{4!}{1!3!}$

$= \dfrac{4\,(3!)}{3!}$

$= 4$

b) Case 1: one coin

${}_4C_1 = \dfrac{4!}{(4-1)!1!}$

$= \dfrac{4!}{3!}$

$= \dfrac{4\,(3!)}{3!}$

$= 4$

Case 2: two coins

${}_4C_2 = \dfrac{4!}{(4-2)!2!}$

$= \dfrac{4!}{2!2!}$

$= \dfrac{4(3)\,(2!)}{2!(2!)}$

$= \dfrac{12}{2(1)}$

$= 6$

Total number of sums: 4 + 6 = 10

Section 11.2 Page 534 Question 11

a) ${}_6C_4 = \dfrac{6!}{(6-4)!4!}$

$= \dfrac{6(5)\,(4!)}{2!4!}$

$= 15$

b) Case 1: four females
${}_6C_4 = 15$ from part a)

Case 2: five females

$$
\begin{aligned}
{}_6C_5 &= \frac{6!}{(6-5)!5!} \\
&= \frac{6!}{1!5!} \\
&= \frac{6\,(5!)}{5!} \\
&= 6
\end{aligned}
$$

Case 3: six females

$$
\begin{aligned}
{}_6C_6 &= \frac{6!}{(6-6)!6!} \\
&= \frac{6!}{0!6!} \\
&= 1
\end{aligned}
$$

Total number of ways of selecting at least four females: 15 + 6 + 1 = 22

Section 11.2 Page 534 Question 12

a) Left Side $= {}_nC_{r-1} + {}_nC_r$

$$
\begin{aligned}
&= \frac{n!}{(n-(r-1))!(r-1)!} + \frac{n!}{(n-r)!r!} \\
&= \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r)!r!} \\
&= \frac{n!(n-r)!r! \; + n!(n-r+1)!(r-1)!}{(n-r+1)!(r-1)!(n-r)!r!} \\
&= \frac{n!(n-r)!r(r-1)! \; + n!(n-r+1)(n-r)!(r-1)!}{(n-r+1)!(r-1)!(n-r)!r!} \\
&= \frac{n!\,\cancel{(n-r)!}\;\cancel{(r-1)!}[r+(n-r+1)]}{(n-r+1)!\,\cancel{(r-1)!}\;\cancel{(n-r)!}\,r!} \\
&= \frac{n!(r+n-r+1)}{(n-r+1)!r!} \\
&= \frac{n!(n+1)}{(n-r+1)!r!} \\
&= \frac{(n+1)!}{(n-r+1)!r!}
\end{aligned}
$$

Right Side $= {}_{n+1}C_r$

$$= \frac{(n+1)!}{(n+1-r)!r!}$$

Left Side = Right Side

Section 11.2 Page 534 Question 13

$$
\begin{aligned}
{}_6C_3 &= \frac{6!}{(6-3)!3!} \\
&= \frac{6!}{3!3!} \\
&= \frac{6(5)(4)\,\cancel{(3!)}^{1}}{3(2)(1)\,\cancel{(3!)}_{1}} \\
&= 20
\end{aligned}
$$

You can order 20 different burgers. This is a combination because the order of the ingredients is not important.

Section 11.2 Page 535 Question 14

a)

$$
\begin{aligned}
{}_{10}C_4 &= \frac{10!}{(10-4)!4!} \\
&= \frac{10!}{6!4!} \\
&= \frac{10(9)(8)(7)\,\cancel{(6!)}^{1}}{\cancel{(6!)}_{1}(4)(3)(2)(1)} \\
&= 210
\end{aligned}
$$

b) This question involves a combination because the order of toppings is not important.

Section 11.2 Page 535 Question 15

a) Method 1: Use a diagram.

Method 2: Use combinations.

$$
\begin{aligned}
{}_5C_2 &= \frac{5!}{(5-2)!2!} \\
&= \frac{5!}{3!2!} \\
&= \frac{5(4)\,\cancel{(3!)}^{1}}{\cancel{(3!)}_{1}(2)(1)} \\
&= 10
\end{aligned}
$$

You can draw 10 line segments connecting any two of the five points.

b) ${}_5C_3 = \dfrac{5!}{(5-3)!3!}$

$= \dfrac{5!}{2!3!}$

$= \dfrac{5(4)\overset{1}{\cancel{(3!)}}}{(2)(1)\underset{1}{\cancel{(3!)}}}$

$= 10$

You can draw 10 triangles from the five given points.

c) ${}_{10}C_3 = \dfrac{10!}{(10-3)!3!}$

$= \dfrac{10!}{7!3!}$

You can draw $\dfrac{10!}{7!3!}$ triangles from ten non-collinear points.

Number of line segments: ${}_{10}C_2 = \dfrac{10!}{(10-2)!2!}$

$= \dfrac{10!}{8!2!}$

The number of triangles is determined by the number of selections of choosing three points, whereas the number of line segments is determined by the number of selections of choosing two points.

Section 11.2 Page 535 Question 16

Left Side = ${}_nC_r$

$= \dfrac{n!}{(n-r)!r!}$

Right Side = ${}_nC_{n-r}$

$= \dfrac{n!}{(n-(n-r))!(n-r)!}$

$= \dfrac{n!}{(n-n+r)!(n-r)!}$

$= \dfrac{n!}{r!(n-r)!}$

Left Side = Right Side

Section 11.2 Page 535 Question 17

a) $${}_{20}C_{12} = \frac{20!}{(20-12)!12!}$$

$$= \frac{20!}{8!12!}$$

$$= \frac{20(19)(18)(17)(16)(15)(14)(13)\ \cancel{(12!)}^{1}}{8(7)(6)(5)(4)(3)(2)(1)\ \underset{1}{\cancel{(12!)}}}$$

$$= 125\ 970$$

There can be 125 970 12-person juries selected.

b) $${}_{12}C_7({}_8C_5) = \frac{12!}{(12-7)!7!}\left(\frac{8!}{(8-5)!5!}\right)$$

$$= \frac{12!8!}{5!7!3!5!}$$

$$= \frac{12(11)(10)(9)(8)\ \cancel{(7!)}^{1}\ (8)(7)(6)\ \cancel{(5!)}^{1}}{5(4)(3)(2)(1)\ \underset{1}{\cancel{(7!)}}\ (3)(2)(1)\ \underset{1}{\cancel{(5!)}}}$$

$$= 44\ 352$$

There can be 44 352 juries containing seven women and five men.

c) Case 1: 10 women

$${}_{12}C_{10}({}_8C_2) = \frac{12!}{(12-10)!10!}\left(\frac{8!}{(8-2)!2!}\right)$$

$$= \frac{12!}{2!10!}\left(\frac{8!}{6!2!}\right)$$

$$= \frac{12(11)(\cancel{10!})^{1}}{2(1)\ \underset{1}{\cancel{10!}}}\left(\frac{8(7)(\cancel{6!})^{1}}{\underset{1}{\cancel{6!}}(2)(1)}\right)$$

$$= 66(28)$$

$$= 1848$$

Case 2: 11 women

$${}_{12}C_{11}({}_8C_1) = \frac{12!}{(12-11)!11!}\left(\frac{8!}{(8-1)!1!}\right)$$

$$= \frac{12!}{1!11!}\left(\frac{8!}{7!1!}\right)$$

$$= \frac{12(\cancel{11!})^{1}}{\underset{1}{\cancel{11!}}}\left(\frac{8(\cancel{7!})^{1}}{\underset{1}{\cancel{7!}}1!}\right)$$

$$= 12(8)$$

$$= 96$$

Case 3: 12 women and 0 men

$${}_{12}C_{12}({}_8C_0) = \frac{12!}{(12-12)!12!}\left(\frac{8!}{(8-0)!0!}\right)$$

$$= \frac{\cancel{12!}^{1}}{0!\underset{1}{\cancel{12!}}}\left(\frac{\cancel{8!}^{1}}{\underset{1}{\cancel{8!}}\,0!}\right)$$

$$= 1$$

There are 1848 + 96 + 1 = 1945 juries containing at least ten women.

Section 11.2 Page 535 Question 18

a) $_{52}C_5 = \dfrac{52!}{(52-5)!5!}$

$= \dfrac{52!}{47!5!}$

$= \dfrac{52(51)(50)(49)(48)\,(\cancel{47!})}{(\cancel{47!})\,(5)(4)(3)(2)(1)}$

$= 2\ 598\ 960$

You can select five cards from a deck of 52 cards in 2 598 96 ways.

b) Determine the number of ways to select three of 13 hearts and 2 cards from the rest of the deck.

$_{13}C_3(\,_{39}C_2) = \dfrac{13!}{(13-3)!3!}\left(\dfrac{39!}{(39-2)!2!}\right)$

$= \dfrac{13!}{10!3!}\left(\dfrac{39!}{37!2!}\right)$

$= \dfrac{13(12)(11)(\cancel{10!})}{\cancel{10!}(3)(2)(1)}\left(\dfrac{39(38)(\cancel{37!})}{\cancel{37!}(2)(1)}\right)$

$= 286(741)$

$= 211\ 926$

There are 211 926 ways to select three hearts from a standard deck of cards.

c) Determine the number of ways to select one black card and four red cards.

$_{26}C_1(\,_{26}C_4) = \dfrac{26!}{(26-1)!1!}\left(\dfrac{26!}{(26-4)!4!}\right)$

$= \dfrac{26!}{25!}\left(\dfrac{26!}{22!4!}\right)$

$= \dfrac{26(\cancel{25!})}{\cancel{25!}}\left(\dfrac{26(25)(24)(23)(\cancel{22!})}{\cancel{22!}(4)(3)(2)(1)}\right)$

$= 388\ 700$

There are 388 700 ways to select one black card and four red cards from a standard deck.

Section 11.2 Page 535 Question 19

a) $${}_6C_4({}_7C_3) = \frac{6!}{(6-4)!4!}\left(\frac{7!}{(7-3)!3!}\right)$$
$$= \frac{6!}{2!4!}\left(\frac{7!}{4!3!}\right)$$
$$= \frac{6(5)\,\cancel{(4!)}^{1}}{2(1)\,\cancel{(4!)}_{1}}\left(\frac{7(6)(5)\,\cancel{(4!)}^{1}}{\cancel{(4!)}_{1}(3)(2)(1)}\right)$$
$$= 525$$

You can select the books in 525 ways.

b) Considering the four science books as one object means there are four objects to arrange in 4! = 24 ways. Since the science books must be together, there are 4!4! = 576 ways to arrange the books.

Section 11.2 Page 535 Question 20

a) $${}_{40}C_{20} = \frac{40!}{(40-20)!20!}$$
$$= \frac{40!}{20!20!}$$

The number of selections possible is $\frac{40!}{20!20!}$.

b) $${}_{20}P_4 = \frac{20!}{(20-4)!}$$
$$= \frac{20!}{16!}$$
$$= \frac{20(19)(18)(17)\,\cancel{(16!)}^{1}}{\cancel{16!}_{1}}$$
$$= 116\ 280$$

Four of the paintings can be displayed in a row in 116 280 ways.

Section 11.2 Page 536 Question 21

a)

$${}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13} = \frac{52!}{(52-13)!13!} \times \frac{39!}{(39-13)!13!} \times \frac{26!}{(26-13)!13!} \times \frac{13!}{(13-13)!13!}$$
$$= \frac{52!}{39!13!} \times \frac{39!}{26!13!} \times \frac{26!}{13!13!} \times \frac{13!}{0!13!}$$

b) $${}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13} = \frac{52!}{39!13!} \times \frac{39!}{26!13!} \times \frac{26!}{13!13!} \times \frac{13!}{0!13!}$$

$$= \frac{52!}{\cancel{39!}_{1}13!} \times \frac{\overset{1}{\cancel{39!}}}{\cancel{26!}_{1}13!} \times \frac{\overset{1}{\cancel{26!}}}{13!13!} \times \frac{\overset{1}{\cancel{13!}}}{0!\cancel{13!}_{1}}$$

$$= \frac{52!}{13!13!13!13!}$$

$$= \frac{52!}{(13!)^4}$$

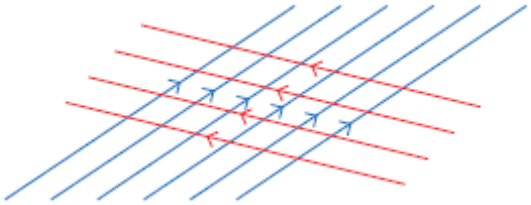
c) $${}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13} = \frac{52!}{(13!)^4}$$

$$= \frac{52(51)(50)...(15)(14)\ \overset{1}{\cancel{(13!)}}}{\cancel{13!}_{1}\left[(13)(12)(11)...(3)(2)(1)\right]^3}$$

$$= 5.364... \times 10^{28}$$

Section 11.2 Page 536 Question 22

Using the given diagram, count the number of possible parallelograms of each dimension.



1 × 1 parallelogram: 15 ways
1 × 2 parallelogram: 22 ways
1 × 3 parallelogram: 14 ways
1 × 4 parallelogram: 6 ways
1 × 5 parallelogram: 3 ways
2 × 2 parallelogram: 8 ways
2 × 3 parallelogram: 10 ways
2 × 4 parallelogram: 4 ways
2 × 5 parallelogram: 2 ways
3 × 3 parallelogram: 3 ways
3 × 4 parallelogram: 2 ways
3 × 5 parallelogram: 1 way

Total number of possibilities: 15 + 22 + 14 + 6 + 3 + 8 + 10 + 4 + 2 + 3 + 2 + 1 = 90
There are 90 possible parallelograms formed.

Section 11.2 Page 536 Question 23

a) Let x represent the number of flavours available.

$$_xC_2 = 630$$

$$\frac{x!}{(x-2)!2!} = 630$$

$$\frac{x(x-1)\,\cancel{(x-2)!}^{1}}{\cancel{(x-2)!}_{1}\,2!} = 630$$

$$\frac{x(x-1)}{(2)(1)} = 630$$

$$x(x-1) = 1260$$

$$x(x-1) = 36(35)$$

Using reasoning, $x = 36$.
There are 36 flavours of ice cream available in the store.

b) If you can duplicate flavours, you can choose from 36 flavours for each scoop.
$36(36) = 1296$
There are 1296 possible two-scoop bowls of ice cream.

Section 11.2 Page 536 Question 24

a)

$$_5C_2 = \frac{5!}{(5-2)!2!}$$

$$= \frac{5!}{3!2!}$$

$$= \frac{5(4)\,\cancel{(3)!}^{1}}{\cancel{(3)!}_{1}(2)(1)}$$

$$= 10$$

$$_{3(5)}C_{3(2)} = {}_{15}C_6$$

$$= \frac{15!}{(15-6)!6!}$$

$$= \frac{15!}{9!6!}$$

$$= \frac{15(14)(13)(12)(11)(10)\,\cancel{(9)!}^{1}}{\cancel{(9)!}_{1}(6)(5)(4)(3)(2)(1)}$$

$$= 5005$$

$$\frac{{}_5C_2}{3} = \frac{10}{3} = 3 \text{ remainder } 1$$

$$\frac{{}_{15}C_6}{3} = \frac{5005}{3} = 1668 \text{ remainder } 1$$

Both expressions have the same remainder, so the statement is true.

b) ${}_{7(5)}C_{7(2)} = {}_{35}C_{14}$

$$= \frac{35!}{(35-14)!14!}$$

$$= \frac{35!}{21!14!}$$

$$= \frac{35(34)(33)(32)(31)(30)(29)(28)(27)(26)(25)(24)(23)(22)\,\cancel{(21)!}^{1}}{\cancel{(21)!}_{1}(14)(13)(12)(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}$$

$$= 2\ 319\ 959\ 400$$

$$\frac{{}_{35}C_{14}}{7} = \frac{2\ 319\ 959\ 400}{7} = 331\ 422\ 771 \text{ remainder } 3$$

$$\frac{{}_{5}C_{2}}{7} = \frac{10}{7} = 1 \text{ remainder } 3$$

Both expressions have the same remainder, so, the statement is true.

c) Seven remainders are possible when dividing by 7. They are 0, 1, 2, 3, 4, 5, and 6.

d) Example: First, I would try a few more cases to try to find a counterexample. Since the statement seems to be true, I would write a computer program to test many cases in an organized way.

Section 11.2 Page 536 Question C1

No, the order of the numbers on the lock matters, so a combination lock would be better called a permutations lock.

Section 11.2 Page 536 Question C2

a) ${}_{a}C_{b} = \frac{a!}{(a-b)!b!}$ is the formula for calculating the number of ways that *b* objects can be selected from a group of *a* objects, if order is not important. For example, if you have a group of 20 students and you want to choose a team of any three people.

b) $a \geq b$ **c)** $b \geq 0$

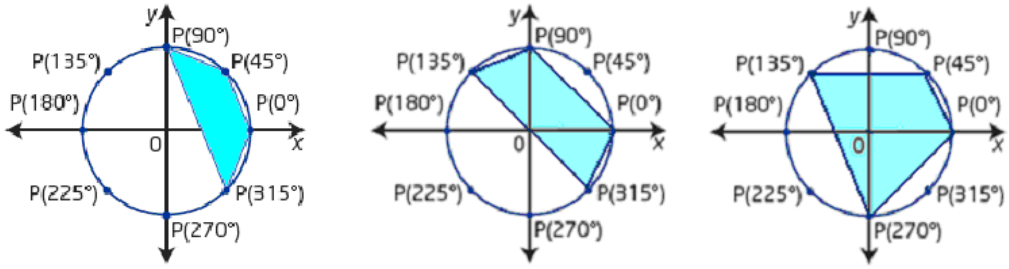
Section 11.2 Page 536 Question C3

Example: Assuming that the rooms are the same and so any patient can be assigned to any of the six rooms, this is a combination situation. Beth is correct.

Section 11.2 Page 536 Question C4

Step 1

Examples:



Step 2

Shape	Number of Quadrilaterals of Each Shape
square	2
rectangle	4
parallelogram	0
isosceles trapezoid	24

Step 3

In general, there are ${}_8C_4 = 70$ possible quadrilaterals. Because of the symmetry of the points on the circle, some of these quadrilaterals are the same.

Section 11.3 The Binomial Theorem

Section 11.3 Page 542 Question 1

a) 1 4 6 4 1

b) 1 8 28 56 70 56 28 8 1

c) 1 11 55 165 330 462 462 330 165 55 11 1

Section 11.3 Page 542 Question 2

a) ${}_2C_0$ ${}_2C_1$ ${}_2C_2$

b) ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

c) ${}_7C_0$ ${}_7C_1$ ${}_7C_2$ ${}_7C_3$ ${}_7C_4$ ${}_7C_5$ ${}_7C_6$ ${}_7C_7$

Section 11.3 Page 542 Question 3

a) $_3C_2 = \dfrac{3!}{(3-2)!2!}$

$= \dfrac{3!}{1!2!}$

b) $_6C_3 = \dfrac{6!}{(6-3)!3!}$

$= \dfrac{6!}{3!3!}$

c) $_1C_0 = \dfrac{1!}{(1-0)!0!}$

$= \dfrac{1!}{1!0!}$

Section 11.3 Page 542 Question 4

The expansion of $(x + y)^n$, contains $n + 1$ terms.

a) $4 + 1 = 5$

b) $7 + 1 = 8$

c) $q + 1$

Section 11.3 Page 542 Question 5

a) $(x + y)^2 = {}_2C_0(x)^2(y)^0 + {}_2C_1(x)^{2-1}(y)^1 + {}_2C_2(x)^{2-2}(y)^2$

$= 1x^2y^0 + 2x^1y^1 + 1x^0y^2$

$= x^2 + 2xy + y^2$

b) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^{3-1}(y)^1 + {}_3C_2(x)^{3-2}(y)^2 + {}_3C_3(x)^{3-3}(y)^3$

Substitute $x = a$ and $y = 1$.

$(a + 1)^3 = {}_3C_0(a)^3(1)^0 + {}_3C_1(a)^{3-1}(1)^1 + {}_3C_2(a)^{3-2}(1)^2 + {}_3C_3(a)^{3-3}(1)^3$

$= 1a^3 + 3a^2 + 3a + 1a^0$

$= a^3 + 3a^2 + 3a + 1$

c) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$

Substitute $x = 1$ and $y = -p$.

$(1 - p)^4 = {}_4C_0(1)^4(-p)^0 + {}_4C_1(1)^{4-1}(-p)^1 + {}_4C_2(1)^{4-2}(-p)^2 + {}_4C_3(1)^{4-3}(-p)^3 + {}_4C_4(1)^{4-4}(-p)^4$

$= 1(1)^4(-p)^0 + 4(1)^3(-p)^1 + 6(1)^2(-p)^2 + 4(1)^1(-p)^3 + 1(1)^0(-p)^4$

$= 1 + 4(-p) + 6p^2 + 4(-p^3) + p^4$

$= 1 - 4p + 6p^2 - 4p^3 + p^4$

Section 11.3 Page 542 Question 6

a) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^{3-1}(y)^1 + {}_3C_2(x)^{3-2}(y)^2 + {}_3C_3(x)^{3-3}(y)^3$

Substitute $x = a$ and $y = 3b$.

$(a + 3b)^3 = {}_3C_0(a)^3(3b)^0 + {}_3C_1(a)^{3-1}(3b)^1 + {}_3C_2(a)^{3-2}(3b)^2 + {}_3C_3(a)^{3-3}(3b)^3$

$= 1(a)^3(1) + 3a^2(3b) + 3(a)^1(3b)^2 + 1a^0(3b)^3$

$= a^3 + 9a^2b + 3a(9b^2) + 1(27b^3)$

$= a^3 + 9a^2b + 27ab^2 + 27b^3$

b) $(x + y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^{5-1}(y)^1 + {}_5C_2(x)^{5-2}(y)^2 + {}_5C_3(x)^{5-3}(y)^3 + {}_5C_4(x)^{5-4}(y)^4 + {}_5C_5(x)^{5-5}(y)^5$

Substitute $x = 3a$ and $y = -2b$.

$(3a - 2b)^5$

$= {}_5C_0(3a)^5(-2b)^0 + {}_5C_1(3a)^{5-1}(-2b)^1 + {}_5C_2(3a)^{5-2}(-2b)^2 + {}_5C_3(3a)^{5-3}(-2b)^3 + {}_5C_4(3a)^{5-4}(-2b)^4 + {}_5C_5(3a)^{5-5}(-2b)^5$

$= 1(3a)^5(1) + 5(3a)^4(-2b) + 10(3a)^3(-2b)^2 + 10(3a)^2(-2b)^3 + 5(3a)^1(-2b)^4 + 1(3a)^0(-2b)^5$

$= 243a^5 + 5(81a^4)(-2b) + 10(27a^3)(4b^2) + 10(9a^2)(-8b^3) + 15a(16b^4) - 32b^5$

$= 243a^5 - 810a^4b + 1080a^3b^2 - 720a^2b^3 + 240ab^4 - 32b^5$

c) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$

Substitute $x = 2x$ and $y = -5$.

$(2x - 5)^4$

$= {}_4C_0(2x)^4(-5)^0 + {}_4C_1(2x)^{4-1}(-5)^1 + {}_4C_2(2x)^{4-2}(-5)^2 + {}_4C_3(2x)^{4-3}(-5)^3 + {}_4C_4(2x)^{4-4}(-5)^4$

$= 1(2x)^4(1) + 4(2x)^3(-5) + 6(2x)^2(-5)^2 + 4(2x)^1(-5)^3 + 1(2x)^0(-5)^4$

$= 16x^4 - 20(8x^3) + 6(4x^2)(25) + 8x(-125) + (1)(625)$

$= 16x^4 - 160x^3 + 6400x^2 - 1000x + 625$

Section 11.3 Page 542 Question 7

a) sixth term of $(a + b)^9$: ${}_9C_5(a)^4(b)^5 = 126a^4b^5$

b) fourth term of $(x - 3y)^6$: ${}_6C_3(x)^3(-3y)^3 = 20x^3(-27y^3)$
$= -540x^3y^3$

c) seventh term of $(1 - 2t)^{14}$: ${}_{14}C_6(1)^8(-2t)^6 = 3003(64t^6)$
$= 192\,192t^6$

d) middle term of $(4x + y)^4$: ${}_4C_2(4x)^2(y)^2 = 6(16x^2)(y^2)$
$= 96x^2y^2$

e) second last term of $(3w^2 + 2)^8$: ${}_8C_7(3w^2)^1(2)^7 = 8(3w^2)(128)$
$= 3072w^2$

Section 11.3 Page 542 Question 8

All outside numbers of Pascal's triangle are 1s. The middle values are determined by adding the number to the left and right in the row above.

Section 11.3 Page 542 Question 9

a) 1; 1 + 1 = 2; 1 + 2 + 1 = 4; 1 + 3 + 3 + 1 = 8; 1 + 4 + 6 + 4 + 1 = 16

b) 2^8 or 256

c) 2^{n-1}, where n is the row number

Section 11.3 Page 542 Question 10

a) The sum of the numbers on the handle equals the number on the blade of each hockey stick.

b) No; the hockey stick handle must begin with 1 from the outside of the triangle and move diagonally down the triangle with each value being in a different row. The number on the blade must be diagonally below the last number on the handle of the hockey stick.

Section 11.3 Page 542 Question 11

a) The expansion of $(x + y)^{12}$ contains 12 + 1 = 13 terms.

b) The coefficient of the fourth term involves ${}_{12}C_3$.

fourth term: ${}_{12}C_3(x)^{12-3}(y)^3 = \frac{12!}{(12-3)!3!}(x)^9 y^3$

$$= \frac{12!}{9!3!}x^9y^3$$

$$= \frac{12(11)(10)\cancel{(9!)}^{1}}{\cancel{9!}_{1}(3)(2)(1)}x^9y^3$$

$$= 220x^9y^3$$

c) The greatest coefficient is in the middle of the row.

$r = \frac{12}{2}$ or 6

$${}_{12}C_6 = \frac{12!}{(12-6)!6!}$$

$$= \frac{12!}{6!6!}$$

$$= \frac{12(11)(10)(9)(8)(7)\cancel{(6!)}^{1}}{\cancel{6!}_{1}(6)(5)(4)(3)(2)(1)}$$

$$= 924$$

Section 11.3 Page 543 Question 12

a) number of terms = $n + 1$

$5 = n + 1$

$4 = n$

Substitute $a = x$, $b = y$ and $n = 4$:

$(a + b)^n = (x + y)^4$

b) number of terms = $n + 1$

$6 = n + 1$

$5 = n$

Substitute $a = 1$, $b = -y$ and $n = 5$:

$(a + b)^n = (1 - y)^5$

Section 11.3 Page 543 Question 13

a) No. While $11^0 = 1$, $11^1 = 11$, $11^2 = 121$, $11^3 = 1331$, and $11^4 = 14\,641$, this pattern only works for the first five rows of Pascal's triangle.

b) m could represent the row number minus 1, $m \leq 4$.

Section 11.3 Page 543 Question 14

a) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
The signs for the second and fourth terms are negative in the expansion of $(x - y)^3$.

b) Left Side $= (x + y)^3 + (x - y)^3$
$= x^3 + 3x^2y + 3xy^2 + y^3 + x^3 - 3x^2y + 3xy^2 - y^3$
$= 2x^3 + 6xy^2$
$= 2x(x^2 + 3y^2)$
= Right Side

c) $(x + y)^3 - (x - y)^3 = x^3 + 3x^2y + 3xy^2\ y^3 - (x^3 - 3x^2y + 3xy^2 - y^3)$
$= x^3 + 3x^2y + 3xy^2\ y^3 - x^3 + 3x^2y - 3xy^2 + y^3$
$= 6x^2y + 2y^3$
$= 2y(3x^2 + y^2)$

The expansion of $(x + y)^3 - (x - y)^3$ has coefficients for x^2 and y^2 that are reversed from the expansion of $(x + y)^3 + (x - y)^3$; also the common factors $2x$ and $2y$ are reversed.

Section 11.3 Page 543 Question 15

a) Case 1: no one attends, Case 2: one person attends, Case 3: two people attend, Case 4: three people attend, Case 5: four people attend, Case 6: all five people attend

b) Case 1: 1 way Case 2: 5 ways

Case 3:

$${}_5C_2 = \frac{5!}{(5-2)!2!}$$
$$= \frac{5!}{3!2!}$$
$$= \frac{5(4)\ (3!)}{3!(2)(1)}$$
$$= 10$$

Case 4:

$${}_5C_3 = \frac{5!}{(5-3)!3!}$$
$$= \frac{5!}{2!3!}$$
$$= \frac{5(4)\ (3!)}{(2)(1)\ 3!}$$
$$= 10$$

Case 5:

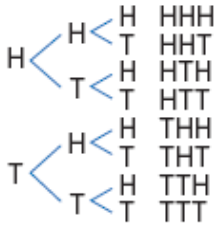
$${}_5C_4 = \frac{5!}{(5-4)!4!}$$
$$= \frac{5!}{1!4!}$$
$$= \frac{5\ (4!)}{4!}$$
$$= 5$$

Case 6: 1 way
Total number of combinations: 1 + 5 +10 + 10 + 5 + 1 = 32

c) The answer is the sum of the terms in the sixth row of Pascal's triangle.

Section 11.3 Page 543 Question 16

a)



b) $(H + T)^3 = HHH + HHT + HTH + HTT + THH + THT + TTH + TTT$
$= H^3 + H^2T + H^2T + HT^2 + H^2T + HT^2 + HT^2 + T^3$
$= H^3 + 3H^2T + 3HT^2 + T^3$

c) H^3 represents getting a head for each of three coin tosses and is the first term of the expansion of $(H + T)^3$. $3H^2T$ represents the possibility of getting two heads and one tail during three coin tosses and is the second term of the expansion of $(H + T)^3$.

Section 11.3 Page 543 Question 17

a) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^{3-1}(y)^1 + {}_3C_2(x)^{3-2}(y)^2 + {}_3C_3(x)^{3-3}(y)^3$
$= {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^2(y)^1 + {}_3C_2(x)^1(y)^2 + {}_3C_3(x)^0(y)^3$
$= {}_3C_0(x)^3 + {}_3C_1(x)^2(y)^1 + {}_3C_2(x)^1(y)^2 + {}_3C_3(y)^3$

Substitute $x = \frac{a}{b}$ and $y = 2$.

$$\left(\frac{a}{b}+2\right)^3 = {}_3C_0\left(\frac{a}{b}\right)^3 + {}_3C_1\left(\frac{a}{b}\right)^2(2)^1 + {}_3C_2\left(\frac{a}{b}\right)^1(2)^2 + {}_3C_3(2)^3$$

$$= 1\left(\frac{a}{b}\right)^3 + (3)\left(\frac{a}{b}\right)^2(2) + 3\left(\frac{a}{b}\right)(4) + 1(8)$$

$$= \frac{a^3}{b^3} + 6\frac{a^2}{b^2} + 12\frac{a}{b} + 8$$

b) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$
$= {}_4C_0(x)^4 + {}_4C_1(x)^3(y) + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$

Substitute $x = \frac{a}{b}$ and $y = -a$.

$$\left(\frac{a}{b}-a\right)^4 = {}_4C_0\left(\frac{a}{b}\right)^4 + {}_4C_1\left(\frac{a}{b}\right)^3(-a) + {}_4C_2\left(\frac{a}{b}\right)^2(-a)^2 + {}_4C_3\left(\frac{a}{b}\right)^1(-a)^3 + {}_4C_4\left(\frac{a}{b}\right)^0(-a)^4$$

$$= 1\left(\frac{a^4}{b^4}\right) + 4\left(\frac{a^3}{b^3}\right)(-a) + 6\left(\frac{a^2}{b^2}\right)(a^2) + 4\left(\frac{a}{b}\right)(-a^3) + (1)(a^4)$$

$$= \frac{a^4}{b^4} - 4\frac{a^4}{b^3} + 6\frac{a^4}{b^2} - 4\frac{a^4}{b} + a^4$$

$$= a^4\left(\frac{1}{b^4} - \frac{4}{b^3} + \frac{6}{b^2} - \frac{4}{b} + 1\right)$$

c) $(x+y)^6 = {}_6C_0(x)^6(y)^0 + {}_6C_1(x)^{6-1}(y)^1 + {}_6C_2(x)^{6-2}(y)^2 + {}_6C_3(x)^{6-3}(y)^3 +$
$${}_6C_4(x)^{6-4}(y)^4 + {}_6C_5(x)^{6-5}(y)^5 + {}_6C_6(x)^{6-6}(y)^6$$
$$= {}_6C_0(x)^6(1) + {}_6C_1(x)^5(y) + {}_6C_2(x)^4(y)^2 + {}_6C_3(x)^3(y)^3 + {}_6C_4(x)^2(y)^4 +$$
$${}_6C_5(x)^1(y)^5 + {}_6C_6(x)^0(y)^6$$

Substitute $x = 1$ and $y = -\frac{x}{2}$.

$$\left(1-\frac{x}{2}\right)^6 = {}_6C_0(1)^6 + {}_6C_1(1)^5\left(-\frac{x}{2}\right) + {}_6C_2(1)^4\left(-\frac{x}{2}\right)^2 + {}_6C_3(1)^3\left(-\frac{x}{2}\right)^3 + {}_6C_4(1)^2\left(-\frac{x}{2}\right)^4$$
$$+ {}_6C_5(1)^1\left(-\frac{x}{2}\right)^5 + {}_6C_6(1)^0\left(-\frac{x}{2}\right)^6$$

$$= 1(1) + 6(1)\left(-\frac{x}{2}\right) + 15(1)\left(\frac{x^2}{2^2}\right) + 20(1)\left(-\frac{x^3}{2^3}\right) + 15(1)\left(\frac{x^4}{2^4}\right) + 6(1)\left(-\frac{x^5}{2^5}\right) + 1(1)\left(\frac{x^6}{2^6}\right)$$

$$= 1 + \cancel{6}^{3}\left(-\frac{x}{\cancel{2}_{1}}\right) + 15\left(\frac{x^2}{4}\right) + \cancel{20}^{5}\left(-\frac{x^3}{\cancel{8}_{2}}\right) + 15\left(\frac{x^4}{16}\right) + \cancel{6}^{3}\left(-\frac{x^5}{\cancel{32}_{16}}\right) + \left(\frac{x^6}{64}\right)$$

$$= 1 - 3x + \frac{15}{4}x^2 - \frac{5}{2}x^3 + \frac{15}{16}x^4 - \frac{3}{16}x^5 + \frac{x^6}{64}$$

d) $(x+y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$
$$= {}_4C_0(x)^4 + {}_4C_1(x)^3(y) + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$$

Substitute $x = 2x^2$ and $y = -\frac{1}{x}$.

$$\left(2x^2-\frac{1}{x}\right)^4 = {}_4C_0(2x^2)^4 + {}_4C_1(2x^2)^3\left(-\frac{1}{x}\right) + {}_4C_2(2x^2)^2\left(-\frac{1}{x}\right)^2 + {}_4C_3(2x^2)^1\left(-\frac{1}{x}\right)^3 + {}_4C_4(2x^2)^0\left(-\frac{1}{x}\right)^4$$

$$= 1(2^4x^8) + 4(2^3x^6)\left(-\frac{1}{x}\right) + 6(2^2x^4)\left(\frac{1^2}{x^2}\right) + 4(2x^2)\left(-\frac{1^3}{x^3}\right) + 1(1)\left(\frac{1^4}{x^4}\right)$$

$$= 16x^8 + 4(8x^6)\left(-\frac{1}{x}\right) + 6(4x^4)\left(\frac{1^2}{x^2}\right) + 8x^2\left(-\frac{1^3}{x^3}\right) + \frac{1}{x^4}$$

$$= 16x^8 - \frac{32x^{\cancel{6}^{5}}}{x^{\cancel{1}^{0}}} + 24x^4\left(\frac{1}{x^2}\right) - \frac{8x^2}{x^3} + \frac{1}{x^4}$$

$$= 16x^8 - 32x^5 + 24x^{\cancel{4}^{2}}\left(\frac{1}{x^{\cancel{2}^{0}}}\right) - \frac{8x^{\cancel{2}^{0}}}{x^{\cancel{3}^{1}}} + \frac{1}{x^4}$$

$$= 16x^8 - 32x^5 + 24x^2 - \frac{8}{x} + \frac{1}{x^4}$$

$$= 16x^8 - 32x^5 + 24x^2 - 8x^{-1} + x^{-4}$$

Section 11.3 Page 543 Question 18

a) middle term of $(a-3b^3)^8$: ${}_8C_4(a)^{8-4}(-3b^3)^4 = 70(a)^4(81b^{12})$

$= 5670a^4b^{12}$

b) $\left(x^2-\frac{1}{x}\right)^{10}$

$$= {}_{10}C_0(x^2)^{10} + {}_{10}C_1(x^2)^9\left(-\frac{1}{x}\right) + {}_{10}C_2(x^2)^8\left(-\frac{1}{x}\right)^2 + {}_{10}C_3(x^2)^7\left(-\frac{1}{x}\right)^3 + {}_{10}C_4(x^2)^6\left(-\frac{1}{x}\right)^4 + {}_{10}C_5(x^2)^5\left(-\frac{1}{x}\right)^5 + {}_{10}C_6(x^2)^4\left(-\frac{1}{x}\right)^6$$
$$+ {}_{10}C_7(x^2)^3\left(-\frac{1}{x}\right)^7 + {}_{10}C_8(x^2)^2\left(-\frac{1}{x}\right)^8 + {}_{10}C_9(x^2)^1\left(-\frac{1}{x}\right)^9 + {}_{10}C_{10}(x^2)^0\left(-\frac{1}{x}\right)^{10}$$

$$= 1(x^{20}) + 10(x^{18})\left(-\frac{1}{x}\right) + 45(x^{16})\left(\frac{1^2}{x^2}\right) + 120(x^{14})\left(-\frac{1^3}{x^3}\right) + 210(x^{12})\left(\frac{1^4}{x^4}\right) +$$
$$252(x^{10})\left(-\frac{1^5}{x^5}\right) + 210(x^8)\left(\frac{1^6}{x^6}\right) + 120(x^6)\left(-\frac{1^7}{x^7}\right) + 45(x^4)\left(\frac{1^8}{x^8}\right) + 10(x^2)\left(-\frac{1^9}{x^9}\right) + 1(1)\left(\frac{1^{10}}{x^{10}}\right)$$

$$= x^{20} - 10(x^{\cancel{18}^{17}})\left(\frac{1}{x^{\cancel{1}^{0}}}\right) + 45(x^{\cancel{16}^{14}})\left(\frac{1}{x^{\cancel{2}^{0}}}\right) - 120(x^{\cancel{14}^{11}})\left(\frac{1}{x^{\cancel{3}^{0}}}\right) + 210(x^{\cancel{12}^{8}})\left(\frac{1}{x^{\cancel{4}^{0}}}\right) -$$
$$252(x^{\cancel{10}^{5}})\left(\frac{1}{x^{\cancel{5}^{0}}}\right) + 210(x^{\cancel{8}^{2}})\left(\frac{1}{x^{\cancel{6}^{0}}}\right) - 120(x^{\cancel{6}^{0}})\left(\frac{1}{x^{\cancel{7}^{1}}}\right) + 45(x^{\cancel{4}^{0}})\left(\frac{1}{x^{\cancel{8}^{4}}}\right) - 10(x^{\cancel{2}^{0}})\left(\frac{1}{x^{\cancel{9}^{7}}}\right) + \frac{1}{x^{10}}$$

$$= x^{20} - 10x^{17} + 45x^{14} - 120x^{11} + 210x^8 - 252x^5 + 210x^2 - \frac{120}{x} + \frac{45}{x^4} - \frac{10}{x^7} + \frac{1}{x^{10}}$$

The fourth term is $-120x^{11}$.

Section 11.3 Page 543 Question 19

a) $\left(x^2-\frac{2}{x}\right)^{12}$

$$= {}_{12}C_0(x^2)^{12} + {}_{12}C_1(x^2)^{11}\left(-\frac{2}{x}\right) + {}_{12}C_2(x^2)^{10}\left(-\frac{2}{x}\right)^2 + {}_{12}C_3(x^2)^9\left(-\frac{2}{x}\right)^3 + {}_{12}C_4(x^2)^8\left(-\frac{2}{x}\right)^4 + {}_{12}C_5(x^2)^7\left(-\frac{2}{x}\right)^5 + {}_{12}C_6(x^2)^6\left(-\frac{2}{x}\right)^6$$
$$+ {}_{12}C_7(x^2)^5\left(-\frac{2}{x}\right)^7 + {}_{12}C_8(x^2)^4\left(-\frac{2}{x}\right)^8 + {}_{12}C_9(x^2)^3\left(-\frac{2}{x}\right)^9 + {}_{12}C_{10}(x^2)^2\left(-\frac{2}{x}\right)^{10} + {}_{12}C_{11}(x^2)^1\left(-\frac{2}{x}\right)^{11} + {}_{12}C_{12}(x^2)^0\left(-\frac{2}{x}\right)^{12}$$
$$= 1(x^{24}) + 12(x^{22})\left(-\frac{2}{x}\right) + 66(x^{20})\left(\frac{2^2}{x^2}\right) + 220(x^{18})\left(-\frac{2^3}{x^3}\right) + 495(x^{16})\left(\frac{2^4}{x^4}\right) + 792(x^{14})\left(-\frac{2^5}{x^5}\right) + 924(x^{12})\left(\frac{2^6}{x^6}\right)$$
$$+ 792(x^{10})\left(-\frac{2^7}{x^7}\right) + 495(x^8)\left(\frac{2^8}{x^8}\right) + 220(x^6)\left(-\frac{29^9}{x^9}\right) + 66(x^4)\left(\frac{2^{10}}{x^{10}}\right) + 12x^2\left(-\frac{2^{11}}{x^{11}}\right) + 1\left(\frac{2^{12}}{x^{12}}\right)$$

By inspection, the fifth term is a constant.

$$495\,\cancel{x^8}^{1}\left(\frac{2^8}{\cancel{x^8}_{1}}\right) = 495(256)$$
$$= 126\ 720$$

The value of the constant is 126 720.

b) $\left(y-\frac{1}{y^2}\right)^{12}$

$$= {}_{12}C_0(y)^{12} + {}_{12}C_1(y)^{11}\left(-\frac{1}{y^2}\right) + {}_{12}C_2(y)^{10}\left(-\frac{1}{y^2}\right)^2 + {}_{12}C_3(y)^9\left(-\frac{1}{y^2}\right)^3 + {}_{12}C_4(y)^8\left(-\frac{1}{y^2}\right)^4 + {}_{12}C_5(y)^7\left(-\frac{1}{y^2}\right)^5 + {}_{12}C_6(y)^6\left(-\frac{1}{y^2}\right)^6$$
$$+ {}_{12}C_7(y)^5\left(-\frac{1}{y^2}\right)^7 + {}_{12}C_8(y)^4\left(-\frac{1}{y^2}\right)^8 + {}_{12}C_9(y)^3\left(-\frac{1}{y^2}\right)^9 + {}_{12}C_{10}(y)^2\left(-\frac{1}{y^2}\right)^{10} + {}_{12}C_{11}(y)^1\left(-\frac{1}{y^2}\right)^{11} + {}_{12}C_{12}(y)^0\left(-\frac{1}{y^2}\right)^{12}$$
$$= 1(y)^{12} + 12(y)^{11}\left(-\frac{1}{y^2}\right) + 66(y)^{10}\left(\frac{1^2}{y^4}\right) + 220(y)^9\left(-\frac{1^3}{y^6}\right) + 495(y)^8\left(\frac{1^8}{y^8}\right) + 792(y)^7\left(-\frac{1}{y^2}\right)^5 + 924(y)^6\left(-\frac{1}{y^2}\right)^6$$
$$+ 792(y)^5\left(-\frac{1}{y^2}\right)^7 + 495(y)^4\left(-\frac{1}{y^2}\right)^8 + 220(y)^3\left(-\frac{1}{y^2}\right)^9 + 66(y)^2\left(-\frac{1}{y^2}\right)^{10} + 12(y)^1\left(-\frac{1}{y^2}\right)^{11} + 1(y)^0\left(-\frac{1}{y^2}\right)^{12}$$

By inspection, the fifth term is a constant.

$$495\,\cancel{y^8}^{1}\left(\frac{1}{\cancel{y^8}_{1}}\right) = 495$$

The value of the constant is 495.

Section 11.3 Page 543 Question 20

$$
\begin{aligned}
&(2x-m)^7 \\
&= {}_7C_0(2x)^7(-m)^0 + {}_7C_1(2x)^6(-m)^1 + {}_7C_2(2x)^5(-m)^2 + {}_7C_3(2x)^4(-m)^3 + {}_7C_4(2x)^3(-m)^4 \\
&\quad + {}_7C_5(2x)^2(-m)^5 + {}_7C_6(2x)^1(-m)^6 + {}_7C_7(2x)^0(-m)^7 \\
&= 1(2^7x^7)(1) + 7(2^6x^6)(-m) + 21(2^5x^5)(m^2) + 35(2^4x^4)(-m^3) + 35(2^3x^3)(m^4) \\
&\quad + 21(2^2x^2)(-m^5) + 7(2x)(m^6) + 1(1)(-m^7) \\
&= 128x^7 - 448x^6m + 672x^5m^2 - 560x^4m^3 + 280x^3m^4 - 84x^2m^5 + 14xm^6 - m^7
\end{aligned}
$$

The term in the expansion that contains x^4 is $-560x^4m^3$.
Solve:

$$
\begin{aligned}
-560x^4m^3 &= -15\,120x^4y^3 \\
m^3 &= \frac{-15\,120\,x^4\,y^3}{-560\,x^4} \\
m^3 &= 27y^3 \\
\sqrt[3]{m^3} &= \sqrt[3]{3^3y^3} \\
m &= 3y
\end{aligned}
$$

Section 11.3 Page 543 Question 21

Example:
Step 1: The numerators start with the second value, 4, and decrease by ones, while the denominators start at 1 and increase by ones to 4.
For the sixth row:

$$\times\frac{5}{1} \quad \times\frac{4}{2} \quad \times\frac{3}{3} \quad \times\frac{2}{4} \quad \times\frac{1}{5}$$

1 5 10 10 5 1

Step 2: The second element in the row is equal to the row number minus 1.

Step 3: The first two terms in the 21st row of Pascal's triangle are 1 and 20. The multipliers for successive terms are $\times\frac{20}{1}$, $\times\frac{19}{2}$, $\times\frac{18}{3}$, ... $\times\frac{3}{18}$, $\times\frac{2}{19}$, $\times\frac{1}{20}$.

Section 11.3 Page 544 Question 22

a) Each entry is the sum of the two values in the row below it to the left and the right.

b)

$$\begin{array}{cccccccc} & \frac{1}{6} & \frac{1}{30} & \frac{1}{60} & \frac{1}{60} & \frac{1}{30} & \frac{1}{6} & \\ \frac{1}{7} & \frac{1}{42} & \frac{1}{105} & \frac{1}{140} & \frac{1}{105} & \frac{1}{42} & \frac{1}{7} \end{array}$$

c) Example: Outside values are the reciprocal of the row number. The product of two consecutive outside rows values gives the value of the second term in the lower row.

Section 11.3 Page 544 Question 23

Consider $a + b = x$ and $c = y$, and substitute in $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

$$\begin{aligned} (a + b + c)^3 &= (a + b)^3 + 3(a + b)^2c + 3(a + b)\,c^2 + c^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 + 3c(a^2 + 2ab + b^2) + 3ac^2 + 3bc^2 + c^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3 \end{aligned}$$

Section 11.3 Page 544 Question 24

a)

Diagram	Points	Line Segments	Triangles	Quadrilaterals	Pentagons	Hexagons
	1					
	2	1				
	3	3	1			
	4	6	4	1		
	5	10	10	5	1	
	6	15	20	15	6	1

b) The numbers are values from row 1 to row 6 of Pascal's triangle with the exception of the first term.

c) The numbers will be values from the 8th row of Pascal's triangle with the exception of the first term: 8 28 56 70 56 28 8 1

Section 11.3 Page 544 Question 25

a) $e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$

$\approx \frac{1}{1} + \frac{1}{1} + \frac{1}{2(1)} + \frac{1}{3(2)(1)} + \frac{1}{4(3)(2)(1)}$

$\approx 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$

$\approx \frac{24}{24} + \frac{24}{24} + \frac{12}{24} + \frac{4}{24} + \frac{1}{24}$

$\approx \frac{24+24+12+4+1}{24}$

$\approx \frac{65}{24}$

$\approx 2.7083...$

The approximate value of e is 2.7083.

b) Using seven terms,

$e \approx \frac{65}{24} + \frac{1}{5!} + \frac{1}{6!}$

$\approx \frac{65}{24} + \frac{1}{5(4)(3)(2)(1)} + \frac{1}{6(5)(4)(3)(2)(1)}$

$\approx \frac{65}{24} + \frac{1}{120} + \frac{1}{720}$

$\approx \frac{1950}{720} + \frac{6}{720} + \frac{1}{720}$

$\approx \frac{1957}{720}$

$\approx 2.7180...$

Using eight terms,

$e \approx \frac{1957}{720} + \frac{1}{7!}$

$\approx \frac{1957}{720} + \frac{1}{7(6)(5)(4)(3)(2)(1)}$

$\approx \frac{1957}{720} + \frac{1}{5040}$

$\approx \frac{13\ 699}{5040} + \frac{1}{5040}$

$\approx \frac{13\ 700}{5040}$

$\approx 2.7182...$

The value of e becomes more precise for the seventh and eighth terms. The more terms used, the more accurate the approximation.

c) The value of e is approximately 2.718 281 828.

d) Using Stirling's approximation,

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

$$15! \approx \left(\frac{15}{e}\right)^{15} \sqrt{2\pi(15)}$$

$$\approx 1.3004 \times 10^{12}$$

Using a calculator,
$15! = 15(14)(13)\ldots(3)(2)(1)$
$= 1.3076\ldots \times 10^{12}$

e) Using the formula from part d),

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

$$50! \approx \left(\frac{50}{e}\right)^{50} \sqrt{2\pi(50)}$$

$$\approx 3.036\ 344... \times 10^{64}$$

Using the formula from part e),

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}\left(1+\frac{1}{12n}\right)$$

$$50! \approx \left(\frac{50}{e}\right)^{50} \sqrt{2\pi(50)}\left(1+\frac{1}{12(50)}\right)$$

$$\approx 3.036\ 344... \times 10^{64}\left(1+\frac{1}{600}\right)$$

$$\approx 3.036\ 344... \times 10^{64}\left(\frac{600}{600}+\frac{1}{600}\right)$$

$$\approx 3.036\ 344... \times 10^{64}\left(\frac{601}{600}\right)$$

$$\approx 3.041\ 404... \times 10^{64}$$

Using a calculator,
$50! = 50(49)(48)\ldots(3)(2)(1)$
$= 3.041\ 409\ 32 \times 10^{64}$

The formula in part e) seems to give a more accurate approximation.

Section 11.3 Page 545 Question C1

The coefficients of the terms in the expansion of $(x + y)^n$ are the same as the numbers in row $n + 1$ of Pascal's triangle.
Examples: $(x + y)^2 = x^2$ and $2xy + y^2$ and row 3 of Pascal's triangle is 1 2 1;
$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^2$ and row 4 of Pascal's triangle is 1 3 3 1.

Section 11.3 Page 545 Question C2

Examples:
a) Permutation: In how many ways can four different chocolate bars be given to two people?

Combination: Steve has two Canadian dimes and two U.S. dimes in his pocket. In how many ways can he draw out two coins?

Binomial expansion: What is the coefficient of the middle term in the expansion of $(a + b)^4$?

b) All three problems have the same answer, 6, but they answer different questions.

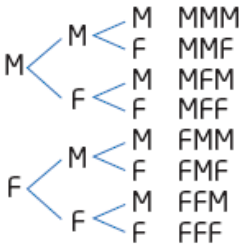
Section 11.3 Page 545 Question C3

Examples:
a), b) For small values of n, it is easier to use Pascal's triangle, but for large values of n, it is easier to use combinations to determine the coefficients in the expansion of $(a + b)^n$.

Chapter 11 Review

Chapter 11 Review Page 546 Question 1

a)



b) There are three outcomes that give them one boy and two girls: MFF, FMF, FFM.

Chapter 11 Review Page 546 Question 2

a) You can leave or enter using any of the gates.
$9(9) = 81$
There are 81 ways to enter and leave the stadium.

b) ways to enter through a north gate: 4 ways
ways to leave through any other gate: $9 - 1 = 8$ ways
$4(8) = 32$
There are 32 ways to enter through a north gate and leave through any other gate.

Chapter 11 Review Page 546 Question 3

a) *Bite* has four different letters. There are 4! = 24 ways to arrange the letters.

b) There are six letters in *bitten*. There are 2! ways to arrange the two *t*'s.

$$\frac{6!}{2!} = \frac{6(5)(4)(3)\,(\cancel{2!})^{1}}{\cancel{2!}_{1}}$$

$$= 360$$

The number of different six-letter arrangements is 360.

c) There are six letters in *mammal*. There are 2! ways to arrange the two *a*'s and 3! ways to arrange the three *m*'s.

$$\frac{6!}{2!3!} = \frac{6(5)(4)\,(\cancel{3!})^{1}}{2(1)\,(\cancel{3!})_{1}}$$

$$= 60$$

The number of different six-letter arrangements is 60.

d) There are 12 letters in *mathematical*. There are 2! ways to arrange the two *m*'s, 3! ways to arrange the three *a*'s, 2! ways to arrange the two *t*'s.

$$\frac{12!}{2!3!2!}$$

The number of different 12-letter arrangements is $\frac{12!}{2!3!2!}$.

Chapter 11 Review Page 546 Question 4

a) There are 2! = 2 ways to arrange Anna and Cleo together. Considering them as one object means there are four objects to arrange in 4! = 24 ways. Since Anna and Cleo must be seated together, there are 4!2! = 48 ways to arrange the people.

b) There are 2! = 2 ways to arrange Anna and Cleo together, and 2! ways to arrange Dina and Eric. Considering each pair as one object means there are three objects to arrange in 3! = 6 ways. Since each pair must be seated together, there are 3!2!2! = 24 ways to arrange the people.

c) Total arrangements of five people: 5! = 120
Ways of seating Anna and Cleo apart = total arrangements – ways of seating them together

= 120 – 48 (from part a))
= 72

There are 72 ways of seating Anna and Cleo apart.

Chapter 11 Review Page 546 Question 5

a) 7! = 5040

b) There are 4! ways to arrange the consonants, and 3! ways to arrange the vowels.
Case 1: start with a vowel
Arrangements with vowels and consonants alternating: 4!3! = 144

Case 2: start with a consonant
Arrangements with vowels and consonants alternating: 4!3! = 144

Total number of possibilities: 144 + 144 = 288

c) There are 3! = 6 ways to arrange the vowels in the middle. There are 4! = 24 ways to arrange two consonants on each end.
4!3! = 144

Chapter 11 Review Page 546 Question 6

a) Case 1: no digits
There are $26^4 = 456\ 976$ ways to create four-digit passwords using letters that may repeat.

Case 2: one digit
There are 10(26)(26)(26) = 175 760 ways to choose the characters, and eight possible positions for the digit.

Total number of possibilities: $26^4 + 4(10)(26^4) = 1\ 160\ 016$

b) Case 1: no digits
There are $26^8 = 208\ 827\ 064\ 576$ ways to create eight-digit passwords using letters that may repeat.

Case 2: one digit
There are $10(26^7)$ ways to choose the characters, and eight possible positions for the digit.

Total number of possibilities: $26^8 + 8(10)(26^7) = 851\ 371\ 878\ 656$

c) 851 371 878 656(10) – 1 160 016(10)

$$= (8\ 513\ 707\ 186\ 400 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ day}}{24 \text{ h}}\right)\left(\frac{1 \text{ year}}{365 \text{ days}}\right)$$

= 270 000 years

It would take about 270 000 years longer to check all of the eight-character passwords than to check the four-character passwords.

Chapter 11 Review Page 546 Question 7

a) $\dfrac{n! + (n-1)!}{n! - (n-1)!} = \dfrac{n(n-1)! + (n-1)!}{n(n-1)! - (n-1)!}$

$= \dfrac{\cancel{(n-1)!}^{1}(n+1)}{\cancel{(n-1)!}_{1}(n-1)}$

$= \dfrac{n+1}{n-1}$

b) $\dfrac{(x+1)! + (x-1)!}{x!} = \dfrac{(x+1)(x)(x-1)! + (x-1)!}{x(x-1)!}$

$= \dfrac{(x-1)![(x+1)(x) + 1]}{x(x-1)!}$

$= \dfrac{\cancel{(x-1)!}^{1}[x^2 + x + 1]}{x\,\cancel{(x-1)!}_{1}}$

$= \dfrac{x^2 + x + 1}{x}$

Chapter 11 Review Page 546 Question 8

a) ${}_{10}C_4 = \dfrac{10!}{(10-4)!4!}$

$= \dfrac{10!}{6!4!}$

$= \dfrac{10(9)(8)(7)\,\cancel{(6!)}^{1}}{\cancel{6!}_{1}(4)(3)(2)(1)}$

$= 210$

You can select four light bulbs in 210 ways.

b) There are $10 - 3 = 7$ good light bulbs.

${}_7C_2 \times {}_3C_2 = \dfrac{7!}{(7-2)!2!} \times \dfrac{3!}{(3-2)!2!}$

$= \dfrac{7!}{5!2!} \times \dfrac{3!}{1!2!}$

$= \dfrac{7(6)\,\cancel{(5!)}^{1}}{\cancel{5!}_{1}(2)(1)} \times \dfrac{3(2)(1)}{1(2)(1)}$

$= 63$

There are 63 ways to select the light bulbs.

Chapter 11 Review Page 546 Question 9

a) $${}_{10}C_3 = \frac{10!}{(10-3)!3!}$$
$$= \frac{10!}{7!3!}$$
$$= \frac{10(9)(8)\cancel{(7!)}^{1}}{\cancel{7!}_{1}(3)(2)(1)}$$
$$= 120$$

b) $${}_{10}P_4 = \frac{10!}{(10-4)!}$$
$$= \frac{10!}{6!}$$
$$= \frac{10(9)(8)(7)\cancel{(6!)}^{1}}{\cancel{6!}_{1}}$$
$$= 5040$$

c) $${}_5C_3 \times {}_5P_2 = \frac{5!}{(5-3)!3!} \times \frac{5!}{(5-2)!}$$
$$= \frac{5!}{2!3!} \times \frac{5!}{3!}$$
$$= \frac{5(4)\cancel{(3!)}^{1}}{2(1)\cancel{3!}_{1}} \times \frac{5(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}}$$
$$= 200$$

d) $$\left(\frac{15!}{4!11!}\right){}_6P_3 = \left(\frac{15(14)(13)(12)\cancel{(11!)}^{1}}{4(3)(2)(1)\cancel{(11!)}_{1}}\right)\left(\frac{6!}{(6-3)!}\right)$$
$$= 1365\left(\frac{6!}{3!}\right)$$
$$= 1365\left(\frac{6(5)(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}}\right)$$
$$= 163\ 800$$

Chapter 11 Review Page 546 Question 10

a) Case 1: Use 1 coin

$$
\begin{aligned}
{}_4C_1 &= \frac{4!}{(4-1)!1!} \\
&= \frac{4!}{3!(1)} \\
&= \frac{4\cancel{(3!)}^{1}}{\cancel{3!}_{1}} \\
&= 4
\end{aligned}
$$

Case 2: Use 2 coins

$$
\begin{aligned}
{}_4C_2 &= \frac{4!}{(4-2)!2!} \\
&= \frac{4!}{2!2!} \\
&= \frac{4(3)\cancel{(2!)}^{1}}{\cancel{2!}_{1}(2)(1)} \\
&= 6
\end{aligned}
$$

Case 3: Use 3 coins

$$
\begin{aligned}
{}_4C_3 &= \frac{4!}{(4-3)!3!} \\
&= \frac{4!}{1!3!} \\
&= \frac{4\cancel{(3!)}^{1}}{(1)\cancel{3!}_{1}} \\
&= 4
\end{aligned}
$$

Case 4: Use 4 coins

$$
\begin{aligned}
{}_4C_4 &= \frac{4!}{(4-4)!4!} \\
&= \frac{4!}{0!4!} \\
&= \frac{\cancel{(4!)}^{1}}{(1)\cancel{4!}_{1}} \\
&= 1
\end{aligned}
$$

Total number of ways: 4 + 6 + 4 + 1 = 15

b) Amounts below are in cents.
From Case 1: 1, 5, 10, 25
From Case 2: 1 + 5 = 6, 1 + 10 = 11, 1 + 25 = 26, 5 + 10 = 15, 5 + 25 = 30, 10 + 25 = 35
From Case 3: 1 + 5 + 10 = 16, 1 + 5 + 25 = 31, 1 + 10 + 25 = 36, 5 + 10 + 25 = 40
From Case 4: 1 + 5 + 10 + 25 = 41

Chapter 11 Review Page 546 Question 11

a)

$$
\begin{aligned}
{}_nC_2 &= 28 \\
\frac{n!}{(n-2)!2!} &= 28 \\
\frac{n(n-1)(n-2)!}{(n-2)!} &= 28(2!) \\
\frac{n(n-1)\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}} &= 28(2)(1) \\
n(n-1) &= 56 \\
8(7) &= 56
\end{aligned}
$$

By inspection, $n = 8$.

${}_8C_2 = 28$

Left Side $= {}_8C_2$

$$= \frac{8!}{(8-2)!2!}$$

$$= \frac{8!}{6!2!}$$

$$= \frac{8(7)\ \cancel{(6!)}^{1}}{\cancel{6!}_{1}(2)(1)}$$

$$= 28$$

Left Side = Right Side

Right Side = 28

b) ${}_nC_3 = 4({}_nP_2)$

$$\frac{n!}{(n-3)!3!} = 4\left(\frac{n!}{(n-2)!}\right)$$

$$\frac{n(n-1)(n-2)\ \cancel{(n-3)!}^{1}}{\cancel{(n-3)!}_{1}\,3!} = 4\left(\frac{n(n-1)\ \cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}}\right)$$

$$\frac{n(n-1)(n-2)}{3!} = 4n(n-1)$$

$$\frac{\cancel{n}^{1}\ \cancel{(n-1)}^{1}\,(n-2)}{\cancel{n}_{1}\ \cancel{(n-1)}_{1}} = 4(3!)$$

$$n-2 = 4(3)(2)(1)$$

$$n = 24+2$$

$$n = 26$$

${}_{26}C_3 = 4({}_nP_2)$

Left Side $= {}_{26}C_3$

$$= \frac{26!}{(26-3)!3!}$$

$$= \frac{26!}{23!3!}$$

$$= \frac{26(25)(24)\ \cancel{(23!)}^{1}}{\cancel{3!}_{1}(3)(2)(1)}$$

$$= 2600$$

Left Side = Right Side

Right Side $= 4({}_{26}P_2)$

$$= 4\left(\frac{26!}{(26-2)!}\right)$$

$$= 4\left(\frac{26!}{24!}\right)$$

$$= 4\left(\frac{26(25)\ \cancel{(24!)}^{1}}{\cancel{24!}_{1}}\right)$$

$$= 2600$$

Chapter 11 Review Page 547 Question 12

$$
\begin{aligned}
{}_{10}C_2 \times {}_8C_3 \times {}_5C_5 &= \frac{10!}{(10-2)!2!} \times \frac{8!}{(8-3)!3!} \times \frac{5!}{(5-5)!5!} \\
&= \frac{10!\overset{1}{\cancel{8!}}\,\overset{1}{\cancel{5!}}}{\underset{1}{\cancel{8!}}\,2!5!3!0!\underset{1}{\cancel{5!}}} \\
&= \frac{10!}{2!5!3!0!} \\
&= \frac{10(9)(8)(7)(6)\,\cancel{(5!)}}{2(1)\,\underset{1}{\cancel{(5!)}}\,(3)(2)(1)(1)} \\
&= 2520
\end{aligned}
$$

There are 2520 ways to break the students into groups.

Chapter 11 Review Page 547 Question 13

a) Example:
Permutation: How many arrangements of the letters AAABB are possible?
Combination: How many ways can you choose three students from a group of five?

b) Yes, since 2 + 3 = 5, the number of ways of selecting two out of a group of five is the same as the number of ways of 'rejecting' the remaining three.

$$
{}_5C_2 = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} \qquad {}_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!}
$$

Chapter 11 Review Page 547 Question 14

a) The row below 1 2 1 is 1 3 3 1.

b) The row below 1 8 28 56 70 56 28 8 1 is 1 9 36 84 126 126 84 36 9 1.

Chapter 11 Review Page 547 Question 15

Examples:
Using multiplication, determine the coefficients by expanding, collecting like terms, and writing the answer in descending order of the exponent of x.

$$
\begin{aligned}
(x+y)^3 &= (x+y)(x+y)(x+y) \\
&= x^3 + 3x^2y + 3xy^2 + y^3
\end{aligned}
$$

Using, Pascal's triangle, coefficients are the terms from row $n + 1$ of Pascal's triangle.
For $(x + y)^3$, row 4 is 1 3 3 1.

Using combinations, coefficients correspond to the combinations as shown:
$(x + y)^3 = {}_3C_0x^3y^0 + {}_3C_1x^2y^1 + {}_3C_2C1x^1y^2 + {}_3C_3x^0y^3$

Chapter 11 Review Page 547 Question 16

a) $(x + y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5$
$= 1(x)^5(1) + 5(x)^4(y) + 10(x)^3(y)^2 + 10(x)^2(y)^3 + 5(x)(y)^4 + 1(1)(y)^5$
$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Substitute $x = a$ and $y = b$.
$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

b) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^2(y)^1 + {}_3C_2(x)^1(y)^2 + {}_3C_3(x)^0(y)^3$
$= 1(x)^3(1) + 3(x)^2(y) + 3(x)(y)^2 + 1(1)(y)^3$
$= (x)^3 + 3(x)^2(y) + 3(x)(y)^2 + (y)^3$

Substitute $y = -3$.
$(x - 3)^3 = (x)^3 + 3(x)^2(-3) + 3(x)(-3)^2 + (-3)^3$
$= x^3 - 9x^2 + 3x(9) + (-27)$
$= x^3 - 9x^2 + 27x - 27$

c) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^3(y)^1 + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$
$= 1(x)^4(1) + 4(x)^3(y) + 6(x)^2(y)^2 + 4(x)(y)^3 + 1(1)(y)^4$
$= (x)^4 + 4(x)^3(y) + 6(x)^2(y)^2 + 4(x)(y)^3 + (y)^4$

Substitute $x = 2x^2$ and $y = -\frac{1}{x^2}$.

$$\left(2x^2 - \frac{1}{x^2}\right)^4 = (2x^2)^4 + 4(2x^2)^3\left(-\frac{1}{x^2}\right) + 6(2x^2)^2\left(-\frac{1}{x^2}\right)^2 + 4(2x^2)\left(-\frac{1}{x^2}\right)^3 + \left(-\frac{1}{x^2}\right)^4$$

$$= (2^4x^8) + 4(2^3x^6)\left(-\frac{1}{x^2}\right) + 6(2^2x^4)\left(\frac{1}{x^4}\right) + 4(2x^2)\left(-\frac{1}{x^6}\right) + \frac{1}{x^8}$$

$$= 16x^8 + 4(8x^{\cancel{6}^{4}})\left(-\frac{1}{\cancel{x^2}^{\,0}}\right) + 6(4\,\cancel{x^4}^{\,1})\left(\frac{1}{\cancel{x^4}^{\,1}}\right) + 4(2x^{\cancel{2}^{\,0}})\left(-\frac{1}{\cancel{x^6}^{\,4}}\right) + \frac{1}{x^8}$$

$$= 16x^8 - 32x^4 + 24 - \frac{8}{x^4} + \frac{1}{x^8}$$

Chapter 11 Review Page 547 Question 17

a) third term of $(a + b)^9$: ${}_9C_2(a)^{9-2}(b)^2 = 36a^7b^2$

b) sixth term of $(x - 2y)^6$: ${}_6C_5(x)^{6-5}(-2y)^5 = 6(x)^1(-2^5y^5)$
$= 6x(-32y^5)$
$= -192xy^5$

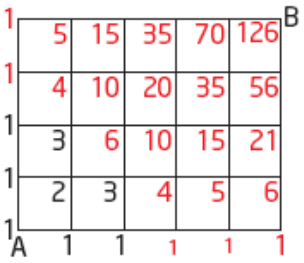
c) middle term of $\left(\frac{1}{x}-2x^2\right)^6$: ${}_6C_3\left(\frac{1}{x}\right)^{6-3}(-2x^2)^3 = 20\left(\frac{1}{x}\right)^3(-2^3x^6)$

$$= 20\left(\frac{1^3}{x^3}\right)(-8x^6)$$

$$= -160x^3$$

Chapter 11 Review Page 547 Question 18

a)



b) Pascal's triangle values are shown with the top of the triangle at point A and the rows appearing diagonally up and right of point A.

c) From the diagram in part a), there are 126 pathways from A to B.

d) There are four identical moves up and five identical moves right.

Number of pathways: $\frac{(4+5)!}{4!5!} = \frac{9!}{4!5!}$

$$= \frac{9(8)(7)(6)\,(5!)}{4(3)(2)(1)\,(5!)}$$

$$= 126$$

There are 126 pathways from A to B.

Chapter 11 Review Page 547 Question 19

a) To move all of the green counters to the left side, each green counter must jump over the yellow counters that are to the left of it.
0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45
It takes 45 moves to separate the green and yellow counters into two groups.

b) Two counters: 0 + 1 = 1 move; three counters: 0 + 1 + 2 = 3 moves; four counters: 0 + 1 + 2 + 3 = 6 moves, and so on up to 12 counters:
0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66 moves
The pattern is the sum of the numbers up to one less than the number of counters of each colour.

c) 25 counters: 0 + 1 + 2 + 3 + … + 22 + 23 + 24 = 300
It takes 300 moves to separate 25 counters of each colour.

Chapter 11 Practice Test

Chapter 11 Practice Test Page 548 Question 1

There are four choices for the first digit (1, 2, 8, 9), four choices for the middle digit, and three choices for the last digit.
$4(4)(3) = 48$
The answer is C.

Chapter 11 Practice Test Page 548 Question 2

The word *sweepers* has eight letters, with 2 *s*'s and 3 *e*'s.

$$\frac{8!}{2!3!} = \frac{8(7)(6)(5)(4)(3)(2)(1)}{2(1)(3)(2)(1)}$$
$$= 3360$$
The answer is D.

Chapter 11 Practice Test Page 548 Question 3

$$ {}_7C_2 \times {}_6C_2 \times {}_5C_1 = \frac{7!}{(7-2)!2!} \times \frac{6!}{(6-2)!2!} \times \frac{5!}{(5-1)!1!}$$
$$= \frac{7!}{\cancel{5!}^{1}\,2!} \times \frac{6!}{4!2!} \times \frac{\cancel{5!}^{1}}{4!1!}$$
$$= \frac{7(6)(5)(4)(3)(2)(1)(6)(5)(4)(3)(2)(1)}{2(1)(4)(3)(2)(1)(2)(1)(4)(3)(2)(1)(1)}$$
$$= 1575$$
The answer is C.

Chapter 11 Practice Test Page 548 Question 4

The exponent is 11. So, there are 11 + 1 = 12 terms in the expansion.
The answer is B.

Chapter 11 Practice Test Page 548 Question 5

third term of $(2x^2 + 3y)^7$: $ {}_7C_2(2x^2)^{7-2}(3y)^2 = 21(2x^2)^5(3y)^2$
$= 21(2^5x^{10})\,(3^2y^2)$
$= 21(32x^{10})\,(9y^2)$
$= 6048x^{10}y^2$
The answer is A.

Chapter 11 Practice Test Page 548 Question 6

The next row in Pascal's triangle is 1 7 21 35 35 21 7 1.
The sixth number in the row is 21.
The answer is C.

Chapter 11 Practice Test Page 548 Question 7

a) $\frac{6!}{2!2!} = \frac{6(5)(4)(3)(2)(1)}{2(1)2(1)}$

$= 180$

There are 180 possible answer keys.

b) AACBDB, ABCADB, ABCBDA, BACBDA, BACADB, BBCADA

Chapter 11 Practice Test Page 548 Question 8

$${}_nP_2 = 72$$

$$\frac{n!}{(n-2)!} = 72$$

$$\frac{n(n-1)\overset{1}{\cancel{(n-2)!}}}{\underset{1}{\cancel{(n-2)!}}} = 72$$

$$n(n-1) = 72$$

$$n(n-1) = 9(8)$$

By inspection, $n = 9$.
No, n does not equal –8.

Chapter 11 Practice Test Page 548 Question 9

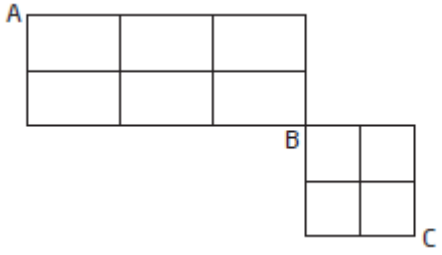
a) Determine the number of arrangements of moving down 2 units and right 3 units.

$$\frac{(2+3)!}{2!3!} = \frac{5!}{2!3!}$$

$$= \frac{5(4)\overset{1}{\cancel{(3!)}}}{2(1)\underset{1}{\cancel{(3!)}}}$$

$$= 10$$

There are 10 pathways from A to B.



b) The number of pathways from A to C is equal to the number of pathways from A to B multiplied by the number of pathways from B to C.

pathways from A to C: $\frac{5!}{2!3!} \times \frac{4!}{2!2!} = 10 \times \frac{4(3)\,\cancel{(2!)}^{1}}{2(1)\,\cancel{(2!)}_{1}}$

$= 10 \times 6$

$= 60$

There are 60 pathways from A to C.

Chapter 11 Practice Test Page 548 Question 10

Case 1: one digit number
5 ways

Case 2: two digit number
4 choices for first digit × 4 choices for last digit
4(4) = 16

Case 3: three digit number
4 choices for first digit × 4 choices for middle digit × 3 choices for last digit
4(4)(3) = 48

Total number of possibilities: 5 + 16 + 48 = 69
There are 69 possible numbers.

Chapter 11 Practice Test Page 548 Question 11

Permutations determine the number of arrangements of *n* items chosen *r* a t a time, when order is important. For example, the number of arrangements of 5 people chosen 2 at a time to ride on a motorcycle is ${}_5P_2 = 20$. A combination determines the number of different selections of *n* objects chosen *r* at a time, when order is not important. For example, the number of selections of 5 objects chosen 2 at a time when order is not important is ${}_5C_2 = 10$.

Chapter 11 Practice Test Page 548 Question 12

$$\left(x^2 + \frac{2}{x}\right)^9$$

$$= {}_9C_0(x^2)^9\left(\frac{2}{x}\right)^0 + {}_9C_1(x^2)^8\left(\frac{2}{x}\right) + {}_9C_2(x^2)^7\left(\frac{2}{x}\right)^2 + {}_9C_3(x^2)^6\left(\frac{2}{x}\right)^3 + {}_9C_4(x^2)^5\left(\frac{2}{x}\right)^4 + {}_9C_5(x^2)^4\left(\frac{2}{x}\right)^5 + {}_9C_6(x^2)^3\left(\frac{2}{x}\right)^6$$
$$+ {}_9C_7(x^2)^2\left(\frac{2}{x}\right)^7 + {}_9C_8(x^2)^1\left(\frac{2}{x}\right)^8 + {}_9C_9(x^2)^0\left(\frac{2}{x}\right)^9$$

$$= 1(x^{18})(1) + 9(x^{16})\left(\frac{2}{x}\right) + 36(x^{14})\left(\frac{2}{x}\right)^2 + 84(x^{12})\left(\frac{2}{x}\right)^3 + 126(x^{10})\left(\frac{2}{x}\right)^4 + 126(x^8)\left(\frac{2}{x}\right)^5 + 84(x^6)\left(\frac{2}{x}\right)^6$$
$$+ 36(x^4)\left(\frac{2}{x}\right)^7 + 9(x^2)\left(\frac{2}{x}\right)^8 + 1(1)\left(\frac{2}{x}\right)^9$$

$$= x^{18} + \frac{18\cancel{x^{16}}^{15}}{\cancel{x}} + 36\cancel{x^{14}}^{12}\left(\frac{2^2}{\cancel{x^2}^{1}}\right) + 84(\cancel{x^{12}}^{9})\left(\frac{2^3}{\cancel{x^3}^{1}}\right) + 126(x^{10})\left(\frac{2}{x}\right)^4 + 126(x^8)\left(\frac{2}{x}\right)^5 + 84(x^6)\left(\frac{2}{x}\right)^6 + 36(x^4)\left(\frac{2}{x}\right)^7 + 9(x^2)\left(\frac{2}{x}\right)^8 + 1(1)\left(\frac{2}{x}\right)^9$$

The term that contains x^9 is $84x^9(2^3) = 672x^9$.

Chapter 11 Practice Test Page 548 Question 13

a) Case 1: Even Numbers That Start With 5 or 9
There are 2 choices for the first digit (5 or 9). There are 4 choices for the last digit (0, 2, 6, or 8). Since two numbers have been used, there are 6 and 5 choices for the second and third digits.
$2(6)(5)(4) = 240$

Case 2: Even Numbers That Start With 6 or 8
There are 2 choices for the first digit (6 or 8). There are 3 choices for the last digit (0, 2, and 6 or 8 since one of these numbers was used for the first digit). Since two numbers have been used, there are 6 and 5 choices for the second and third digits.
$2(6)(5)(3) = 180$

There are $240 + 180 = 420$ 4-digit even numbers greater than 5000 that can be formed.

b) Case 1: Numbers That Start With 5 or 9
There are 2 choices for the first digit (5 or 9). There is 1 choice for the last digit (0). There are 6 and 5 choices for the second and third digits.
$2(6)(5)(1) = 60$

Case 2: Numbers That Start With 6 or 8
There are 2 choices for the first digit (6 or 8). There is 1 choice for the last digit (0). There are 6 and 5 choices for the second and third digits.
$2(6)(5)(1) = 60$

You can form $60 + 60 = 120$ numbers.

Chapter 11 Practice Test Page 548 Question 14

a)

$$ {}_nP_3 = 120$$

$$\frac{n!}{(n-3)!} = 120$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}^{1}}{\cancel{(n-3)!}_{1}} = 120$$

$$n(n-1)(n-2) = 6(5)(4)$$

By inspection, $n = 6$.

b)

$$3({}_nC_2) = 12({}_nC_1)$$

$$\frac{n!}{(n-2)!2!} = \frac{12}{3}\left(\frac{n!}{(n-1)!1!}\right)$$

$$\frac{n(n-1)\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}(2)(1)} = 4\left(\frac{n\cancel{(n-1)!}^{1}}{\cancel{(n-1)!}_{1}(1)}\right)$$

$$n(n-1) = 4(2)(n)$$

$$\frac{\cancel{n}^{1}(n-1)}{\cancel{n}_{1}} = 8$$

$$n-1 = 8$$

$$n = 9$$

Chapter 11 Practice Test Page 548 Question 15

$$(x+y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5$$
$$= 1(x)^5(1) + 5(x)^4(y) + 10(x)^3(y)^2 + 10(x)^2(y)^3 + 5(x)(y)^4 + 1(1)(y)^5$$
$$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

Substitute $x = y$ and $y = -\frac{2}{y^2}$.

$$\left(y - \frac{2}{y^2}\right)^5 = (y)^5 + 5(y)^4\left(-\frac{2}{y^2}\right) + 10(y)^3\left(-\frac{2}{y^2}\right)^2 + 10(y)^2\left(-\frac{2}{y^2}\right)^3 + 5(y)\left(-\frac{2}{y^2}\right)^4 + \left(-\frac{2}{y^2}\right)^5$$
$$= y^5 + 5y^{\cancel{4}^{2}}\left(-\frac{2}{\cancel{y^2}}\right) + 10y^{\cancel{3}^{0}}\left(\frac{2^2}{\cancel{y^4}^{1}}\right) + 10y^{\cancel{2}^{0}}\left(-\frac{2^3}{\cancel{y^6}^{4}}\right) + 5\cancel{y}\left(\frac{2^4}{\cancel{y^8}^{7}}\right) + \left(-\frac{2^5}{y^{10}}\right)$$
$$= y^5 - 10y^2 + 10\left(\frac{4}{y}\right) - 10\left(\frac{8}{y^4}\right) + 5\left(\frac{16}{y^7}\right) - \left(\frac{32}{y^{10}}\right)$$
$$= y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}$$

Chapter 11 Practice Test Page 548 Question 16

a) Considering the *a*'s as one object means there are four letters to arrange in $4! = 24$ ways.

b) There are $2! = 2$ ways of arranging the two *a*'s. The total number of arrangements of the letters in *aloha* is

$$\frac{5!}{2!} = \frac{5(4)(3)(2)(1)}{2(1)}$$
$$= 60$$

If the *a*'s cannot be together, there are $60 - 24 = 36$ ways of arranging the letters.

c) Determine the total arrangements with a vowel as the first letter.
There are 3 choices for the first vowel and 4! ways of arranging the remaining letters. Since there are 2! ways of arranging the two identical *a*'s, divide by 2!.

Total arrangements: $\frac{3(4!)}{2!} = \frac{3(4)(3)(2)(1)}{2(1)}$
$$= 36$$

Then, subtract the number of arrangements with a vowel as the first letter and the consonants together.
Consider the consonants together as one object. Since a vowel must be first, there are 3! ways of arranging the final letters. Multiply by 2 since there are 2! ways of arranging the consonants together, and divide by 2! since there are 2! ways of arranging the two identical *a*'s.

Arrangements with consonants together: $\frac{3(3!)\,\cancel{(2!)}^{1}}{\cancel{2!}_{1}} = 3(3)(2)(1)$

$= 18$

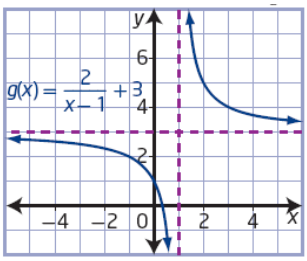
Arrangements beginning with a vowel and consonants not together: 36 – 18 = 18

Cumulative Review, Chapters 9–11

Cumulative Review, Chapters 9–11 Page 550 Question 1

a) The transformations involve a vertical stretch by a factor of 2 about the *x*-axis and a translation of 1 unit right and 3 units up.

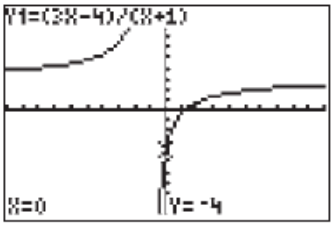
b)



c) domain: $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range: $\{y \mid y \neq 3, y \in \mathrm{R}\}$, *x*-intercept: $\frac{1}{3}$, *y*-intercept: 1, horizontal asymptote: $y = 3$, vertical asymptote: $x = 1$

Cumulative Review, Chapters 9–11 Page 550 Question 2

a)



b) domain: $\{x \mid x \neq -1, x \in \mathrm{R}\}$, range: $\{y \mid y \neq 3, y \in \mathrm{R}\}$, *x*-intercept: $\frac{4}{3}$, *y*-intercept: –4, horizontal asymptote: $y = 3$, vertical asymptote: $x = -1$

Cumulative Review, Chapters 9–11 Page 550 Question 3

a) The graph of $y = \frac{x^2 - 3x}{x^2 - 9}$ has a vertical asymptote at $x = -3$, a point of discontinuity at (3, 0.5), and an *x*-intercept of 0. Therefore, graph C represents the function.

b) The graph of $y = \dfrac{x^2 - 1}{x + 1}$ has no vertical asymptote, a point of discontinuity at (–1, –2), and an x-intercept of 1. Therefore, graph A represents the function.

c) The graph of $y = \dfrac{x^2 + 4x + 3}{x^2 + 1}$ has no vertical asymptote, no point of discontinuity, and x-intercepts of –3 and –1. Therefore, graph B represents the function.

Cumulative Review, Chapters 9–11 Page 550 Question 4

a)

$$\frac{1}{x^2 - 9} + \frac{1}{x + 3} = 0$$

$$\frac{1}{(x - 3)(x + 3)} + \frac{1}{x + 3} = 0$$

$$\frac{1}{(x - 3)(x + 3)} + \frac{x - 3}{(x - 3)(x + 3)} = 0$$

$$\frac{1 + x - 3}{(x - 3)(x + 3)} = 0$$

$$x - 2 = 0$$

$$x = 2$$

Check:

Left Side $= \dfrac{1}{x^2 - 9} + \dfrac{1}{x + 3}$ Right Side = 0

$= \dfrac{1}{(2)^2 - 9} + \dfrac{1}{2 + 3}$

$= \dfrac{1}{4 - 9} + \dfrac{1}{5}$

$= \dfrac{1}{-5} + \dfrac{1}{5}$

Left Side = Right Side

b) $\dfrac{8x}{x - 3} = x + 3$

$8x = (x + 3)(x - 3)$

$8x = x^2 - 9$

$0 = x^2 - 8x - 9$

$0 = (x - 9)(x + 1)$

$x - 9 = 0$ or $x + 1 = 0$

$x = 9$ $x = -1$

Check $x = 9$:

$$\text{Left Side} = \frac{8x}{x-3} \qquad \text{Right Side} = x+3$$
$$= \frac{8(9)}{9-3} \qquad = 9+3$$
$$= \frac{72}{6} \qquad = 12$$
$$= 12 \qquad = 12$$

Left Side = Right Side

Check $x = -1$:

$$\text{Left Side} = \frac{8x}{x-3} \qquad \text{Right Side} = x+3$$
$$= \frac{8(-1)}{-1-3} \qquad = -1+3$$
$$= \frac{-8}{-4} \qquad = 2$$
$$= 2$$

Left Side = Right Side

c)

$$\frac{x+4}{4} = \frac{x+5}{x^2+6x+5}$$
$$\frac{x+4}{4} = \frac{\overset{1}{\cancel{x+5}}}{(\underset{1}{\cancel{x+5}})(x+1)}$$
$$\frac{x+4}{4} = \frac{1}{x+1}$$
$$(x+4)(x+1) = 4$$
$$x^2+4x+x+4 = 4$$
$$x^2+5x = 4-4$$
$$x(x+5) = 0$$
$$x = 0 \quad \text{or} \quad x+5 = 0$$
$$x = -5$$

Check $x = 0$:

$$\text{Left Side} = \frac{x+4}{4} \qquad \text{Right Side} = \frac{0+5}{(0)^2+6(0)+5}$$
$$= \frac{0+4}{4} \qquad = \frac{5}{0+0+5}$$
$$= 1 \qquad = 1$$

Left Side = Right Side

Check $x = -5$:

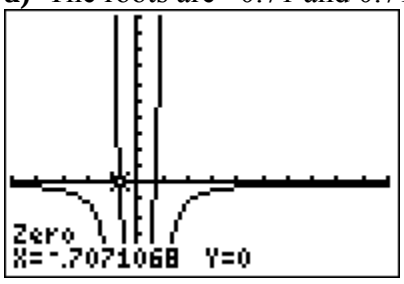
$$\text{Left Side} = \frac{x+4}{4} \qquad \text{Right Side} = \frac{-5+5}{(-5)^2+6(-5)+5}$$
$$= \frac{-5+4}{4} \qquad = \frac{5}{25-30+5}$$
$$= \frac{-1}{4} \qquad = \frac{5}{0}$$

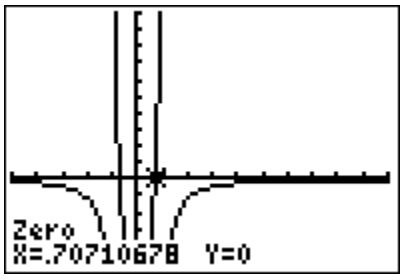
Left Side $\neq$ Right Side

The answer is $x = 0$.

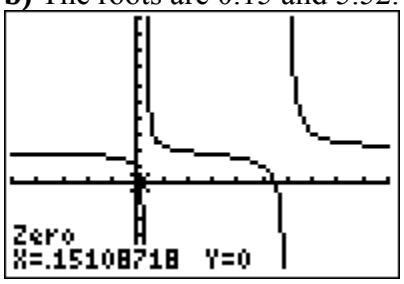
Cumulative Review, Chapters 9–11 Page 550 Question 5

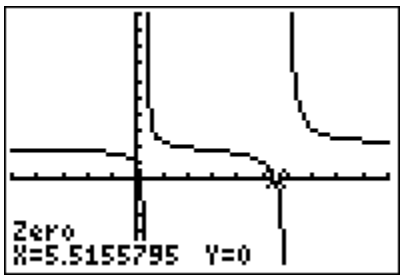
a) The roots are –0.71 and 0.71.





b) The roots are 0.15 and 5.52.





Cumulative Review, Chapters 9–11 Page 550 Question 6

a) $h(x) = (f + g)(x)$

$h(x) = f(x) + g(x)$

$h(x) = \sqrt{x+2} + x - 2$

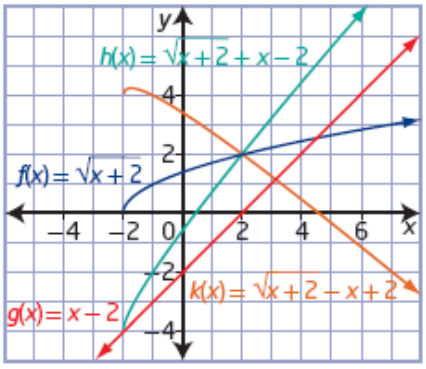
$k(x) = (f - g)(x)$

$k(x) = f(x) - g(x)$

$k(x) = \sqrt{x+2} - (x-2)$

$k(x) = \sqrt{x+2} - x + 2$

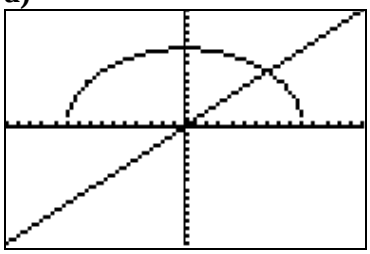
b)



c) $f(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \leq 4.25, y \in \mathrm{R}\}$

Cumulative Review, Chapters 9–11 **Page 550** **Question 7**

a)

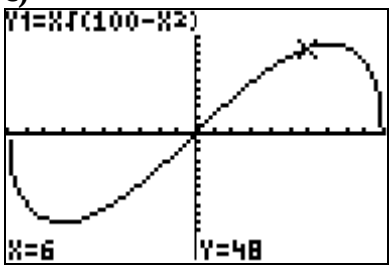


$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid -10 \le x \le 10, x \in \mathrm{R}\}$,
range $\{y \mid 0 \le y \le 10, y \in \mathrm{R}\}$

b) $h(x) = (f \cdot g)(x)$

$h(x) = x\sqrt{100 - x^2}$

c)



$h(x)$: domain $\{x \mid -10 \le x \le 10, x \in \mathrm{R}\}$, range $\{y \mid -50 \le y \le 50, y \in \mathrm{R}\}$

Cumulative Review, Chapters 9–11 **Page 550** **Question 8**

a)

$$h(x) = \frac{f(x)}{g(x)}$$

$$h(x) = \frac{x^2 + 3x + 2}{x^2 - 4}$$

$$h(x) = \frac{\cancel{(x+2)}^{1}(x+1)}{(x-2)\cancel{(x+2)}_{1}}$$

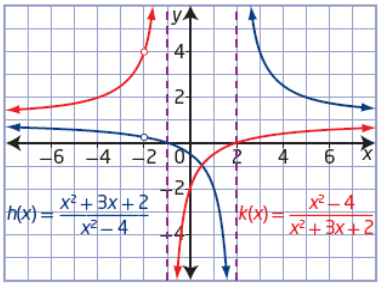
$$h(x) = \frac{x+1}{x-2},\ x \neq -2,\ 2$$

$$k(x) = \frac{g(x)}{f(x)}$$

$$k(x) = \frac{x^2 - 4}{x^2 + 3x + 2}$$

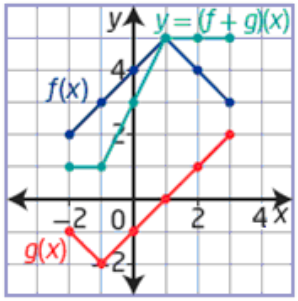
$$k(x) = \frac{(x-2)\cancel{(x+2)}^{1}}{\cancel{(x+2)}_{1}(x+1)}$$

$$k(x) = \frac{x-2}{x+1},\ x \neq -2,\ -1$$



b) The two functions have different domains but the same range.
$h(x)$: domain $\{x \mid x \neq -2, 2, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq -2, -1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$

Cumulative Review, Chapters 9–11 Page 551 Question 9



Cumulative Review, Chapters 9–11 Page 551 Question 10

a) Determine the value of $h(3)$.
$h(x) = x^2 - 9$
$h(3) = (3)^2 - 9$
$h(3) = 9 - 9$
$h(3) = 0$

Substitute $h(3) = 0$ into $f(x)$.
$f(h(3)) = f(0)$
$f(h(3)) = 0 - 3$
$f(h(3)) = -3$

b) Determine the value of $f(5)$.
$f(x) = x - 3$
$f(5) = 5 - 3$
$f(5) = 2$

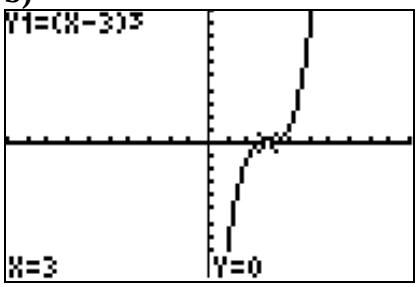
Substitute $f(5) = 2$ into $g(x)$.
$g(f(5)) = g(2)$
$g(f(5)) = \frac{1}{2}$

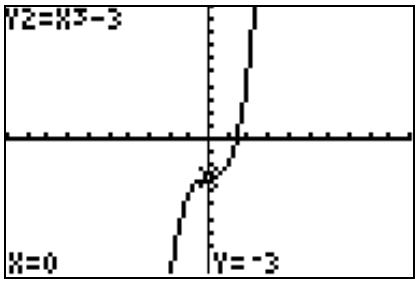
Cumulative Review, Chapters 9–11 Page 551 Question 11

a) $(f \circ g)(x) = f(x - 3)$
$(f \circ g)(x) = (x - 3)^3$

$(g \circ f)(x) = g(x^3)$
$(g \circ f)(x) = x^3 - 3$

b)





c) Both of the composite functions increase up into quadrant I and decrease down into quadrant III. The graph of $(f \circ g)(x) = (x - 3)^3$ is a translation of 3 units right of the graph of $f(x)$. The graph of $(g \circ f)(x) = x^3 - 3$ is a translation of 3 units down of the graph of $f(x)$.

Cumulative Review, Chapters 9–11 Page 551 Question 12

a) Substitute $g(x) = 10^x$ into $f(g(x))$.
$f(g(x)) = f(10^x)$
$f(g(x)) = \log 10^x$
$f(g(x)) = x$
The domain of the composite function is $\{x \mid x \in \mathrm{R}\}$.

b) Substitute $f(x) = \sin x$ into $g(f(x))$.
$g(f(x)) = g(\sin x)$
$g(f(x)) = \dfrac{1}{\sin x}$
$g(f(x)) = \csc x$
The domain of the composite function is $\{x \mid x \neq \pi n, n \in \mathrm{R}\}$.

c) Substitute $g(x) = x^2$ into $f(g(x))$.
$f(g(x)) = f(x^2)$
$f(g(x)) = \dfrac{1}{x^2 - 1}$
The domain of the composite function is $\{x \mid x \neq \pm 1, x \in \mathrm{R}\}$.

Cumulative Review, Chapters 9–11 Page 551 Question 13

total possibilities: 2(3)(2)(2)(4) = 96
There are 96 different meals possible.

Cumulative Review, Chapters 9–11 Page 551 Question 14

Total number of arrangements: 6! = 720
Arrangements with vowels together: Consider the vowels as one object. There are 5! ways to arrange the letters and 2! ways to arrange the vowels.
5!2! = 240
Since vowels cannot be together, there are 720 – 240 = 480 possible arrangements.

Cumulative Review, Chapters 9–11 Page 551 Question 15

$$\begin{aligned}
{}_7C_3 + {}_5P_2 &= \frac{7!}{(7-3)!3!} + \frac{5!}{(5-2)!} \\
&= \frac{7!}{4!3!} + \frac{5!}{3!} \\
&= \frac{7(6)(5)\,\cancel{(4!)}^{1}}{\cancel{4!}_{1}(3)(2)(1)} + \frac{5(4)\,\cancel{(3!)}^{1}}{\cancel{3!}_{1}} \\
&= 35 + 20 \\
&= 55
\end{aligned}$$

Cumulative Review, Chapters 9–11 Page 551 Question 16

There are 5 – 2 = 3 men on the committee.

$$
\begin{aligned}
{}_6C_2 \times {}_7C_3 &= \frac{6!}{(6-2)!2!} \times \frac{7!}{(7-3)!3!} \\
&= \frac{6!}{4!2!} \times \frac{7!}{4!3!} \\
&= \frac{6(5)\,(\cancel{4!})}{\cancel{4!}(2)(1)} \times \frac{7(6)(5)\,(\cancel{4!})}{\cancel{4!}(3)(2)(1)} \\
&= 525
\end{aligned}
$$

There are 525 ways to select the committee.

Cumulative Review, Chapters 9–11 Page 551 Question 17

a) There are 3! ways to arrange the three mathematics books, 5! ways to arrange the five history books, and 4! ways to arrange the four French books. The three sets of books can be arranged in 3! ways.

Total number of arrangements: 3!5!4!3! = 103 680

b) There are 3 ways of choosing the mathematics book for one end and 2 ways of choosing the mathematics book for the other end. There are 4! ways to arrange the four French books together. Consider the four French books as one object. There are 7 objects to arrange between the mathematics books, and this can be done in 7! ways.

Total number of arrangements: 3(2)(4!)(7!) = 725 760

Cumulative Review, Chapters 9–11 Page 551 Question 18

a)

$$
\begin{aligned}
\frac{(n+4)!}{(n+2)!} &= 42 \\
\frac{(n+4)(n+3)\,(\cancel{n+2)!}}{\cancel{(n+2)!}} &= 42 \\
(n+4)(n+3) &= 7(6)
\end{aligned}
$$

By inspection, $n + 4 = 7$ and $n + 3 = 6$. Therefore, $n = 3$.

b)

$$ {}_nP_3 = 20n $$

$$ \frac{n!}{(n-3)!} = 20n $$

$$ \frac{\overset{1}{\cancel{n}}(n-1)(n-2)\overset{1}{\cancel{(n-3)!}}}{\underset{1}{\cancel{(n-3)!}}} = 20\overset{1}{\cancel{n}} $$

$$ (n-1)(n-2) = 20 $$

$$ (n-1)(n-2) = 5(4) $$

By inspection, $n - 1 = 5$ and $n - 2 = 4$. Therefore, $n = 6$.

c)

$$ {}_{n+2}C_n = 21 $$

$$ \frac{(n+2)!}{(n+2-n)!n!} = 21 $$

$$ \frac{(n+2)!}{2!n!} = 21 $$

$$ \frac{(n+2)(n+1)\overset{1}{\cancel{(n)!}}}{2(1)\underset{1}{\cancel{(n)!}}} = 21 $$

$$ \frac{(n+2)(n+1)}{2} = 21 $$

$$ (n+2)(n+1) = 21(2) $$

$$ (n+2)(n+1) = 42 $$

$$ (n+2)(n+1) = 7(6) $$

By inspection, $n + 2 = 7$ and $n + 1 = 6$. Therefore, $n = 5$.

Cumulative Review, Chapters 9–11 Page 551 Question 19

Examples:
Pascal's triangle:
$(x + y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$; the coefficients are values from the fifth row of Pascal's triangle.
$(x + y)^6 = 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$; the coefficients are values from the seventh row of Pascal's triangle.

Combinations:
$(x + y)^4 = {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3x^1y^3 + {}_4C_3x^0y^4$; the coefficients ${}_4C_0$, ${}_4C_1$, ${}_4C_2$, ${}_4C_3$, ${}_4C_4$ have the same values as in the fifth row of Pascal's triangle.
$(x + y)^6 = {}_6C_0x^6y^0 + {}_6C_1x^5y^1 + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5x^1y^5 + {}_6C_6x^0y^6$; the coefficients ${}_6C_0$, ${}_6C_1$, ${}_6C_2$, ${}_6C_3$, ${}_6C_4$, ${}_6C_5$, ${}_6C_6$ have the same values as in the seventh row of Pascal's triangle.

Cumulative Review, Chapters 9–11 Page 551 Question 20

a) $(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$
$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Substitute $x = 3x$ and $y = -5$.
$(3x-5)^4 = (3x)^4 + 4(3x)^3(-5) + 6(3x)^2(-5)^2 + 4(3x)(-5)^3 + (-5)^4$
$= 3^4x^4 - 20(3^3x^3) + 6(3^2x^2)(25) + 12x(-125) + 625$
$= 81x^4 - 20(27x^3) + 150(9x^2) - 1500x + 625$
$= 81x^4 - 540x^3 + 1350x^2 - 1500x + 625$

b) $(x+y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4$
$+ {}_5C_5(x)^0(y)^5$
$= 1(x)^5(1) + 5(x)^4(y) + 10(x)^3(y)^2 + 10(x)^2(y)^3 + 5(x)(y)^4 + 1(1)(y)^5$
$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Substitute $x = \frac{1}{x}$ and $y = -2x$.

$$\left(\frac{1}{x}-2x\right)^5 = \left(\frac{1}{x}\right)^5 + 5\left(\frac{1}{x}\right)^4(-2x) + 10\left(\frac{1}{x}\right)^3(-2x)^2 + 10\left(\frac{1}{x}\right)^2(-2x)^3 + 5\left(\frac{1}{x}\right)(-2x)^4 + (-2x)^5$$

$$= \frac{1}{x^5} + 5\left(\frac{1}{x^4}\right)(-2x) + 10\left(\frac{1}{x^3}\right)(4x^2) + 10\left(\frac{1}{x^2}\right)(-2^3x^3) + 5\left(\frac{1}{x}\right)(2^4x^4) + (-2^5x^5)$$

$$= \frac{1}{x^5} - 10\cancel{x}\left(\frac{1}{\cancel{x^4}^{3}}\right) + 10\left(\frac{1}{x^{\cancel{3}^{1}}}\right)(4\cancel{x^2}^{0}) + 10\left(\frac{1}{\cancel{x^2}^{0}}\right)(-8x^{\cancel{3}^{1}}) + 5\left(\frac{1}{\cancel{x}}\right)(16\cancel{x^4}^{3}) - 32x^5$$

$$= \frac{1}{x^5} - \frac{10}{x^3} + \frac{40}{x} - 80x + 80x^3 - 32x^5$$

Cumulative Review, Chapters 9–11 Page 551 Question 21

a) Fourth term of $(5x+y)^5$:
${}_5C_3(5x)^2(y)^3 = 10(25x^2)y^3$
$= 250x^2)y^3$
The coefficient of the fourth term is 250.

b) Fourth term of $\left(\frac{1}{x^2} - x^3\right)^8$:

$${}_8C_3\left(\frac{1}{x^2}\right)^5(-x^3)^3 = 56\left(\frac{1}{x^{10}}\right)(-x^9)$$

$$= -56\left(\frac{1}{x^{10}}\right)(x^9)$$

The coefficient of the fourth term is –56.

Cumulative Review, Chapters 9–11 Page 551 Question 22

a) In combination notation, ${}_nC_r$, the second value in the row is n, and r represents the term 0, 1, 2, ….
The next term is ${}_{25}C_4$.

b) The number of terms in this row is 25 + 1 = 26.

c) 2300 is the sum of the two terms in the row above, to the right and left of ${}_{25}C_3$. The values in the row above are ${}_{24}C_0$, ${}_{24}C_1$, ${}_{24}C_2$, ${}_{24}C_3$, ….
${}_{25}C_3 = {}_{24}C_2 + {}_{24}C_3$

Unit 4 Test

Unit 4 Test Page 552 Question 1

Factor the denominator.
$x^2 - 3x - 10 = (x - 5)(x + 2)$
$x + 2$ is a factor of both the numerator and denominator, so it does not correspond to a vertical asymptote. $x - 5$ is a factor of the denominator only, so it does correspond to a vertical asymptote and not a point of discontinuity. In statement C, the range should refer to y, not x.
The correct answer is **D**.

Unit 4 Test Page 552 Question 2

$y = \dfrac{-4}{x+2} + 3$ has a horizontal asymptote at $y = 3$, a vertical asymptote at $x = -2$, and a y-intercept of 1.
The answer is **B**.

Unit 4 Test Page 552 Question 3

$P(x) = R(x) - C(x)$
$P(x) = 10x - 0.001x^2 - (2x + 5000)$
$P(x) = 10x - 0.001x^2 - 2x - 5000$
$P(x) = -0.001x^2 + 8x - 5000$
$0 = -0.001x^2 + 8x - 5000$
$0 = 0.001x^2 - 8x + 5000$
Use the quadratic formula, with $a = 0.001$, $b = -8$, and $c = 5000$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(0.001)(5000)}}{2(0.001)}$$

$$x = \frac{8 \pm \sqrt{64 - 20}}{0.002}$$

$$x = \frac{8 + \sqrt{64 - 20}}{0.002} \quad \text{or} \quad x = \frac{8 - \sqrt{64 - 20}}{0.002}$$

$x = 7316.624...$ $\qquad$ $x = 683.375...$

Check $x \approx 683.375$:

Left Side = 0

Right Side $= 0.001x^2 - 8x + 5000$
$= 0.001(683.375)^2 - 8(683.375) + 5000$
$= 467.001\ 39 - 5467 + 5000$
$= 0.00$

Left Side = Right Side

The answer is **A**.

Unit 4 Test Page 552 Question 4

$$h(x) = (f \cdot g)(x)$$
$$= \frac{1}{x}(x+1)^2$$

Since $x \neq 0$, the answer is **B**.

Unit 4 Test Page 552 Question 5

$g(x) = x^2 - 2$

$$f(g(x)) = f(x^2 - 2)$$
$$= \sqrt{(x^2 - 2) + 1}$$
$$= \sqrt{x^2 - 1}$$

$m = \sqrt{x^2 - 1}$

The answer is **B**.

Unit 4 Test Page 552 Question 6

A set of five desks can be assigned to two students in ${}_5P_2$ ways.

The answer is D.

Unit 4 Test Page 552 Question 7

$$
\begin{aligned}
{}_6C_2 + {}_4P_3 &= \frac{6!}{(6-2)!2!} + \frac{4!}{(4-3)!} \\
&= \frac{6!}{4!2!} + \frac{4!}{1!} \\
&= \frac{6(5)\overset{1}{\cancel{(4!)}}}{\underset{1}{\cancel{4!}}(2)(1)} + \frac{4(3)(2)(1)}{1} \\
&= 15 + 24 \\
&= 39
\end{aligned}
$$

The answer is **C**.

Unit 4 Test Page 552 Question 8

Factor the denominator.
$2x^2 - 5x - 3 = (2x + 1)(x - 3)$
$x - 3$ is a factor of both the numerator and denominator. So, it corresponds to a point of discontinuity.
Find the x-coordinate:
$x - 3 = 0$
$x = 3$
Find the y-coordinate:

$$y = \frac{x-3}{2x^2 - 5x - 3}$$

$$y = \frac{\overset{1}{\cancel{x-3}}}{(2x+1)\underset{1}{\cancel{(x-3)}}}$$

$$y = \frac{1}{2x+1}$$

$$y = \frac{1}{2(3)+1}$$

$$y = \frac{1}{7}$$

The point of discontinuity is $\left(3, \frac{1}{7}\right)$.

Unit 4 Test Page 552 Question 9

$$\frac{x^2}{x^2+1} = \frac{x}{4}$$

$$4x^2 = x(x^2+1)$$

$$4x^2 = x^3 + x$$

$$0 = x^3 + x - 4x^2$$

$$0 = x(x^2 + 1 - 4x)$$

$x = 0$ or $x^2 - 4x + 1 = 0$

Use the quadratic formula, with $a = 1$, $b = -4$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$x = \frac{4 \pm \sqrt{12}}{2}$$

$$x = \frac{4 + \sqrt{12}}{2} \quad \text{or} \quad x = \frac{4 - \sqrt{12}}{2}$$

$x = 3.7320...$ $x = 0.2679...$

The roots are $x = 0$, 3.73, and 0.27, to the nearest hundredth.

Unit 4 Test Page 552 Question 10

$$(2x + 5y^2)^4 = {}_4C_0(2x)^4(5y^2)^0 + {}_4C_1(2x)^3(5y^2)^1 + {}_4C_2(2x)^2(5y^2)^2 + {}_4C_3(2x)^1(5y^2)^3 + {}_4C_4(2x)^0(5y^2)^4$$

third term: ${}_4C_2(2x)^2(5y^2)^2 = 6(2^2x^2)(5^2y^4)$

$= 6(4x^2)(25y^4)$

$= 600x^2y^4$

Unit 4 Test Page 552 Question 11

$$(x-2)^5$$

$$= {}_5C_0(x)^5(-2)^0 + {}_5C_1(x)^4(-2)^1 + {}_5C_2(x)^3(-2)^2 + {}_5C_3(x)^2(-2)^3 + {}_5C_4(x)^1(-2)^4 + {}_5C_5(x)^0(-2)^5$$

$$= 1(x)^5(1) + 5(x)^4(-2) + 10(x)^3(4) + 10(x)^2(-8) + 5(x)(16) + 1(1)(-32)$$

$$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

The sum of the coefficients is $1 - 10 + 40 - 80 + 80 - 32 = -1$.

Unit 4 Test Page 552 Question 12

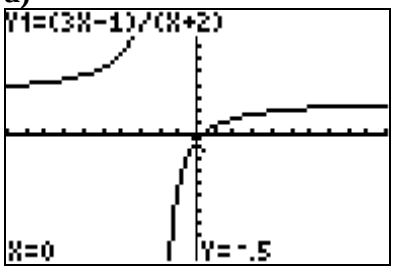
a) The transformations involve a vertical stretch by a factor of 2 and a translation of 1 unit left and 3 units down.

b) The equations of the asymptotes are $x = -1$ and $y = -3$.

c) For the graph of $g(x)$, as x approaches -1, $|y|$ becomes very large.

Unit 4 Test Page 553 Question 13

a)



b)
domain: $\{x \mid x \neq -2, x \in \mathrm{R}\}$,
range: $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept: $\frac{1}{3}$,
y-intercept: $-\frac{1}{2}$

c) $0 = \frac{3x-1}{x+2}$

$0 = 3x - 1$

$1 = 3x$

$\frac{1}{3} = x$

The root of the equation is $x = \frac{1}{3}$.

d) The x-intercept of the graph of the function $y = \frac{3x-1}{x+2}$ is the root of the equation $0 = \frac{3x-1}{x+2}$.

Unit 4 Test Page 553 Question 14

a) Factor the denominator.

$$f(x) = \frac{x-4}{x^2-2x-8}$$

$$f(x) = \frac{x-4}{(x-4)(x+2)}$$

Since $x - 4$ is a factor of both the numerator and denominator, it corresponds to a point of discontinuity.

$x - 4 = 0$

$x = 4$

To determine the y-coordinate, substitute $x = 4$ into the simplified function.

$$f(x) = \frac{\cancel{x-4}^{1}}{\cancel{(x-4)}^{1}(x+2)}$$

$$f(x) = \frac{1}{(x+2)}$$

$$f(x) = \frac{1}{(4+2)}$$

$$f(x) = \frac{1}{6}$$

The point of discontinuity occurs at $\left(4, \frac{1}{6}\right)$.

Since $x + 2$ is a factor of only the denominator, it corresponds to a vertical asymptote.

$x + 2 = 0$

$x = -2$

The vertical asymptote occurs at $x = -2$.

To determine y-intercepts, substitute $x = 0$.

$$f(x) = \frac{1}{x+2}$$

$$y = \frac{1}{0+2}$$

$$y = \frac{1}{2}$$

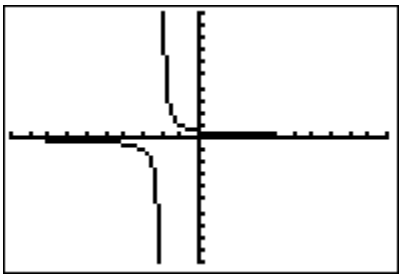
The y-intercept is $\left(0, \frac{1}{2}\right)$.

To determine x-intercepts, substitute $y = 0$.

$$0 = \frac{1}{x+2}$$

$0 \neq 1$

There are no x-intercepts.



b) Factor the numerator and denominator.

$$f(x) = \frac{x^2 + x - 6}{x^2 + 2x - 3}$$

$$f(x) = \frac{(x+3)(x-2)}{(x+3)(x-1)}$$

Since $x + 3$ is a factor of both the numerator and denominator, it corresponds to a point of discontinuity.

$x + 3 = 0$

$x = -3$

To determine the y-coordinate, substitute $x = -3$ into the simplified function.

$$f(x) = \frac{\cancel{(x+3)}^{1}(x-2)}{\cancel{(x+3)}_{1}(x-1)}$$

$$f(x) = \frac{x-2}{x-1}$$

$$f(x) = \frac{-3-2}{-3-1}$$

$$f(x) = \frac{-5}{-4}$$

$$f(x) = \frac{5}{4}$$

The point of discontinuity occurs at $\left(-3, \frac{5}{4}\right)$.

Since $x - 1$ is a factor of only the denominator, it corresponds to a vertical asymptote.

$x - 1 = 0$

$x = 1$

The vertical asymptote occurs at $x = 1$.

To determine y-intercepts, substitute $x = 0$.

$$f(x) = \frac{x-2}{x-1}$$

$$y = \frac{0-2}{0-1}$$

$$y = \frac{-2}{-1}$$

$$y = 2$$

The y-intercept is (0, 2).

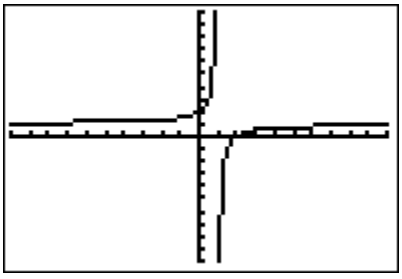
To determine x-intercepts, substitute $y = 0$.

$$0 = \frac{x-2}{x-1}$$

$$0 = x - 2$$

$$2 = x$$

The x-intercept is (2, 0).



c) Factor the numerator and denominator.

$$f(x) = \frac{x^2 - 5x}{x^2 - 2x - 3}$$

$$f(x) = \frac{x(x-5)}{(x-3)(x+1)}$$

Since the numerator and denominator do not have a common factor, there is no point of discontinuity.

Since $x - 3$ and $x + 1$ are a factors of the denominator, they correspond to vertical asymptotes.

$x - 3 = 0$ $\qquad$ $x + 1 = 0$

$x = 3$ $\qquad$ $x = -1$

The vertical asymptotes occur at $x = 3$ and $x = -1$.

To determine y-intercepts, substitute $x = 0$.

$$f(x) = \frac{x^2 - 5x}{x^2 - 2x - 3}$$

$$y = \frac{0^2 - 5(0)}{0^2 - 2(0) - 3}$$

$$y = 0$$

The y-intercept is (0, 0).

To determine x-intercepts, substitute $y = 0$.

$$0 = \frac{x^2 - 5x}{x^2 - 2x - 3}$$

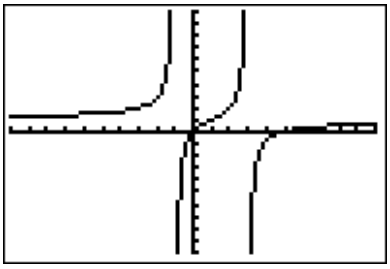
$$0 = x^2 - 5x$$

$$0 = x(x-5)$$

$x = 0$ or $x - 5 = 0$

$x = 5$

The x-intercepts are at (0, 0) and (5, 0).



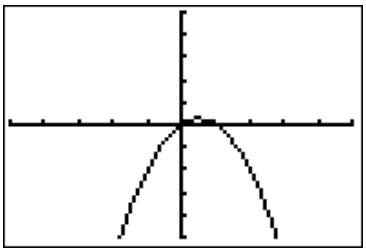
Unit 4 Test Page 553 Question 15

$f(x) = 1 - x^2$, $g(x) = x - 1$

a) $y = (f + g)(x)$

$y = 1 - x^2 + x - 1$

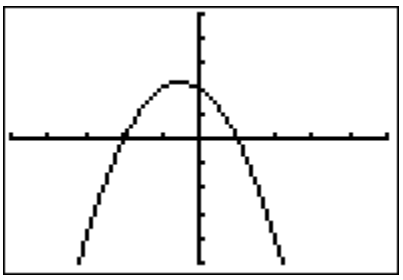
$y = x - x^2$



domain: $\{x \mid x \in \mathrm{R}\}$,

range: $\{y \mid y \leq 0.25, y \in \mathrm{R}\}$

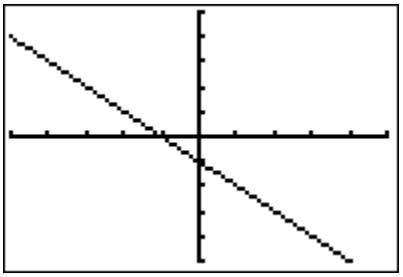
b) $y = (f - g)(x)$
$y = 1 - x^2 - (x - 1)$
$y = 1 - x^2 - x + 1$
$y = 2 - x^2 - x$



domain: $\{x \mid x \in \mathrm{R}\}$,
range: $\{y \mid y \leq 2.25, y \in \mathrm{R}\}$

c) $y = \left(\frac{f}{g}\right)(x)$

$y = \frac{1 - x^2}{x - 1}$



Restrictions on domain and range:
$x - 1 \neq 0$
$x \neq 1$

$y = \frac{1 - x^2}{x - 1}$

$y = \frac{(1 - x)(1 + x)}{x - 1} \times \frac{-1}{-1}$

$y = \frac{\cancel{(1 - x)}^{1}(-1 - x)}{\cancel{-x + 1}_{1}}$

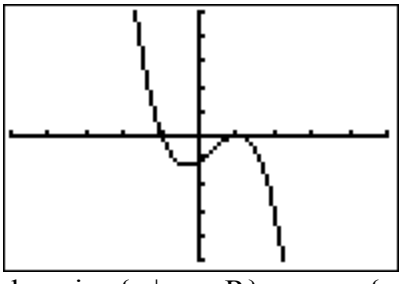
$y = -1 - x$

$y = -1 - 1$

$y = -2$

domain: $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range: $\{y \mid y \neq -2, y \in \mathrm{R}\}$

d) $y = (f \bullet g)(x)$
$y = (1 - x^2)(x - 1)$
$y = x - x^3 - 1 + x^2$ or
$y = -x^3 + x^2 + x - 1$



domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$

Unit 4 Test Page 553 Question 16

$f(x) = x - 3,\ g(x) = \sqrt{x-1}$

a) $h(x) = f(x) + g(x)$

$h(x) = x - 3 + \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

b) $h(x) = f(x) - g(x)$

$h(x) = x - 3 - \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

c) $h(x) = \left(\frac{f}{g}\right)(x)$

$h(x) = \frac{x-3}{\sqrt{x-1}}$

$\sqrt{x-1} > 0$

$x - 1 > 0$

$x > 1$

domain: $\{x \mid x > 1, x \in \mathrm{R}\}$

d) $h(x) = (f \cdot g)(x)$

$h(x) = (x-3)\sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

Unit 4 Test Page 553 Question 17

a) $g(x) = |x|$

$g(2) = |2|$

$g(2) = 2$

$f(g(2)) = f(2)$

$f(g(2)) = 2^2 - 3$

$f(g(2)) = 4 - 3$

$f(g(2)) = 1$

b) $(f \circ g)(2) = f(g(2))$

From part a), $(f \circ g)(2) = 1$.

c) $f(g(x)) = f(|x|)$

$f(g(x)) = |x|^2 - 3$

$f(g(x)) = x^2 - 3$

d) $(g \circ f)(x) = g(f(x))$

$(g \circ f)(x) = g(x^2 - 3)$

$(g \circ f)(x) = |x^2 - 3|$

Unit 4 Test Page 553 Question 18

a) $f(g(x)) = 2^{3x+2}$

Let $g(x)$ represent the inner function.

$g(x) = 3x + 2$

Therefore, $f(x) = 2^x$.

b) $f(g(x)) = \sqrt{\sin x + 2}$

Let $g(x)$ represent the inner function.

$g(x) = \sin x + 2$

Therefore, $f(x) = \sqrt{x}$.

Unit 4 Test Page 553 Question 19

a)

$$\frac{n!}{(n-2)!} = 420$$

$$\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}} = 420$$

$$n(n-1) = 420$$

$$n(n-1) = 21(20)$$

By inspection, $n = 21$.

b)

$${}_nC_2 = 78$$

$$\frac{n!}{(n-2)!2!} = 78$$

$$\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}(2)(1)} = 78$$

$$\frac{n(n-1)}{2} = 78$$

$$n(n-1) = 156$$

$$n(n-1) = 13(12)$$

By inspection, $n = 13$.

c)

$${}_nC_{n-2} = 45$$

$$\frac{n!}{(n-(n-2))!(n-2)!} = 45$$

$$\frac{n!}{(n-n+2)!(n-2)!} = 45$$

$$\frac{n!}{2!(n-2)!} = 45$$

$$\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{2(1)\,\cancel{(n-2)!}_{1}} = 45$$

$$\frac{n(n-1)}{2} = 45$$

$$n(n-1) = 90$$

$$n(n-1) = 10(9)$$

By inspection, $n = 10$.

Unit 4 Test Page 553 Question 20

a) There are 4! = 24 arrangements possible without repeating letters.

b) With repeating letters, there are $4^4 = 256$ possible arrangements. Therefore, there are 256 – 24 = 232 more arrangements.

c) No, there are fewer ways. Because the letter C is repeated, half of the arrangements will be repeats.

Unit 4 Test Page 553 Question 21

a) The sub-committee will have 5 – 3 = 2 boys.

$$
\begin{aligned}
{}_4C_2 \times {}_5C_3 &= \frac{4!}{(4-2)!2!} \times \frac{5!}{(5-3)!3!} \\
&= \frac{4!}{2!2!} \times \frac{5!}{2!3!} \\
&= \frac{4(3)\,\cancel{(2!)}^{1}}{2(1)\,\cancel{(2)!}^{1}} \times \frac{5(4)\,\cancel{(3!)}^{1}}{2(1)\,\cancel{(3)!}^{1}} \\
&= 60
\end{aligned}
$$

There are 60 ways the sub-committee can be formed.

b) Case 1: Sub-committee has three girls
From part a), 60 arrangements

Case 2: Sub-committee has four girls
The sub-committee will have 5 – 4 = 1 boy.

$$
\begin{aligned}
{}_4C_1 \times {}_5C_4 &= \frac{4!}{(4-1)!1!} \times \frac{5!}{(5-4)!4!} \\
&= \frac{4!}{3!(1)} \times \frac{5!}{1!4!} \\
&= \frac{4\,\cancel{(3!)}^{1}}{\cancel{(3)!}_{1}} \times \frac{5\,\cancel{(4!)}^{1}}{1\,\cancel{(4)!}_{1}} \\
&= 20
\end{aligned}
$$

Case 3: Sub-committee has five girls
The sub-committee has zero boys.

$$
\begin{aligned}
{}_4C_0 \times {}_5C_5 &= \frac{4!}{(4-0)!0!} \times \frac{5!}{(5-5)!5!} \\
&= \frac{4!}{4!(1)} \times \frac{5!}{0!5!} \\
&= 1
\end{aligned}
$$

Total possibilities: 60 + 20 + 1 = 81

Unit 4 Test Page 553 Question 22

$(x + y)^7 = {}_7C_0(x)^7(y)^0 + {}_7C_1(x)^6(y)^1 + {}_7C_2(x)^5(y)^2 + {}_7C_3(x)^4(y)^3 + {}_7C_4(x)^3(y)^4 + {}_7C_5(x)^2(y)^5 + {}_7C_6(x)^1(y)^6 + {}_7C_7(x)^0(y)^7$

The term that contains x^5 is ${}_7C_2(x)^5(y)^2$.

${}_7C_2(x)^5(y)^2 = 81\ 648x^5$

$21(3x)^5(a)^2 = 81\ 648x^5$

$21(3^5x^5)a^2 = 81\ 648x^5$

$21(243x^5)a^2 = 81\ 648x^5$

$$a^2 = \frac{81\ 648\ \cancel{x^5}^{1}}{21(243\ \cancel{x^5}_{1})}$$

$$a^2 = 16$$

$$a = \pm 4$$